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Original Article

Disjoint Restrained Domination in the Join and Corona of Graphs

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Abstract *- Let $G=(V(G),E(G))$ be a connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $u\in V(G\backslash S)$, there exists $v\in S$ such that $uv\in E(G)$. A dominating set D is called a restrained dominating set if for each $u\in V(G)\backslash D$ there exist $v\in V(D)$ and $z\in V(G)\backslash D(z\neq u)$ such that u is adjacent to v and z. Further, if D is a minimum restrained dominating set of G, then a restrained dominating set $S\subseteq V(G)\backslash D$ is called an inverse restrained dominating set of G with respect to D. A disjoint restrained dominating set of G is the set $C=D\cup S\subseteq V(G)$. In this paper, we investigate the concept and give some important results on disjoint restrained domination arising from the join and corona of two graphs.*

Keywords - *binary operations on graphs, disjoint restrained dominating set, dominating set, inverse restrained dominating set, restrained dominating set.*

I. INTRODUCTION

Suppose that $G=(V(G),E(G))$ is a simple graph with vertex set $V(G)$ and edge set $E(G)$. By simple graph, we mean a finite and undirected graph with neither loops nor multiple edges. For the general graph theoretic terminology, the readers may refer to [1].

A vertex v is said to dominate a vertex u if uv is an edge of G or $v=u$. A set of vertices $S\subseteq V(G)$ is called a dominating set of G if every vertex not in S is dominated by at least one member of S. The size of a set of least cardinality among all dominating sets for G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is called γ −set of G. Domination in a graph has been a huge area of research in graph theory. It was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. Domination in graphs has been studied in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

A dominating set S is called a restrained dominating set of G if for each $u\in V(G)\backslash S$ there exists $v\in S$ such that $uv\in E(G)$ and there exists $z\in V(G)\backslash(S\cup\{u\})$ such that $uz\in E(G)$. The restrained domination number of G, is the minimum cardinality of a restrained dominating set of G and is denoted by $\gamma_r(G)$. A restrained dominating set of cardinality $\gamma_r(G)$ is called γ_r −set of G. Restrained domination has been studied in [12, 14, 15, 16, 17, 18, 19, 20, 31].

A restrained dominating set S is called an inverse restrained dominating set of G if each $S\subseteq V(G)\backslash D$ with D is a γ_r −set of G. The inverse restrained domination number of G, is the minimum cardinality of an inverse restrained dominating set of G and is denoted by $\gamma_r^{-1}(G)$. An inverse restrained dominating set of cardinality $\gamma_r^{-1}(G)$ is called γ_r^{-1} −set of G. The inverse domination has been studied in [21, 22, 23, 34, 25, 26, 27, 28, 31, 32, 33].

Motivated by [21] and the idea of disjoint domination in graphs [29, 30], we initiate the study of disjoint restrained dominating set. Let D be a minimum restrained dominating set and S be an inverse restrained dominating set of G with respect to D. A disjoint restrained dominating set of G is the set $C=D\cup S\subseteq V(G)$. The disjoint restrained domination number of G, is the minimum cardinality of a disjoint restrained dominating set of G and is denoted by $\gamma\gamma_r(G)$. A disjoint fair dominating set of cardinality $\gamma\gamma_r(G)$ is called $\gamma\gamma_r$ − set of G.

In this paper, we investigate the concept and give some important results. We further give the characterization of a disjoint restrained dominating set in the join and corona of two graphs.

II. RESULTS

Remark 2.1 [29] Let G be a connected graph of order $n\geq 3$. If D is a \$ γ_r −set and S is an inverse restrained dominating set (or γ_r^{-1} −set) of G, then $D\cap S=\emptyset$ (and $C=D\cup S$ is a $\gamma\gamma_r$ −set of G). Further, the set C need not be a restrained dominating set.

Remark 2.2 [29] Let G be a connected graph of order $n\geq 3$. Then

(i) $2\leq\gamma\gamma_r(G)\leq n$, and

(ii) $\gamma(G)\leq\gamma_r(G)\leq\gamma\gamma_r(G)$.

