

An Upper Bound For Volatility Swaps Pricing Under Stochastic Volatility Model With Jump-Diffusion

Wen-Jun Du

School of Information Science and Technology & Department of Mathematics & Jinan University, Tianhe District, Guangzhou, Guangdong, China

Abstract - During the development of volatility derivatives, volatility swaps become one of the most popular volatility derivatives. Volatility swaps are a kind of volatility derivatives and its essence are forward contracts on annualized realized volatility that provide an easy way for investors to trade future realized volatility against the current implied volatility. This article discusses the valuation of discretely sampled volatility swaps within the frame of Heston's stochastic volatility model under jump-diffusion model. Due to the independence of a Brownian motion and a compound Poisson process, the realized volatility can be decomposed into two parts. On the jump diffusion model, we introduce the S.G.Kou model with jump sizes double exponentially distributed, and finally work out an upper bound of fair strike price for volatility-average swaps pricing

Keywords - volatility swaps, stochastic volatility, jump-diffusion, double exponential distribution

I. INTRODUCTION

In the modern financial market, the study of finance is fundamentally about the trade-off between risk and expected return. Though various measures have been proposed to characterize the risk existing in the trade-off, but the standard deviation of an asset's return has been the most commonly used measure of risk since the middle of the last century, and the term volatility is undoubtedly a well-known form of this standard deviation. While asset volatilities play an important role in portfolio theory, they are of even greater significance for derivatives pricing. Since the mid-1990s, a subset of derivative securities has emerged that places even greater emphasis on volatility. These contingent claims elevate volatility to a more significant role in determining the payoff of the derivative security. These types of derivative securities are referred to as volatility derivatives. Carr and Lee [16] outlined an overview of the current market for volatility derivatives and gave a survey of the literature. In short, volatility derivatives are a class of derivative securities where the payoff explicitly depends on some measure of the volatility of an underlying asset.

Volatility swaps and variance swaps are two common types of volatility derivatives, the pricing of volatility derivatives can be separated into two categories: pricing volatility swaps and variance swaps. Volatility swaps are essentially forward contracts on the future realized variance of the returns of the specified underlying asset. The payoff at expiry for the long position of a volatility swap is equal to the floating amounts of annualized realized volatility minus a fixed delivery price of the contract multiplied by a notional amount of the swap in dollars per annualized volatility, whereas the short position is just the opposite. More specially, the value of a volatility swap at expiry can be written as $(RV - K_{vol}) \times L$, where the RV is the annualized realized volatility over the contract life $[0, T]$ and K_{vol} is the annualized delivery price for the volatility swap, which is set to make the value of a volatility swap equal to zero for both long and short positions at the time the contract is initially entered. T is the life time of the contract and L is the notional amount of the swap in dollars per annualized variance point. Back to the definition RV , there are at least two different measures, one measure is called standard deviation swap which is calculated as the square root of average realized variance, the other is called volatility-average swap. Specific forms of these two definitions will be given in the later sections. When the definition of annualized realized volatility is defined as the square root of average realized variance, since the payoff function involves a square root operator, pricing volatility swaps is clearly viewed to be more difficult to price analytically than variance swaps.

Due to the increasing popularity of volatility and variance swaps in the financial market, there have been numerous works on valuation of such derivatives. He and Zhu [13] presented a series-form solution for pricing variance and volatility swaps with stochastic volatility and stochastic interest rate. Elliott and Lian [8] proposed a set of closed-form exact solutions for pricing discretely sampled variance swaps and volatility swaps, based on the Heston stochastic volatility model with regime switching. Rujivan and Rakwongwan [14] presented an analytical pricing formula for volatility swaps and volatility options with discrete sampling under the standard Black-Scholes model with time varying risk-free interest rate. In addition, there are some other classical papers include [7, 11, 12] etc.

Now we mainly focus on analytically pricing volatility swaps. Zhu and Lian [2] presented a closed-form exact solution for



the pricing of discretely-sampled volatility swaps, under the framework of Heston stochastic volatility model, based on the definition of the so-called average of realized volatility. Yang et al.[10] delivered an analytical pricing formula for volatility swaps when the underlying asset follows a stochastic volatility model with jumps and stochastic intensity. Broadie and Jain [15] derived fair discrete volatility strikes by using simulation and variance reduction techniques and numerical integration techniques in different models of the underlying evolution of the asset price, including the Black-Scholes model, the Heston stochastic volatility model and the Merton jump-diffusion model etc.

