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Radio Number of Some Path Related Graph

Alamgir Rahaman Basunia*[1], Laxman Saha[2], Kalishankar Tiwary[3]

[1]*Research Scholar, Department of mathematics, Raiganj University, Raiganj-733134, India*
[2]*Assistant Professor, Department of Mathematics, Balurghat College, Balurghat-733101, India.*
[3]*Professor, Department of mathematics, Raiganj University, Raiganj-733134, India*

Abstract - *A radio labeling of a graph G is a function f from the vertex set $V\ (G)$ to the set of non negative integers such that $|f(u)\ -\ f(v)|\ \geq\ diam(G)\ +\ 1\ -\ d_G(u,v)$, where $diam(G)$and $d_G(u,v)$ are diameter and distance between u and v in graph G, respectively. The radio number $rn(G)$ of G is the smallest number k such that G has radio labeling with $max\{f(v)\ :\ v\ \in V\ (G)\}\ =\ k$. We investigate the radio number of some special type of path related graph.*

Keywords - *Tree, Radio number, Span.*

I. INTRODICTION

In a telecommunication system to design radio networks,the interference constraints between a pair of transmitters play a vital role. For the transmitters of radio network, we seek to assign channels to transmitters such that it satisfies all interference constraints. The assignment of channels to the transmitters is popularly known as channel assignment problem which was introduced by Hale [3].In 2005, Chartrand et al. [1] introduced the concept of radio labeling and put the level of interference at largest possible-the diameter of graph. For a simple connected graph G. The distance between any two vertices u and v is denoted by $d_G(u,v)$ or simply$d(u,v)$.The diameter of G, denoted by $diam(G)$, is the maximum value of $d(u,v)$ for all $u,v\ \in G$. A radio labeling$f$ of G is an assignment of non-negative integers to the vertices of G satisfying $|f(u) - f(v)|\ \geq\ diam(G) + 1 - d_G(u,v)$,for all $u,v\ \in V\ (G)$. The integer$f(u)$ is called the label of u under f, and the *span* of f is defined as $span(f)\ =\ max\{|f(u)\ -\ f(v)|\ :\ u,v\ \in V\ (G)\}$. The radio number of G, denoted by $rn(G)$, is the minimum span among all radio labelling of G. A radio labeling induces an ordering $u_0,u_1,\ldots,u_{p-1}(p\ =\ |V\ (G)|)$ of vertices such that $0\ =\ f(u_0)\ <\ f(u_1)\ <\ldots<\ f(u_{p-1})\ =\ span(f)$. The radio number of graph is studied by several authors (see, [5]-[22]). The first result on the radio number of trees was given by Chartrand et all [1]. They gave an upper bound for the radio number of paths and trees. Later, Liu and Zhu[11] gave the exact radio number of paths. In this paper, we determine the radio number of some special type of path related graph.

II. PRELIMINARIES

Let T be a tree with centroid S. For any two vertices u and v, if u is on the (S,v) −path, then u is an ancestor of v, and v is a descendent of u. The centroid S is an ancestor of every vertex,and every vertex is its own ancestor and descendent. Define level function on $V\ (T)$ by $L_S(u)\ =\ d(S,u)$ for any $u\ \in V\ (T)$, we use $L(u)$ instead of $L_S(u)$. For any $u,v\ \in V\ (T)$ define ϕ as

$$\phi(u,v)\ =\ max\{\ L(t): t\ is\ a\ common\ ancestor\ of\ u\ and\ v\ with\ respect\ to\ centroid\ S\}.$$

Theweight of T at the centroid S is denoted by $w(T)$ and defined by $w(T) = \sum_{u\in V(T)} L(u)$.

Let P_{2k+1}be a path with $2k\ +\ 1$ number of vertices. Now we associate r vertices to both of the nearest two vertices of the centroid of P_{2k+1}. Let the vertices which are newly associated to the left side of the centroid be $x_1,x_2,\ldots,x_r$and those which are newly associated to the right side of the centroid be $y_1,y_2,\ldots,y_r$. We name this graph as T^r_{2k+1}. We have the following observations for the treeT^r_{2k+1}.