Remark 2.3 If G is a complete graph of order γ_r, then $\gamma\gamma_r(G) = 2$.

The *join* of two graphs G and H is the graph $G + H$ with vertex-set $V(G+H) = V(G) \cup V(H)$ and edge-set

$$E(G+H) = E(G) \cup E(H) \cup \{uv: u \in V(G), v \in V(H)\}.$$

Let G and H be non-complete graphs. Since $\gamma_r(G+H) = 1$ if $\gamma(G) = 1$ or $\gamma(H) = 1$, and $\gamma_r(G+H) = 2$ if otherwise, the following remarks holds.

Remark 2.4 If G and H are non-complete graphs, then $\gamma_r(G+H) \geq 2$.

In view of Remark 2.4, the following remark is immediate.

Remark 2.5 If G and H are non-complete graphs, then $\gamma\gamma_r(G+H) \geq 2$.

We need the following results for the characterization of the disjoint restrained dominating set of the join of two graphs.

Lemma 2.6 *Let G and H be connected non-complete graphs. If $D \cap S = \emptyset$, $|D| \leq 2$, and D and S are dominating sets of G or H, then a subset $C = D \cup S$ is a disjoint restrained dominating set of $G + H$.*

Proof. Suppose that D and S are dominating sets of G. Then D and S are dominating sets of $G + H$. Let $x \in D$. Since H is a connected non-complete graph, there exist $u, v \in V(H)$ such that $uv \in E(H)$. Thus, $xu, uv \in E(G+H)$. Since G is a connected non-complete graph, $V(G)\backslash D \neq \emptyset$. Let $y \in V(G)\backslash D$. Since D is a dominating set of G, there exists $v \in D$ such that $vy \in E(G)$. Clearly, $yu \in E(G+H)$. Thus, $vy, yu \in E(G+H)$. Hence, for each $a \in V(G+D)\backslash D$ there exists $b \in D$ such that $ba \in E(G+H)$ and there exists $c \in V(G+H)(D \cup \{a\})$ such that $ac \in E(G+H)$, that is, D is a restrained dominating set of $G+H$. Similarly, S is a restrained dominating set of $G+H$. If $D \cap S = \emptyset$ then $S \subseteq V(G+H)\backslash D$. Without loss of generality, assume $|D| \leq |S|$. Since $|D| \leq 2$, D is a γ_r-set of $G+H$ and S is an inverse restrained dominating set of $G+H$. Accordingly, $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. Similarly, if D and S are dominating sets of H, then $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. ■

Lemma 1.7 *Let G and H be connected non-complete graphs. If $D \cap S = \emptyset$, $|D| \leq 2$, and D is a dominating set of G and S is a dominating set of H (or $S = S_G \cup S_H$, where $S_G \subset V(G)$ and $S_H \subset V(H)$), then a subset $C = D \cup S$ is a disjoint restrained dominating set of $G + H$.*

Proof. Suppose that D is a dominating set of G and S is a dominating set of H. Then D and S are dominating sets of $G+H$. By similar proof in Lemma 2.6, D is a γ_r −set of $G+H$ and S is an inverse restrained dominating set of $G+H$. Thus, $C = D \cup S$ is a disjoint restrained dominating set of $G+S$. Further, if D is a dominating set of G and $S = S_G \cup S_H$, where $S_G \subset V(G)$ and $S_H \subset V(H)$. Let $x \in S_G$ and $y \in S_H$. Then x dominates $V(H)$ and y dominates $V(G)$. Thus, S is a dominating set of $G+H$. By similar proof in Lemma 2.6, $C = D \cup S$ is a disjoint restrained dominating set of $G+S$. ■

Lemma 2.8 *Let G and H be connected non-complete graphs. If $D \cap S = \emptyset$, $|D| \leq 2$, and D is a dominating set of H and S is a dominating set of G (or $S = S_G \cup S_H$, where $S_G \subset V(G)$ and $S_H \subset V(H)$), then a subset $C = D \cup S$ is a disjoint restrained dominating set of $G + H$.*

Proof. Suppose that D is a dominating set of H and S is a dominating set of G. By similar proof in Lemma 2.7, $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. ■.