In the typical Black-Scholes option-pricing framework, two empirical phenomena have received much attention: the leptokurtic feature that the return distribution of assets may have a higher peak and asymmetric heavier tails than those of the normal distribution, and an empirical phenomenon called volatility smile. In order to incorporate both of them and to strike a balance between reality and tractability, Kou [1] proposes a double exponential jump-diffusion model. This paper mainly considers the valuation of discretely sampled volatility swaps within the frame of Heston's stochastic volatility model under jump-diffusion model, and mainly based on the work of [2, 3], we finally derive an upper bound of fair strike price for volatility-average swaps pricing.

II. Preliminary

In this section, we will introduce some definitions and formulas that will be used in the following sections, include Brownian motion, double exponential jump-diffusion model, Ito-Deblin formula and Heston model.

A. Definition [9] Assuming (Ω, F, P) is a probability space. A stochastic process $\{W(t), t > 0\}$ is said to be a Brownian motion process if

- (1) $W(0) = 0$
- (2) $\{W(t), t > 0\}$ has stationary independent increments.
- (3) for every $t \geq 0$, $W(t)$ is normally distributed with mean 0 and variance $c^2 t$

the Brownian motion process, sometimes called the Wiener, is one of the most useful stochastic processes in applied probability theory.

B. Definition [1] Under the double exponential jump-diffusion model, the underlying asset S_t is modeled by the following dynamic:

$$\frac{dS(t)}{S(t-)} = \mu dt + \sigma dB_t + dJ(t) \quad (1)$$

where B_t is a standard Brownian motion, the drift μ and the volatility σ are assumed to be constants. $J(t) = \sum_{j=1}^{N(t)} (M(j) - 1)$, $N(t)$ is a Poisson process with rate λ , and $M(j)$ is a sequence of independent identically distributed (i.i.d.) nonnegative random variables such that $Y = \ln(M)$ has an asymmetric double exponential distribution with the density:

$$f_Y(y) = p\eta_1 e^{-\eta_1 y} I_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} I_{\{y < 0\}}, \quad \eta_1 > 1, \eta_2 > 0 \quad (2)$$

where $p, q \geq 0, p + q = 1$, represent the probabilities of upward and downward jumps.

C. Definition [5] In the risk-neutral world, the Heston model assumes that the underlying asset S_t is modeled by the following diffusion process with a stochastic instantaneous variance V_t

$$\begin{cases} dS_t = rS_t dt + \sqrt{V_t} S_t d\widetilde{B}_t^S \\ dV_t = k^Q(\theta^Q - V_t) dt + \sigma_V \sqrt{V_t} d\widetilde{B}_t^V \end{cases} \quad (3)$$

where r is the risk-free interest rate, θ^Q and k^Q are the long-term mean of variance and mean-reverting speed parameter of the variance respectively. σ_V is the so-called volatility of volatility. The two Wiener processes \widetilde{B}_t^S and \widetilde{B}_t^V describe the random noise in asset and variance respectively. They are assumed to be correlated with a constant correlation coefficient ρ , that is $(d\widetilde{B}_t^S, d\widetilde{B}_t^V) = \rho dt$. To ensure the variance is always positive, it is required that $2k^Q\theta^Q \geq \sigma_V^2$.

D. Theorem [4] Assuming the partial derivatives $f_t(t, x), f_x(t, x)$ and $f_{xx}(t, x)$ of the function $f(t, x)$ are defined and continuous, $W(t)$ is Brownian motion, then for each $T \geq 0$:

$$f(T, W(T)) = f(0, W(0)) + \int_0^T f_t(t, W(t))dt + \int_0^T f_x(t, W(t))dW(t) + \frac{1}{2} \int_0^T f_{xx}(t, W(t))dt \quad (4)$$

The above formula is called the Ito-Deblin formula of Brownian motion, and can be rewritten as:

$$df(t, W(t)) = f_t(t, W(t))dt + f_x(t, W(t))dW(t) + \frac{1}{2} f_{xx}(t, W(t))dt \quad (5)$$