Observation

(a) The number of vertices of $T^r_{2k+1}$is$n\ =\ (2k\ +\ 1)\ +\ 2r$.
(b) The diameter of T^r_{2k+1} is $2k$.
(c) The centroid T^r_{2k+1} is at v_k.
(d) The weight of T^r_{2k+1} is given by

$$\begin{aligned} W(\ T^r_{2k+1})\ &=\ \{1\ \cdot\ 2\ +\ 2\ \cdot\ 2+\ldots+k\ \cdot .2\}\ +\ 2\ \cdot\ 2r. \\ &=\ 2\ \cdot\ (1\ +\ 2\ +\ 3+\ldots+k) \\ &= 2\ \cdot \frac{k(k+1)}{2} + 4r. \\ &=\ k(k\ +\ 1) +\ 4r. \end{aligned}$$

In next two sections deal with the radio number of T^r_{2k+1} In the immediate section we give alower bound for radio number of T^r_{2k+1} From here to onward, we denote q is the diameter of T^r_{2k+1}.

III. LOWER BOUND OF RADIO NUMBER OF T^r_{2k+1}

In this section, we give a lower bound for radio number of T^r_{2k+1} Recall that $V\,(T^r_{2k+1}) = \{v_0, v_1, \dots, v_{2k}\} \cup \{x_0, x_1, \dots, x_r\} \cup \{y_1, y_2, \dots, y_r\}$. A radio labelling f is a one-to-one function.On the other hand, any one-to-one integral function$f$ on $V\,(T^r_{2k+1})$, with $0 \in f(V)$, induces an ordering of $V\,(T^r_{2k+1})$, which is a line-up of the vertices with increasing images. We denote thisordering by $U(f)$, where $V\,(T^r_{2k+1}) = U(f) = \{u_0, u_1, u_2, \dots, u_{n-1}\}$ with

$$0 = f(u_0) < f(u_1) < f(u_2) < \dots < f(u_{n-1}).$$

Notice, if f is a radio labelling, then the span of f is $f(u_{n-1})$. Now from the radio conditions we have the following for $0 \le i \le n-2$

$$f(u_{i+1}) - f(u_i) \ge q + 1 - d(u_i, u_{i+1}). \qquad (1)$$

To make it an equality, we add a positive quantity $J_f(u_i, u_{i+1})$, called *jump* of f from $u_i$to u_{i+1}, in right side of the inequality (1). Therefore,

$$f(u_{i+1}) - f(u_i) = q + 1 - d(u_i, u_{i+1}) + J_f(u_i, u_{i+1}).$$

Summing up these $n-1$ equations,

$$f(u_{n-1}) = \sum_{i=0}^{n-2} [f(u_{i+1}) - f(u_i)] + f(u_0)$$

$$= \sum_{i=0}^{n-2} [q + 1 - d(u_i, u_{i+1})] + J_f(u_i, u_{i+1}) + f(u_0)$$

$$\ge (n-1)(q+1) - 2\sum_{i=0}^{n-2} L(u_i) + L(u_0) + L(u_{n-1}) + \sum_{i=0}^{n-2} J_f(u_i, u_{i+1}) + 2\phi(u_i, u_{i+1}) + f(u_0)$$

$$= (n-1)(q+1) - 2W(T^r_{2k+1}) + f(u_0) + L(u_0) + L(u_{n-1}) + \sigma(f) \qquad (2)$$

Where $\sigma(f) = \sum_{i=0}^{n-2} \sigma_f(u_i, u_{i+1})$ and $\sigma_f(u_i, u_{i+1}) = J_f(u_i, u_{i+1}) + 2\phi(u_i, u_{i+1})$. Here total jump $J(f) = \sum_{i=0}^{n-2} \mathrm{J_f(u_i, u_{i+1})}$. So the relation between $\sigma(f)$ and$J(f)$ is $\sigma(f) = J(f) + 2\sum_{i=0}^{n-2} \phi(\mathrm{u_i, u_{i+1}})$.

Lemma 3.1 *If u_i and u_{i+2} are in the same branch of T^r_{2k+1} and u_{i+1} is in a different branch of T^r_{2k+1}, then $\sigma_f(u_i, u_{i+1}) + \sigma_f(u_{i+1}, u_{i+2}) \ge max\{2L(u_{i+1}) + 2\phi(u_i, u_{i+2}) - 2k - 1, 0\}$*

Proof:

$$\begin{aligned} f(u_{i+1}) - f(u_i) &= 2k + 1 - d(u_i, u_{i+1}) + J_f(u_i, u_{i+1}) \\ &= 2k + 1 - L(u_i) - L(u_{i+1}) + 2\phi(u_i, u_{i+1}) + J_f(u_i, u_{i+1}) \\ &= 2k + 1 - L(u_i) - L(u_{i+1}) + \sigma_f(u_i, u_{i+1}) \end{aligned}$$

and

$$f(u_{i+2}) - f(u_{i+1}) = 2k + 1 - L(u_{i+1}) - L(u_{i+2}) + \sigma_f(u_{i+1}, u_{i+2})$$

