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A Higher-Dimensional LRS Bianchi Type-V String Cosmological Model in Brans-Dicke Scalar-Tensor Theory of Gravitation

Diksha Trivedi[1], A. K. Bhabor[2]

[1,2] *Department of Mathematics and Statistics, University College of Science M.L. Sukhadia University, Udaipur-313001, India*

Abstract- *Five dimensional LRS Bianchi type-V string cosmological models in scalar-tensor theory of gravitation proposed by Brans-Dicke (Phys. Rev. 124:925, 1961) containing cosmic string is investigated. Berman's law,* $T = -(\rho + \lambda) = 0$, *proportionality of scalar expansion and shear scalar are the conditions, used in order to find solutions of the model. Some physical and geometrical properties of the obtained model are also discussed.*

Keywords — *Bianchi type-V, cosmic strings, Brans-Dicke theory.*

I. Introduction

The standard model of gravitation and cosmology is based on the theory of general relativity. Einstein's General theory of Relativity has all the potential to describing gravitational phenomena, but it has been criticized due to lack of certain desirable features. Einstein himself pointed out that Mach's principle is not verified by general relativity. Hence, to overcome such singularities in the general theory of relativity, scalar-tensor theories of gravitation have been investigated by several astronomers. Brans and Dicke theory (1961) referred to as the scalar- tensor theory of gravitation, is a modified version of Einstein's general theory of relativity found on Mach's principle by introducing a scalar field ϕ coupled to the mass density of the universe [1]. The theory states that scalar field is reciprocal of time varying gravitational constant G, but this theory does not permit the scalar field to cooperate with elementary particles and photons.

BD field equations for the combined scalar and tensor fields are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)$$

$$-\phi^{-1}(\phi_{i,j}g_{ij}\phi^{,k}_{;k}) \tag{1}$$

and

$$\phi = \phi^{,k}_{;k} = 8\pi(3+2\omega)^{-1}T \tag{2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is an Einstein tensor, R is the Ricci tensor, ϕ is the Brans-Dicke scalar field, ω is the dimensionless constant, and T_{ij} is the energy momentum tensor.
The equation of motion is given by

$$T^{ij}_{;j} = 0 \tag{3}$$

It is a consequence of the field Eqs. (1) and (2).

Nariai [2], Belinskii and Khalatnikov [3], Reddy and Rao [4], Banerjee and Santos [5], Singh and Rai [6], Singh *et al.* [7], Shri ram [8], Berman *et al.* [9], Reddy [42], Reddy *et al.* [11], Adhav *et al.* [12], and Rao *et al.* [13] are some of the authors who worked on the several aspects of this theory. The study of Bianchi type-V cosmological models has attracted many researchers in recent time as these models contain some specific isotropic cases and allow arbitrary small anisotropy level at any instant of cosmic time. These models are generalized version of FRW models with negative curvature. Lorenz [14], Ram and Singh [15], Baillie and Madsen [16], Beesham [17], Banerjee and Sanyal [18], Venkateswarulu and Reddy [19], Roy and Prasad [20], Camci *et al.* [21], Pradhan *et al.* [22], Bali and Singh [23], Ram *et al.* [24] and Singh [25] are some authors who have investigated Bianchi type-V cosmological models. In recent years, string cosmology has become centre of attraction for researchers. Zeldovich [26] has proposed that cosmic strings give rise to density perturbations, which is the reason behind the formulation of galaxies. Chakraborty and Nandy [27] have studied string theory in five dimensional flat space time by using barotropic equation of state and p-string model. Krori *et al.* [28] have investigated that cosmic strings do not exist in Bianchi type-V cosmology. Yadav [29] has studied Bianchi type-V string cosmological model and late time acceleration. Recently, Rasouli *et al.* [30] have produced Modified Brans-Dicke theory. MBDT in D dimensions is formulated by applying a dimensional reduction procedure within IMT setting for the standard BD theory in (D+1) dimensional space time. Jyotsna and Tiwari [31] have studied Bianchi type-V string cosmological model filled with perfect fluid in self creation cosmology. Recently, Rao and Sudha [32] have studied Bianchi type-V dark energy model in Brans-Dicke theory of gravitation. Yadav *et al.* [33], Rao and Rao [34], Bishi and Mahanta [35], Rao *et al.* [36], Humad and Shrimali (2014) and Deo *et al.* [37] have studied Bianchi type-V string cosmological models in different conditions.. Higher dimensional string cosmological models have been studied by Samanta *et al.* [38], Rathore and Mandawat [39], Mohanty and Sahoo [40] and Singh and Singh [41]. String cosmological models in Brans-Dicke theory of gravitation have been studied by Reddy [42] and Vidyasagar *et. al.* [43].

This paper is organized as follows. Section 2 represents field equations of the Brans-Dicke theory of gravitation containing cosmic string. Section 3 contains solution of field equation obtained in section 2, using some conditions. Physical parameters of the model and discussion are given in section 3. In section 4 conclusions are given.