E. Theorem Suppose $X(t)$ is jump process, the partial derivatives $f'(x)$ and $f''(x)$ of the function $f(x)$ are continuous, then the Ito-Deblin formula of jump process is:

$$f(X(t)) = f(X(0)) + \int_0^t f'(X(s))dX^c(s) + \frac{1}{2} \int_0^t f''(X(s))dX^c(s)dX^c(s) + \sum_{0 < s \leq t} f(X(s)) - f(X(s-)) \quad (6)$$

where $X^c(t)$ is the continuous part of $X(t)$

III. Discussion on the pricing of volatility swaps

In this section, we will discuss the fair strike price of volatility swaps, under the Heston stochastic volatility model with jump, and obtain an upper bound of the fair strike price for discretely-sampled volatility-average swaps. For the rest of this paper, the conditional expectation at time t is denoted by $E_t^Q = E^Q[\cdot | \mathcal{F}_t]$, where \mathcal{F}_t is the filtration up to time t

A. Volatility swaps

A volatility swap is a forward contract on realized volatility, and the amount paid at expiration of a volatility swap is based on the difference between the realized volatility and implied volatility. Assuming the current time is 0, the value of a volatility swap at expiry can be written as $(RV(0, N, T) - K_{vol}) \times L$, where the $RV(0, N, T)$ is the annualized realized volatility over the contract life $[0, T]$, N is the number of time steps in sampling, K_{vol} is the annualized delivery price for the volatility swap, which is set to make the value of a volatility swap equal to zero at the time the contract is initially entered. L is the notional amount of the swap in dollars per annualized volatility point.

Before discussing our pricing approach for the determination of the fair price of volatility swap, we will introduce the details of how the realized volatility should be calculated. As illustrated in [7], there are at least two different measures of realized volatility:

$$RV_{d1}(0, N, T) = \sqrt{\frac{AF}{N} \sum_{i=1}^N \left(\frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right)^2} \times 100 \quad (7)$$

or

$$RV_{d2}(0, N, T) = \sqrt{\frac{\pi}{2NT} \sum_{i=1}^N \left| \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right|} \times 100 \quad (8)$$

where $t_i, i = 0 \dots N$ is the i th observation time of the realized volatility in the time period $[0, T]$. S_{t_i} is spot price of the underlying asset at the i th observation time t_i , and there are total N observations. AF is the annualized factor converting volatility to an annualized volatility.

Although both of these two definitions can be used to measure the realized volatility, but they are slightly different. The definition $RV_{d1}(0, N, T)$ is essentially calculated as the square root of average realized variance, and the definition $RV_{d2}(0, N, T)$ is the average of realized volatility. [6] studied the properties of the average of realized volatility and found that the definition $RV_{d2}(0, N, T)$ is a more robust measurement of realized volatility. As a result, we will focus on the calculation of the definition $RV_{d2}(0, N, T)$.

B. Pricing approach

We now discuss our pricing approach for the determination of the upper bound of fair strike price for volatility-average swaps. In the risk-neutral world, assuming that the underlying asset $S(t)$ is modeled by the following process:

$$\begin{cases} \frac{dS(t)}{S(t-)} = rdt + \sqrt{V_t}d\widetilde{B}_t^S + dJ(t) \\ dV(t) = k^Q(\theta^Q - V(t))dt + \sigma_V\sqrt{V(t)}d\widetilde{B}_t^V \end{cases} \quad (9)$$

where $S(t)$ is the underlying asset price, $S(t-)$ is the underlying asset price before a possible jump occurs. Other parameters here are the same as those in Heston model and double exponential jump diffusion model described previously.

Proposition [3] Suppose $W(t)$ is a standard Brownian motion, $J(t) = \sum_{j=1}^{N(t)}(M(j) - 1)$ is compound Poisson process, where $N(t)$ is Poisson process with rate λ , $M(j)$ is the relative range of the underlying asset price jump. $W(t)$ and $J(t)$ are defined in the same probability space, and corresponding to the same filtration $\mathcal{F}_t, t \geq 0$. Then $W(t)$ and $J(t)$ are independent.

The main steps of the proof procedure in [3] will be given below, and if reader is interested in the specific details, please consult the references for more help.