Lemma 2.9 *Let G and H be connected non-complete graphs. If $D \cap S = \emptyset$, $|D| \leq 2$, S is a dominating set of G (or H) and $D = \{x, y\}$ where $x \in V(G)$, $y \in V(H)$, then a subset $C = D \cup S$ is a disjoint restrained dominating set of $G + H$.*

Proof. Suppose that S is a dominating set of G and $D = \{x, y\}$ where $x \in V(G)$ and $y \in V(H)$. Then S is a dominating set of $G+H$. Since $x \in V(G)$ and $y \in V(H)$, D is a dominating set of $G+H$. By similar proof in Lemma 2.7, $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. Similarly, if S is a dominating set of H and $D = \{x, y\}$ where $x \in V(G), y \in V(H)$, then $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. ■

Lemma 2.10 *Let G and H be connected non-complete graphs. If $D \cap S = \emptyset$, $|D| \leq 2$, and $S = S_G \cup S_H$, where $S_G \subset V(G)$ and $S_H \subset V(H)$, and $D = \{x, y\}$ where $x \in V(G)$ and $y \in V(H)$, then a subset $C = D \cup S$ is a disjoint restrained dominating set of $G + H$.*

Proof. Suppose that $S = S_G \cup S_H$ where $S_G \subset V(G)$ and $S_H \subset V(H)$, and $D = \{x, y\}$, where $x \in V(G)$ and $y \in V(H)$. Since $S_G \subset V(G)$ and $S_H \subset V(H)$, clearly, S is a dominating set of $G+S$. Since $x \in V(G)$ and $y \in V(H)$, D is a dominating set of $G+H$. By similar proof in Lemma 2.7, $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. ■

The following theorem provides the characterization of the disjoint restrained dominating set in the join of two graphs. Since the join of two complete graphs is a complete graph, the graphs under study are connected non-complete graphs.

Theorem 2.11 *Let G and H be connected non-complete graphs. Then a subset $C = D \cup S$ of $V(G+H)$ is a disjoint restrained dominating set in $G+H$ if and only if $D \cap S = \emptyset, |D| \leq 2$ and one of the following statements holds:*

(i) *D and S are dominating sets of G or H.*

(ii) *D is a dominating set of G and S is a dominating set of H (or $S = S_G \cup S_H$ where $S_G \subset V(G)$ and $S_H \subset V(H)$).*

(iii) *D is a dominating set of H and S is a dominating set of G (or $S = S_G \cup S_H$ where $S_G \subset V(G)$ and $S_H \subset V(H)$).*

(iv) *S is a dominating set of G (or H) and $D = \{x, y\}$ where $x \in V(G)$ and $y \in V(H)$.*

(v) *$S = S_G \cup S_H$ where $S_G \subset V(G)$ and $S_H \subset V(H)$, and $D = \{x, y\}$ where $x \in V(G)$ and $y \in V(H)$.*

Proof. Suppose that a subset $C = D \cup S$ of $V(G+H)$ is a disjoint restrained dominating set in $G+H$. By definition, $S = V(G+H) \backslash D$ implies that $D \cap S = \emptyset$, and by Remark 2.4, $|D| = \gamma_r(G+H) \leq 2$. If $C \cap V(H) = \emptyset$ then $C \subseteq V(H)$. This implies that D and S are dominating sets of G. If $C \cap V(G) = \emptyset$ then $C \subseteq V(H)$. This implies that D and S are dominating sets of H. This proves statement *(i)*. Now, suppose that $C \cap V(H) \neq \emptyset$ and $C \cap V(G) \neq \emptyset$. Then $(D \cup S) \cap V(H) = \emptyset$ and $(D \cup S) \cap V(G) \neq \emptyset$. Consider the following cases.