II. Metric and field equations

We consider the five-dimensional LRS Bianchi type-V metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}(dy^2 + dz^2) + C^2\, dm^2 \tag{4}$$

where A, B, C are functions of cosmic time t.

The energy momentum tensor containing cosmic strings is given by Letelier (1983) as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{5}$$

where ρ is the rest energy density of strings with particle fixed up to them, λ be the tension density of strings which may be positive or negative, the vector u_i indicates the cloud four velocity and x_i denotes the direction of string satisfies the relation.

$$u^i u_i = x^i x_i = -1 \;\; and \;\; u^i x_i = 0 \tag{6a}$$

For the comoving coordinate system, (5) leads to

$$T_1^1 = -\lambda, \qquad T_2^2 = T_3^3 = T_5^5 = 0, \qquad T_4^4 = -\rho$$

$$T = -(\rho + \lambda), \;\; T_j^i = 0 \;\; \text{for} \;\; i \neq j \tag{6b}$$

We consider

$$\rho = \rho_p + \lambda$$

Where ρ_p is the rest energy density of the particles. The quantities ρ, λ and ϕ are regarded as the functions of cosmic time t only.

The Brans-Dicke field equations (1), (2) and (3) for the metric (4), with the help of (6a) and (6b), lead to

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}^2}{B^2} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{1}{A^2} = 8\pi\phi^{-1}\lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{1}{A^2} = 0 \tag{8}$$

$$\frac{\dot{A}\dot{C}}{AC}+2\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\frac{3}{A^2}=8\pi\phi^{-1}\rho \tag{9}$$

$$\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}+2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)+\frac{\ddot{\phi}}{\phi}-\frac{3}{A^2}=0 \tag{10}$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \tag{11}$$

$$\ddot{\phi}+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{8\pi(\rho+\lambda)}{\phi(3+2\omega)} \tag{12}$$

Also, the energy conservation equation (3) leads to

$$\dot{\rho}+\rho\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\lambda\frac{\dot{A}}{A}=0 \tag{13}$$

where overhead dot represents differentiation with respect to the cosmic time t.

We define the following physical parameters for solving the above field equations.
The spatial volume V and average scale factor is given by

$$V=a^4(t)=AB^2C \tag{14}$$

The mean Hubble parameter$H$, scalar expansion θ and shear scalar σ^2 are given by

$$H=\frac{\dot{a}}{a}=\frac{1}{4}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{15}$$

$$\theta=4H=\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{16}$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{4}H_i^2-4H^2\right) \tag{17}$$

The mean anisotropy parameter A_h is defined as

$$A_h=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2 \tag{18}$$

where H_i denotes the directional Hubble parameters in x, y, z, m directions.

III. Solution of field equations

By integrating equation (11) we get

$$B = kA, \tag{19}$$

where k is a constant of integration. Without loss of generality we can take k to be unity, therefore we have,

$$B = A, \tag{20}$$

Now, using equation (20), the field equations (7)-(13) lead to the following equations:

$$2\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + 2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}^2}{A^2} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{1}{A^2} = 8\pi\phi^{-1}\lambda, \tag{21}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + 2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}^2}{A^2} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{1}{A^2} = 0, \tag{22}$$

$$3\frac{\dot{A}\dot{C}}{AC} + 3\frac{\dot{A}^2}{A^2} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(3\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) - \frac{3}{A^2} = 8\pi\phi^{-1}\rho, \tag{23}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(3\frac{\dot{A}}{A}\right) + \frac{\ddot{\phi}}{\phi} - \frac{3}{A^2} = 0, \tag{24}$$

$$\ddot{\phi} + \left(3\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) = \frac{8\pi(\rho + \lambda)}{\phi(3 + 2\omega)}. \tag{25}$$

Equations (21)-(25) are a system of five independent equations in five unknowns A,C,ϕ,ρ and λ. Moreover, the above five equations are highly non-linear equations. Because of this, we take the help of following physically plausible conditions to obtain a determinate solution.

(i) We use the condition

$$\boldsymbol{T = -(\rho + \lambda) = 0}, \tag{26}$$

Which, physically represents the trace free energy momentum tensor containing cosmic strings under consideration (Naidu et al. 2013) [44].

(ii) The shear scalar σ^2 is proportional to scalar expansion θ so that we can take (Collins et al. 1980) [45]

$$C = A^n, \tag{27}$$

Where $n \neq 0$ is a constant.

(iii) We obtain the solution using the special law of variation for Hubble's parameter given by Berman (1983) [46] that yields constant deceleration parameter models of the universe defined by

$$q = -a\frac{\ddot{a}}{\dot{a}^2} = constant, \tag{28}$$

Now equation (14) permits the solution

$$a(t) = (AB^2C)^{\frac{1}{4}} = (ct + d)^{\frac{1}{1+q}}, \tag{29}$$

where $c = 0$ and d are constants of integration. This equation gives rise to the fact that the accelerating expansion of the universe should be represented by the condition $1+q>0$.