Summing up we get

$$f(u_{i+2}) - f(u_i) = 4k + 2 - L(u_i) - L(u_{i+2}) - 2L(u_{i+1}) + \sigma_f(u_i, u_{i+1}) + \sigma_f(u_{i+1}, u_{i+2})$$

where $\sigma_f(u_t, u_{t+1}) = J_f(u_t, u_{t+1}) + 2\varphi(u_t, u_{t+1})$ for $t = i, i+1$. On the other hand, since f is a radio labeling, we have

$$\begin{aligned} f(u_{i+2}) - f(u_i) &\ge 2k + 1 - d(u_i, u_{i+2}) \\ &= 2k + 1 - L(u_i) - L(u_{i+2}) + 2\phi(u_i, u_{i+2}) \end{aligned}$$

Combining the two expressions above, we get where $\sigma_f(u_i, u_{i+1}) + \sigma_f(u_{i+1}, u_{i+2}) \ge 2L(u_{i+1}) + 2\phi(u_i, u_{i+2}) - 2k - 1$. Since the value of $\sigma_f(u_t, u_{t+1}) \ge 0$ for $t = i, i+1$, the result follows immediately.

Remark 3.1 *From the above lemma we see that if a vertex u_{i+1} is at level k, then $\sigma_f(u_i, u_{i+1}) + \sigma_f(u_{i+1}, u_{i+2}) \geq 1$, provided $s \notin \{u_i, u_{i+2}\}$.*
Theorem 1. *For the tree T^r_{2k+1}, $rn(T^r_{2k+1}) \geq (n-1)(2k+1) - \{2k(k+1) + 8r\} + 2$.*

Proof: Let f be an arbitrary radio labelling of T^r_{2k+1} and $u_0, u_1, \ldots, u_n$ be the ordering of the vertices of T^r_{2k+1} such that $0 = f(u_0) < f(u_1) < \cdots < f(u_{n-1}) = span(f)$. Then from Eq. (2) with $f(u_0) = 0$ and $W(T^r_{2k+1}) = k(k+1) + 4r, f(u_{n-1})$ satisfies the following

$$f(u_{n-1}) \geq (n-1)(2k+1) - \{2k(k+1) + 8r\} + L(u_0) + L(u_{n-1}) + \sigma(f) \quad (3)$$

Where $\sigma(f) = \sum_{i=0}^{n-2} \sigma_f(u_i, u_{i+1})$ and $\sigma_f(u_i, u_{i+1}) = J_f(u_i, u_{i+1}) + 2\varphi(u_i, u_{i+1})$. If possible, let $span(f) = (n-1)(q+1) - \{2k(k+1) + 8r\} + 1$. Then from Eq. (3), we must have

$$L(u_0) + L(u_{n-1}) + \sigma(f) = 1$$

As $L(u_0) + L(u_{n-1}) \geq 1$ so Eq.(4) implies $L(u_0) + L(u_{n-1}) = 1$ and $\sigma(f) = 0$. But the equality is true only when one of u_0 and u_{n-1} is the centroid s and other is adjacent to s. But then Remark 3.1 implies $\sigma(f) \geq 1$ as T^r_{2k+1} has two vertices which are level k. So we get a contradiction. Hence we obtain the result.

IV. UPPER BOUND FOR RADIO NUMBER OF T^r_{2k+1}

To find an optimal radio labelling, we need to rearrange the vertices of T^r_{2k+1}. In the below we give a vertex index scheme for the same.

4.1 Vertex Arrangement Scheme of T^r_{2k+1}:

Recall that $V(T^r_{2k+1}) = \{v_0, v_1, \ldots, v_{2k}\} \cup \{x_0, x_1, \ldots, x_r\} \cup \{y_0, y_1, \ldots, y_r\}$. Now we rename the vertices of $V(T^r_{2k+1})$ by u_i's, where u_i's are defined as below.

(a) For $0 \leq i \leq 2k-1$

$$u_i = \begin{cases} v_k, & i = 0 \\ v_{2k}, & i = 1 \\ v_{\frac{i-2}{2}}, & i \text{ is even and } 2 \leq i \leq 2k-2 \\ v_{k+\frac{i-1}{2}}, & i \text{ is odd and } 3 \leq i \leq 2k-1. \end{cases}$$

(b) For $0 \leq i \leq n-2k-2$, $u_{2k+i} = \begin{cases} x_{\frac{i+2}{2}}, & i \text{ is even} \\ y_{\frac{i+1}{2}}, & i \text{ is odd} \end{cases}$

(c) $u_{n-1} = v_{k-1}$.