Proof: Suppose u_1 and u_2 are fixed constant, and defines

$$Y(t) = \exp\left[u_1W(t) + u_2J(t) - \frac{1}{2}u_1^2t - \lambda(\varphi_M(u_2) - 1)t\right]$$

where $\varphi_M(u_2)$ is moment generating function with jump size $(M(j) - 1)$. In the meantime, we assume that:

$$X(s) = u_1W(s) + u_2J(s) - \frac{1}{2}u_1^2s - \lambda(\varphi_M(u_2) - 1)s$$

and $f(x) = e^x$. Obviously, $Y(s) = f(X(s))$

Let X^c be continuous part of X , we have

$$dX^c(s) = u_1dW(s) - \frac{1}{2}u_1^2ds - \lambda(\varphi_M(u_2) - 1)ds, \quad dX^c(s)dX^c(s) = u_1^2ds$$

Utilizing the Ito-Deblin formula of jump process, it can find out that the function $Y(t)$ should satisfy

$$\begin{aligned} Y(t) &= f(X(t)) \\ &= f(X(0)) + \int_0^t f'(X(s))dX^c(s) + \frac{1}{2} \int_0^t f''(X(s))dX^c(s)dX^c(s) + \sum_{0 < s \leq t} [f(X(s)) - f(X(s-))] \\ &= 1 + u_1 \int_0^t Y(s)dW(s) + (\varphi_M(u_2) - 1) \int_0^t Y(s-)d(N(s) - \lambda s) \end{aligned} \quad (10)$$

Note the above formula, which can be derived that $Y(t)$ is a martingale. Since $Y(0) = 1, EY(t) = 1$ for every $t \geq 0$, hence

$$E\left(\exp\left[u_1W(t) + u_2J(t) - \frac{1}{2}u_1^2t - \lambda(\varphi_M(u_2) - 1)t\right]\right) = 1, \quad t \geq 0$$

Move the term of above formula, it is obvious that

$$E(e^{u_1W(t)+u_2J(t)}) = e^{\frac{1}{2}u_1^2t} \cdot e^{\lambda t(\varphi_M(u_2)-1)}$$

Since the moment generating function of $W(t)$ and $J(t)$ are $Ee^{u_1W(t)} = e^{\frac{1}{2}u_1^2t}, Ee^{u_2J(t)} = e^{\lambda t(\varphi_M(u_2)-1)}$ respectively, $W(t)$ and $J(t)$ are independent.

Due to the independence of Brownian motion $W(t)$ and compound Poisson process $J(t)$, the realized volatility of volatility swap can be decomposed into two parts. The first is the accumulated realized volatility contributed from the diffusive component of the underlying asset price process and second is the contribution from jumps. Hence we can decompose the model as follows:

$$\begin{cases} dS(t) = rS(t)dt + \sqrt{V_t}S(t)d\widetilde{B}_t^S \\ dV(t) = k^Q(\theta^Q - V(t))dt + \sigma_V\sqrt{V(t)}d\widetilde{B}_t^V \end{cases} \quad (11)$$

and pure jump process $\frac{dS(t)}{S(t-)} = dJ(t)$

In the above discussion, we decompose the origin model into two parts. Therefore, we next focused on the calculations of the fair delivery price for these two parts separately. In the following, the realized volatility of continuous part is denoted by $RV_{d2}^1(0, N, T)$ and pure jump part is denoted by $RV_{d2}^2(0, N, T)$

Proposition [2] If the underlying asset follows the dynamics (11), then the pricing formula for the volatility swaps in the form of:

$$\begin{aligned}
 K_{vol}^1 &= E_0^Q[RV_{d2}^1(0, N, T)] = E_0^Q \left[\sqrt{\frac{\pi}{2NT}} \sum_{i=1}^N \left| \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right| \right] \times 100 \\
 &= \sqrt{\frac{2}{\pi NT}} \int_0^\infty \sum_{i=1}^N Re \left[\frac{f(\phi j + 1; t_{i-1}, t_i, V_0) - f(\phi j; t_{i-1}, t_i, V_0)}{\phi j} \right] d\phi \times 100 \tag{12}
 \end{aligned}$$

where $f(\phi; t, T, V_0)$ is the forward characteristic function of the stochastic variable $y_{t,T} = \log S_T - \log S_t (t < T)$ and defined as the Fourier transform of the probability density function of $y_{t,T}$ i.e.

$$f(\phi; t, T, V_0) = E_0^Q[e^{\phi y_{t,T}} | y_0, V_0], \quad t < T$$

If someone is interested in the specific content mentioned above, please refer to [2] for more details.