Case 1. Suppose that $D \cap V(H) = \emptyset$ and $S \cap V(G) = \emptyset$. Then $D \subseteq V(G)$ and $S \subseteq V(H)$. Thus, D is a dominating set of G and S is a dominating set of H. If $S \cap V(G) \neq \emptyset$ and $S \cap V(H) \neq \emptyset$, then let $S_G = S \cap V(G)$ and $S_H = S \cap V(H)$. This implies that $S = S_G \cup S_H$, where $S_G \subset V(G)$ and $S_H \subset V(H)$. This proves statement *(ii)*.

Case 2. Suppose that $D \cap V(G) = \emptyset$ and $S \cap V(H) = \emptyset$. Then $D \subseteq V(H)$ and $S \subseteq V(G)$. Thus, D is a dominating set of H and S is a dominating set of G. Similarly, if $S \cap V(G) \neq \emptyset$ and $S \cap V(H) \neq \emptyset$, then $S = S_G \cup S_H$, where $S_G \subset V(G)$ and $S_H \subset V(H)$. This proves statement *(iii)*.

Case 3. Suppose that $S \cap V(H) = \emptyset$,. Then $S \subset V(G)$, that is, S is a dominating set of G. Further, $D \cap V(H) \neq \emptyset$. If $D \cap V(G) \neq \emptyset$, then $D \cap V(G) \subset V(G)$ and $D \cap V(H) \subset V(H)$. Now, $|D| \leq 2$. Since G and H are connected non-complete graphs, $|D|$ cannot be 1. Thus, |D|=2. Set $D = \{x, y\}$ where $D \cap V(G) = \{x\}$ and $D \cap V(H) = \{y\}$. Thus, $x \in V(G)$ and $y \in V(H)$. Similarly, if $S \cap V(G) = \emptyset$, then S is a dominating set of H and $D = \{x, y\}$ where $x \in V(G)$, $y \in V(H)$. This proves statement *(iv)*.

Case 4. Suppose that $S \cap V(G) \neq \emptyset$, $S \cap V(H) \neq \emptyset$, $D \cap V(G) \neq \emptyset$, and D$\cap V(H) \neq \emptyset$. Let $S_G = S \cap V(G)$ and $S_H = S \cap V(H)$. Then $S = S_G \cup S_H$ where $S_G \subset V(G)$ and $S_H \subset V(H)$. Let $D = \{x, y\}$ where $x \in V(G)$, $y \in V(H)$. This proves statement *(v)*.

For the converse, let G and H be connected non-complete graphs. Suppose that statement *(i)* is satisfied. Then by Lemma 2.6, $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. Similarly, if statement *(ii), (iii), (iv)* or *(v)* are satisfied, then by Lemma 2.7, Lemma 2.8, Lemma 2.9, or Lemma 2.10, $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. ■

Corollary 2.12 *Let G and H be connected non-complete graphs. Then*

$$\gamma\gamma_r(G+H) = \begin{cases} 2, & if\ \gamma(G) = 1\ and\ \gamma(H) = 1 \\ 3, & if\ D = \{x\}\ is\ the\ only\ \gamma-set\ of\ G\ (or\ H\ exclusive) \\ 4, & if\ \gamma(G) \neq 1\ and\ \gamma(H) \neq 1 \end{cases}$$

Proof. Suppose that $C = D \cup S$ is a disjoint restrained dominating set of $G+H$. Then $\gamma\gamma_r(G+H) \leq |C|$. By Theorem 2.11, D is a dominating set of G and S is a dominating set of H (or Then $S = S_G \cup S_H$ where $S_G \subset V(G)$ and $S_H \subset V(H)$).

Case 1. If $\gamma(G) = 1$ and $\gamma(H) = 1$, then let $D = \{x\}$ and $S = \{y\}$. This implies that $\gamma\gamma_r(G+H) \leq |C| = |D \cup S| = |D| + |S| = 2$. By Remark 2.5, $\gamma\gamma_r(G+H) \geq 2$. Thus, $2 \leq \gamma\gamma_r(G+H) \leq 2$, that is, $\gamma\gamma_r(G+H) = 2$.