Remark 4.1 From the above new arrangement of vertices $u_0, u_1, \ldots u_{n-1}$ forms an alternating sequences i.e., $\phi(u_t, u_{t+1}) = 0$ for every i.
From here no onwards by the consecutive vertices we understand that u_t, u_{t+1} are consecutive if $f(u_t) < f(u_{t+1})$.
Lemma 4.1 *Let u_t and u_{t+1} be any two consecutive vertices of $T^r_{2k+1} \backslash \{u_1\}$, then $L(u_t) + L(u_{t+1}) \leq k+1$.*

Proof : Let us partition the vertex set $V(T^r_{2k+1})$ into four disjoin sets S_1, S_2, S_3 *and* S_4, where

$$S_1 = \{v_0, v_1, \ldots, v_{k-1}\}.$$
$$S_2 = \{v_{k+1}, v_{k+2}, \ldots, v_{2k}\}$$
$$S_3 = \{x_1, x_2, \ldots, x_r\}.$$
$$S_4 = \{y_1, y_2, \ldots, y_r\}.$$

Also we have,

$$L(v_i) = k - i, \quad 0 \leq i \leq k-1$$
$$L(v_i) = 1, \quad 1 \leq i \leq k.$$
$$L(x_i) = 2, \quad 1 \leq i \leq r$$
$$L(y_i) = 2, \quad 1 \leq i \leq r$$

Now consider the following cases.

Case-1 : $u_t \in S_1$. If $u_t \in S_1$, then $u_t = v_i$ for some i satisfying $0 \leq i \leq k-2$ and $u_{t+1} \in S_2$. Also $u_{t+1} = v_{k+i+1}$. Therefore,

$$L(u_t) + L(u_{t+1}) = L(v_i) + L(v_{k+i+1}) = (k-i) + (i+1) = k+1$$

Case-2 : $u_t \in S_2 \setminus \{v_{2k}\}$. Let $u_t \notin \{v_{2k-1}, v_{2k}\}$. Then $u_t \in S_2$ and this implies $u_t = v_{k+i}$ with $1 \le i \le k-2$ and $u_{t+1} \in S_1$. Also $u_{t+1} = v_i$. Thus we have

$$L(u_t) + L(u_{t+1}) = L(v_{k+i}) + L(v_i) = i + (k-i) = k.$$

Again if $u_t = v_{2k-1}$, then $u_{t+1} \in S_3$ and $u_{t+1} = x_1$. So we obtain

$$L(u_t) + L(u_{t+1}) = L(v_{2k-1}) + L(x_i) = (k-1) + 2 = k+1.$$

Case-3 : $u_t \in S_3$. If $u_t \in S_3$, then $u_t = x_i$ for some i satisfying $1 \le i \le r$; $u_{t+1} \in S_4$ and $u_{t+1} = y_i$. Therefore, $L(u_t) + L(u_{t+1}) = L(x_i) + L(y_i) = 2 + 2 = 4.$

Case-4 : $u_t \in S_4$. If $u_t \in S_4 \{y_r\}$, then $u_t = y_i$ for some i with $1 \le i \le r$-1 and $u_{t+1} = x_{i+1}$.
Hence $L(u_t) + L(u_{t+1}) = L(y_i) + L(x_{i+1}) = 2 + 2 = 4$

Again if $u_t = y_r$, then $u_{t+1} = v_{k-1}$ and for this values of u_t ,we have

$$L(u_t) + L(u_{t+1}) = L(y_r) + L(v_{k-1}) = 3.$$

On account of all the above cases, we obtain $L(u_t) + L(u_{t+1}) \le k + 1.$

4.2 Radio Labeling of T^r_{2k+1}

Now we are in a position to give a radio labelling of T^r_{2k+1} .
Theorem 2 *For the tree T^r_{2k+1}, the mapping f defined as; $f: V(T^r_{2k+1}) \to \{0,1,2, \dots, \}$.*

$$f(u_i) = \begin{cases} 0, & i = 0 \\ f(u_{i-1}) + 2k + 1 - L(u_{i-1}) - L(u_i), & i \ge 1, i \ne 3 \\ f(u_{i-1}) + 2k + 1 - L(u_{i-1}) - L(u_i) + 1, & i = 3 \end{cases}$$

is an optimal radio labeling.