If the price of underlying asset jumps at time t_{i-1} , since $J(t) = \sum_{j=1}^{N(t)} (M(j) - 1)$, then

$$\frac{S(t_i) - S(t_{i-1})}{S(t_{i-1})} = M(j) - 1$$

Reviewing the definition of $RV_{d2}(0, N, T)$, we have

$$\left| \frac{S(t_i) - S(t_{i-1})}{S(t_{i-1})} \right| = \left| \frac{S(t_i^+) - S(t_{i-1})}{S(t_{i-1})} \right| = M(j) - 1$$

If the price of underlying asset jump $N(t)$ times at $[0, T]$, then the realized volatility accumulated by jump-diffusion process is

$$RV_{d2}^2(0, N, T) = \sqrt{\frac{\pi}{2NT}} \sum_{j=1}^{N(t)} |M(j) - 1| \times 100 \tag{13}$$

Proposition. Suppose $M(j), j = 1, 2, 3 \dots$ is a sequence of independent identically distributed (i.i.d.) nonnegative random variables such that $Y = \log(M)$ has an asymmetric double exponential distribution with the density:

$$f_Y(y) = p\eta_1 e^{-\eta_1 y} I_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} I_{\{y < 0\}}, \quad \eta_1 > 1, \eta_2 > 0$$

where $p, q \geq 0, p + q = 1$, represent the probabilities of upward and downward jumps. If the underlying asset follows the dynamics (9), then the fair delivery price of the volatility swap satisfies the following formula:

$$\begin{aligned}
 K_{vol} &\leq \left[\sqrt{\frac{2}{\pi NT}} \int_0^\infty \sum_{i=1}^N Re \left[\frac{f(\phi j + 1; t_{i-1}, t_i, V_0) - f(\phi j; t_{i-1}, t_i, V_0)}{\phi j} \right] d\phi + \lambda T \sqrt{\frac{\pi}{2NT}} \left(\frac{p\eta_1}{\eta_1 - 1} - \frac{q\eta_2}{\eta_2 + 1} + q - p \right) \right] \\
 &\times 100 \tag{14}
 \end{aligned}$$

Proof : Because of the independence of Brownian motion $W(t)$ and compound Poisson process $J(t)$, when the underlying asset follows the dynamics (9), the following formula can be easily derived through utilizing the absolute value inequality

$$RV_{d2}(0, N, T) \leq RV_{d2}^1(0, N, T) + RV_{d2}^2(0, N, T)$$

Take expectations on both sides of the above formula, we have

$$E_0^Q[RV_{d2}(0, N, T)] \leq E_0^Q[RV_{d2}^1(0, N, T)] + E_0^Q[RV_{d2}^2(0, N, T)]$$

in which

$$\begin{aligned}
 E_0^Q[RV_{d2}^2(0, N, T)] &= E_0^Q \left[\sqrt{\frac{\pi}{2NT}} \sum_{j=1}^{N(t)} |M(j) - 1| \times 100 \right] \\
 &= \sqrt{\frac{\pi}{2NT}} E_0^Q \left[\sum_{j=1}^{N(t)} |M(j) - 1| \right] \times 100 \\
 &= \lambda T \sqrt{\frac{\pi}{2NT}} E_0^Q[|M(j) - 1|] \times 100
 \end{aligned}$$

where

$$\begin{aligned}
 E_0^Q[|M(j) - 1|] &= \int_{-\infty}^{+\infty} |e^y - 1| f_Y(y) dy = \int_{-\infty}^0 (1 - e^y) f_Y(y) dy + \int_0^{+\infty} (e^y - 1) f_Y(y) dy \\
 &= \int_{-\infty}^0 (1 - e^y) q\eta_2 e^{\eta_2 y} dy + \int_0^{+\infty} (e^y - 1) p\eta_1 e^{-\eta_1 y} dy
 \end{aligned}$$

$$= \frac{p\eta_1}{\eta_1 - 1} - \frac{q\eta_2}{\eta_2 + 1} + q - p$$

therefore

$$E_0^Q[RV_{d2}^2(0, N, T)] = \lambda T \sqrt{\frac{\pi}{2NT}} \left(\frac{p\eta_1}{\eta_1 - 1} - \frac{q\eta_2}{\eta_2 + 1} + q - p \right) \times 100$$

According to the formula (12), we knew that

$$E_0^Q[RV_{d2}^1(0, N, T)] = \sqrt{\frac{2}{\pi NT}} \int_0^\infty \sum_{i=1}^N Re \left[\frac{f(\phi j + 1; t_{i-1}, t_i, V_0) - f(\phi j; t_{i-1}, t_i, V_0)}{\phi j} \right] d\phi \times 100$$

hence

$$K_{vol} = E_0^Q[RV_{d2}(0, N, T)] \leq \left[\sqrt{\frac{2}{\pi NT}} \int_0^\infty \sum_{i=1}^N Re \left[\frac{f(\phi j + 1; t_{i-1}, t_i, V_0) - f(\phi j; t_{i-1}, t_i, V_0)}{\phi j} \right] d\phi + \lambda T \sqrt{\frac{\pi}{2NT}} \left(\frac{p\eta_1}{\eta_1 - 1} - \frac{q\eta_2}{\eta_2 + 1} + q - p \right) \right] \times 100$$

The above equation gives an upper bound of fair strike price for volatility- average swaps, based on the definition of $RV_{d2}(0, N, T)$.