Case 2. If $D = \{x\}$, then D is a γ_r −set of $G+H$ is clear. Since G is a connected non-complete graph, $V(G) \backslash D \neq \emptyset$. Let $v \in V(G) \backslash D$ and $u \in V(H)$. Set $S = \{u, v\}$ such that $S_G = \{v\} \subset V(G)$ and $S_H = \{u\} \subset V(H)$. Thus, $\gamma\gamma_r(G+H) \leq |C| = |D \cup S| = |D| + |S| = 1|2 = 3$. Since $D = \{x\}$ is the only γ_r −set of $G+H$, it follows that the S is a γ_r^{-1} −set of $G+H$ with respect to D (by Remark 1.4). By Remark 2.1, $\gamma\gamma_r(G+H) = \gamma_r(G+H) + \gamma_r^{-1}(G+H)$. Thus $3 = 1 + 2 = \gamma_r(G+H) + \gamma_r^{-1}(G+H) = \gamma\gamma_r(G+H) = \gamma_r \leq |C| = 3$, that is, $\gamma\gamma_r(G+H) = 3$.

Case 3. If $\gamma(G) \neq 1$ and $\gamma(H) \neq 1$, then $\gamma_r(G+H) = 2$ (Remark 2.4). Let $S_G = \{u\} \subset V(G)$ and $S_H = \{v\} \subset V(H)$, and $D = \{x, y\}$, where $x \in V(G)$ and $y \in V(H)$ (by Theorem 2.11). Then D and $S = S_G \cup S_H$ are minimum restrained dominating sets of $G+H$. Set D be a γ_r −set of $G+H$ and S be a γ_r^{-1} −set of $G+H$. Then $\gamma\gamma_r(G+H) = \gamma_r(G+H) + \gamma_r^{-1}(G+H) = 2 + 2 = 4$. Thus, $\gamma\gamma_r(G+H) = 4$. ■

Let G and H be graphs of order m and n, respectively. The *corona* of two graphs G and H is the graph $G \circ H$ obtained by taking one copy of G and m copies of H, and then joining the ith vertex of G to every vertex of the ith copy of

H. The join of vertex v of G and a copy H^v of H in the corona of G and H is denoted by $v + H^v$.

Remark 1.13 Let G be a connected graph. Then $V(G)$ is a minimum dominating set of $G \circ H$.

Theorem 1.14 *Let G be a nontrivial connected graph and H has no isolated vertices. Then a subset $C = D \cup S$ of $V(G \circ H)$ is a disjoint restrained dominating set of $G \circ H$ if and only if $D = V(G)$ and $S = \bigcup_{v \in V(G)} S_v$ where S_v is a dominating set of H^v for each $v \in V(G)$.*

Proof. Suppose that a subset $C = D \cup S$ of $V(G \circ H)$ is a disjoint restrained dominating set of $G \circ H$. Let D be a minimum restrained dominating set of $G \circ H$. Then $\gamma(G \circ H) \leq |D|$, that is, $|V(G)| \leq |D|$ by Remark 2.13. Since $\langle V(G \circ H \backslash V(G)\rangle = H$ has no isolated vertices, it follows that $V(G)$ is a restrained dominating set of $G \circ H$. This means that $|D| \leq |V(G)|$, that is, $D = V(G)$. Further, S is an inverse restrained dominating set of $G \circ H$. Thus, $S \subseteq V(G \circ H)\backslash D = V(G \circ H)\backslash V(G) = \bigcup_{v \in V(G)} V(H^v)$. Let $S_v \subseteq V(H^v)$ for each $v \in V(G)$. Then $\bigcup_{v \in V(G)} S_v \subseteq \bigcup_{v \in V(G)} V(H^v)$. Set $S = \bigcup_{v \in V(G)} S_v$. Since S is a dominating set of $G \circ H$, S_v must be a dominating set of H^v for each $v \in V(G)$.