Proof: To show f is a radio labelling of T^r_{2k+1}, we need to prove $f(u_j) - f(u_i) \ge 2k + 1 - d(u_i, u_{i+1})$ for all $j > i$. First we take $j = i + 1$. Then from definition of f , we have $f(u_{i+1}) \ge f(u_i) + 2k + 1 - L(u_i) - L(u_{i+1})$ and hence $f(u_{i+1}) - f(u_i) \ge 2k + 1 - d(u_{i+1}, u_i)$ as $\varphi(u_i, u_{i+1}) = 0$ implies that $L(u_i) + L(u_{i+1}) = d(u_i, u_{i+1})$. Now for $j = i + 2.$, we calculate the difference $f(u_{i+2}) - f(u_i)$ in the below.

$$\begin{aligned} f(u_{i+2}) - f(u_i) &= 2(2k+1) - L(u_i) - L(u_{i+1}) - L(u_{i+1}) - l(u_{i+2}) \\ &= 2(2k+1) - \{L(u_i) + L(u_{i+1})\} - \{L(u_{i+1}) + L(u_{i+2})\} \\ &\ge 2(2k+1) - 2(k+1) \\ &= 2k \end{aligned}$$

Therefore the radio condition is satisfies for $j = i + 2.$ Similarly, for $j \ge i + 3$, we have

$$\begin{aligned} f(u_{i+1}) - f(u_i) &= 2k + 1 - L(u_i) - L(u_{i+1}) \\ f(u_{i+2}) - f(u_{i+1}) &= 2k + 1 - L(u_{i+1}) - L(u_{i+2}) \\ f(u_{i+3}) - f(u_{i+2}) &= 2k + 1 - L(u_{i+2}) - L(u_{i+3}) \\ ... &= \cdots \\ f(u_j) - f(u_{j-1}) &= q + 1 - L(u_{j-1}) - L(u_j). \end{aligned}$$

Adding all these we get,

$$\begin{aligned} f(u_j) - f(u_i) &= (j-i)(2k+1) - \sum_{t=i}^{j-i} \{L(u_t) + L(u_{t+1})\} \\ &\ge (j-i)(2k+1) - (j-i)(k+1) \\ &= (j-i)k \\ &= 3k, \quad \text{as } j - i \ge 3 \end{aligned}$$

Thus for all i and j , we have $f(u_j) - f(u_i) \ge 2k + 1 - d(u_i, u_j)$. i.e., f is a radio labelling of T^r_{2k+1}. To show optimality, we need show that the span of f coincides with the lower bound presented in Theorem 1.
$span(f)$: From definition of f ,We have

$$f(u_i) = \begin{cases} 0, & i = 0 \\ f(u_{i-1}) + 2k + 1 - L(u_{i-1}) - L(u_i), & i \geq 1, i \neq 3 \\ f(u_{i-1}) + 2k + 1 - L(u_{i-1}) - L(u_i) + 1, & i = 3 \end{cases}$$

Now the difference $f(u_{n-1}) - f(u_0)$ is given by
$f(u_{n-1}) - f(u_0) = (n-1)(2k+1) - 2W(T^r_{2k+1}) + L(u_0) + L(u_{n-1}) + 1$ and putting the values $f(u_0)$, $W(T^r_{2k+1})$ and $L(u_0) + L(u_{n-1})$, we have

$$\begin{aligned} f(u_{n-1}) &= (n-1)(2k+1) - 2\{k(k+1) + 4r\} + L(u_0) + L(u_{n-1}) + 1 \\ &= (n-1)(2k+1) - \{2k(k+1) + 8r\} + 0 + 1 + 1 \\ &= (n-1)(2k+1) - \{2k(k+1) + 8r\} + 2 \end{aligned}$$

Therefore, f is a radio labelling T^r_{2k+1} with $span(f) = f(u_{n-1}) = (n-1)(2k+1) - \{2k(k+1) + 8r\} + 2\}$. In Figure 1, we give an optimal radio labelling of T^4_{11} according to our rule stated in this theorem.

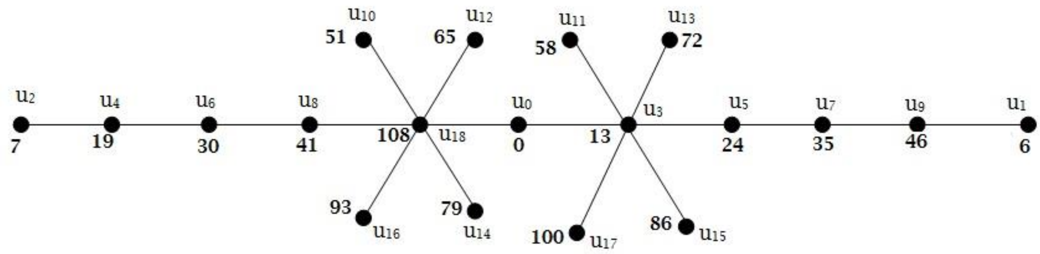

Figure 1

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