IV. CONCLUSIONS

In this paper, we have applied the Heston stochastic volatility model with jump to describe the underlying asset price, and by demonstrating the independence of a Brownian motion and a compound Poisson process, finally we derived an upper bound of fair strike price for discretely-sampled volatility swaps that the definition used to measure the realized volatility is the average of realized volatility. The original intention of this article is to calculate a closed-form exact solution for the pricing of discretely-sampled volatility swaps, but due to the existence of the absolute value symbols in the definition of the realized volatility, we cannot completely divide the realized volatility into two parts, and therefore temporarily obtain an upper bound of fair strike price. Under the same conditions, when we consider the pricing problem of the variance swaps, a closed-form solution for discretely-sampled variance swaps can be derived, but it's difficult for volatility swaps and deserves our further investigation, for example, whether closed-form solution can be obtained under some conditions. Furthermore, there are additional questions worth considering, such as: the pricing problem for continuous samples volatility swaps under the Heston stochastic volatility model with jump, and when the realized volatility is defined as the square root of the average of realized variance, can we come up with a pricing formula.

REFERENCES

- [1] Kou S G. A jump-diffusion model for option pricing[J]. Management science, 48(8) (2002) 1086-1101.
- [2] Zhu S P, Lian G H. Analytically pricing volatility swaps under stochastic volatility[J]. Journal of computational and applied mathematics, 288 (2015) 332-340.
- [3] Min Wang, Kaili Xiang., Pricing variance swaps with stochastic volatility under jump-diffusion. Dynamics of continuous, discrete & impulsive systems, Series B. Applications and algorithms, 21(2/3) (2014) 121-136.
- [4] Steven E. Shreve. Financial Stochastic Analysis[M]. Shanghai University of Finance and Economics Press. 6 (2015).
- [5] Heston S L. A closed-form solution for options with stochastic volatility with applications to bond and currency options[J]. The review of financial studies, 6(2) (1993) 327-343.
- [6] Barndorff-Nielsen O E, Shephard N. Realized power variation and stochastic volatility models[J]. Bernoulli, 9(2) (2003) 243-265.
- [7] Howison S, Rafailidis A, Rasmussen H. On the pricing and hedging of volatility derivatives[J]. Applied Mathematical Finance, 11(4) (2004) 317-346.
- [8] Elliott R J, Lian G H. Pricing variance and volatility swaps in a stochastic volatility model with regime switching, discrete observations case[J]. Quantitative Finance, 13(5) (2013) 687-698.
- [9] Ross S M, Kelly J J, Sullivan R J, et al. Stochastic processes[M]. New York, Wiley, (1996).
- [10] Yang B Z, Yue J, Wang M H, et al. Volatility swaps valuation under stochastic volatility with jumps and stochastic intensity[J]. Applied Mathematics and Computation, 355 (2019) 73-84
- [11] Carr P, Geman H, Madan D B, et al. Pricing options on realized variance[J]. Finance and Stochastics, 9(4) (2005) 453-475.
- [12] Swishchuk A. Modeling of variance and volatility swaps for financial markets with stochastic volatilities[J]. WILMOTT magazine, 2 (2004) 64-72.
- [13] He X J, Zhu S P. A series-form solution for pricing variance and volatility swaps with stochastic volatility and stochastic interest rate[J]. Computers & Mathematics with Applications, 76(9) (2018) 2223-2234.
- [14] Rujivan S, Rakwongwan U. Analytically pricing volatility swaps and volatility options with discrete sampling, Nonlinear payoff volatility derivatives[J]. Communications in Nonlinear Science and Numerical Simulation, 100 (2021) 105849.
- [15] Broadie M, Jain A. The effect of jumps and discrete sampling on volatility and variance swaps[J]. International Journal of Theoretical and Applied Finance, 11(08) (2008) 761-797.
- [16] Carr P, Lee R. Volatility derivatives[J]. Annu. Rev. Financ. Econ., 1(1) (2009) 319-339.