For the converse, suppose that $D = V(G)$ and $S = \bigcup_{v \in V(G)} S_v$ where S_v is a dominating set of H^v for each $v \in V(G)$. Then $D \cap S = V(G) \cap S = \emptyset$. Since G is connected, D is a minimum dominating set of $G \circ H$ by Remark 1.13. Since $\langle V(G \circ H)\backslash D\rangle = \langle V(G \circ H)\backslash V(G)\rangle = H$ has no isolated vertices, it follows that D is a minimum restrained dominating set of $G \circ H$ (since D is a minimum dominating set). Since $S_v \subseteq V(H^v)$ is a dominating set of H^v for each $v \in V(G)$, it follows that $S = \bigcup_{v \in V(G)} S_v$ is a dominating set in $G \circ H$. Since G is connected, $\langle V(G \circ H)\backslash S\rangle = \langle V(G \circ H)\backslash \bigcup_{v \in V(G)} S_v\rangle = \langle V(G) \cup [\bigcup_{v \in V(G)} V(H^v)\backslash S_v]\ \rangle$ has no isolated vertices. This implies that S is a restrained dominating set of $G \circ H$. Since D is a minimum restrained dominating set of $G \circ H$ and $D \cap S = \emptyset$, it follows that $S \subseteq V(G \circ H)\backslash D$ is an inverse restrained dominating set of $G \circ H$ with respect to D. Accordingly, $C = D \cup S$ is a disjoint restrained dominating set of $G \circ H$. ■

The next result follows from Theorem 2.14.

Corollary 2.15 *Let G be a nontrivial connected graph and H has no isolated vertices. Then $\gamma\gamma_r(G \circ H) = |V(G)|(\gamma_r(H) + 1)$.*

Proof. Suppose that $C = D \cup S$ is a disjoint restrained dominating set of $G \circ H$. By Theorem 2.14, $D = V(G)$ and $S = \bigcup_{v \in V(G)} S_v$ where S_v is a dominating set of H^v for each $v \in V(G)$. Thus,

$$\begin{aligned}\gamma\gamma_r(G \circ H) &\leq |C| = |D \cup S| = |D| \cup |S| \\ &= |V(G)| + \left|\bigcup_{v \in V(G)} S_v\right| = |V(G)| + |V(G)||S_v| \\ &= |V(G)|(|S_v| + 1)\end{aligned}$$

for all dominating set $S_v \subseteq V(H^v)$. That is, $\gamma\gamma_r(G \circ H) \leq |V(G)|(\gamma(H) + 1)$. By remark 2.13, $D = V(G)$ is the γ_r −set of $G \circ H$. Further, $S = \bigcup_{v \in V(G)} S_v$, where S_v is a dominating set of H^v for each $v \in V(G)$, is a minimum inverse restrained dominating set of $G \circ H$ if S_v is a γ −set of H^v. Thus, $|S| = |\bigcup_{v \in V(G)} S_v| = |V(G)||S_v|$ for all dominating set S_v of H^v, that is, $|S| = |V(G)|\gamma(H)$ is a γ_r −set of $G \circ H$. By Remark 2.1, $\gamma\gamma_r(G \circ H) = \gamma_r(G + H) + \gamma_r^{-1}(G \circ H)$. Thus,

$$\begin{aligned}|V(G)|(\gamma(H) + 1) &= |V(G)|\gamma(H) + |V(G)| \\ &= \gamma_r^{-1}(G \circ H) + \gamma_r(G \circ H) \\ &= \gamma\gamma_r(G \circ H) \leq |V(G)||\gamma(H) + 1).\end{aligned}$$

Hence, $\gamma\gamma_r(G \circ H) = |V(G)|(\gamma(H) + 1)$. ■

III. CONCLUSIONS

In this paper, we introduced the concept of disjoint restrained domination in graphs and provided the characterization of a disjoint restrained dominating set in the join and corona of two graphs. Moreover, the disjoint restrained domination number of the join and corona of two graphs were calculated. Further research can be done on some other related topics.

1. Characterize the disjoint restrained dominating sets of the Cartesian product and lexicographic product of two graphs.

2. Find the disjoint restrained domination number of the Cartesian product and lexicographic product of two graphs.

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