Now, from equations (20), (27) and (29), we get the solutions for the metric potentials as

$$A=B=(ct+d)^{\frac{8}{(n+3)(1+q)}},\quad C=(ct+d)^{\frac{4n}{(n+3)(1+q)}}. \tag{30}$$

Using Eq.(30) and suitable choice of constants of integration, we can write the metric (4) as

$$ds^2=-dt^2+(ct+d)^{\frac{8}{(n+3)(1+q)}}(dx^2+e^{2x}dy^2+e^{2x}dz^2)+(ct+d)^{\frac{8n}{(n+3)(1+q)}}dm^2. \tag{31}$$

Now, using Eqs. (27) and (30) in Eq. (25) the scalar field can be written as

$$\phi=\phi_0\left(\frac{q+1}{q-3}\right)(ct+d)^{\frac{q-3}{1+q}}+C_0, \tag{32}$$

where ϕ_0 and C_0 are constants of integration.

IV. Physical properties of the model

Equation (31) represents the LRS Bianchi type-V string cosmological model within the framework of Brans-Dicke scalar-tensor theory of gravitation. It is significant to know about the evolution of the universe. The physical and kinematical parameters of the cosmological model are given by

Spatial volume,

$$V^4=(ct+d)^{\frac{4}{1+q}}. \tag{33}$$

The Hubble parameter,

$$H=\frac{c}{(1+q)(ct+d)}. \tag{34}$$

The Scalar expansion,

$$\theta=\frac{4c}{(1+q)(ct+d)}. \tag{35}$$

The shear scalar,

$$\sigma^2=\frac{6c^2(n-1)^2}{(n+3)^2(1+q)^2(ct+d)^2}. \tag{36}$$

The average anisotropy parameter,

$$A_h=\frac{3(n-1)^2}{(n+3)^2}. \tag{37}$$

The energy density,

$$\rho=\frac{\phi_0(1+q)}{8\pi(q-3)}(ct+d)^{\frac{q-3}{1+q}}\left[\frac{(n+1)48c^2+\frac{1}{2}(q-3)(n+2)^2(-3\omega-\omega q+8)}{(n+3)^2(1+q)^2(ct+d)^2}-3(ct+d)^{\frac{-8}{(1+q)(ct+d)}}\right] \tag{38}$$

Tension density in the string,

$$\lambda=0. \tag{39}$$

The Ratio,

$$\frac{\sigma^2}{\theta^2} = \frac{3(n-1)^2}{8(n+3)^2} \neq 0, \quad \text{for } n > 1 \tag{40}$$

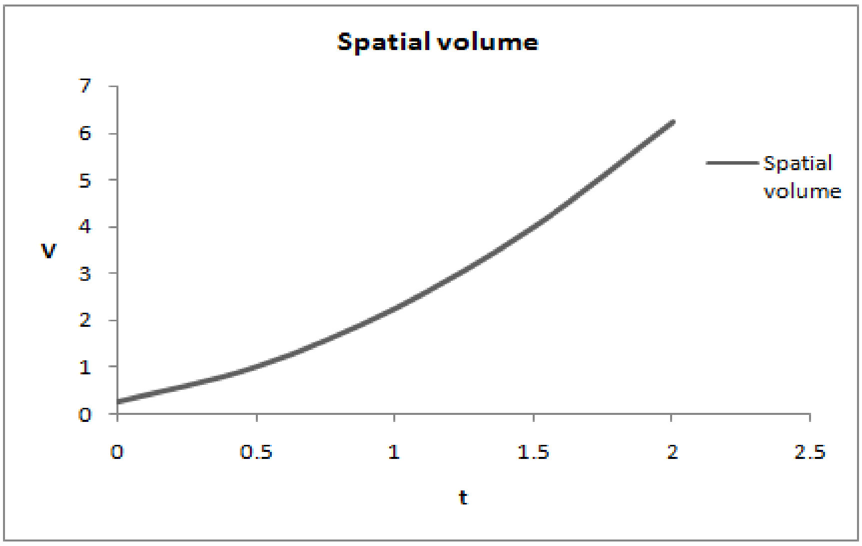

Figure 1: Spatial volume *V* as a function of time *t* with *q* = 1

The above observations can be used in order to study ab- out the behavior of the universe. From Eq. (31), it is observable that model does not admit initial singularity but it has point type singularity at $t = t_0 = -\frac{d}{c}$. Spatial volume tend to zero at singular point whereas it becomes infinitely large when limit $t \to \infty$ ($\because\ 1 + q > 0$) (see Figure (1)). It implies that cosmological model has expanding behavior. The physical parameters Hubble constant (H), scalar expansion(θ), shear scalar (σ^2) and energy density (ρ) tend to infinity at this point type singularity. Also, this parameters approach finite limit as t tends to zero (see figure (2), (3) and (4)). Hence, the beginning of universe happens from initial singularity with infinite rate of Hubble constant, infinite rate of scalar expansion and shear scalar and infinite energy density.

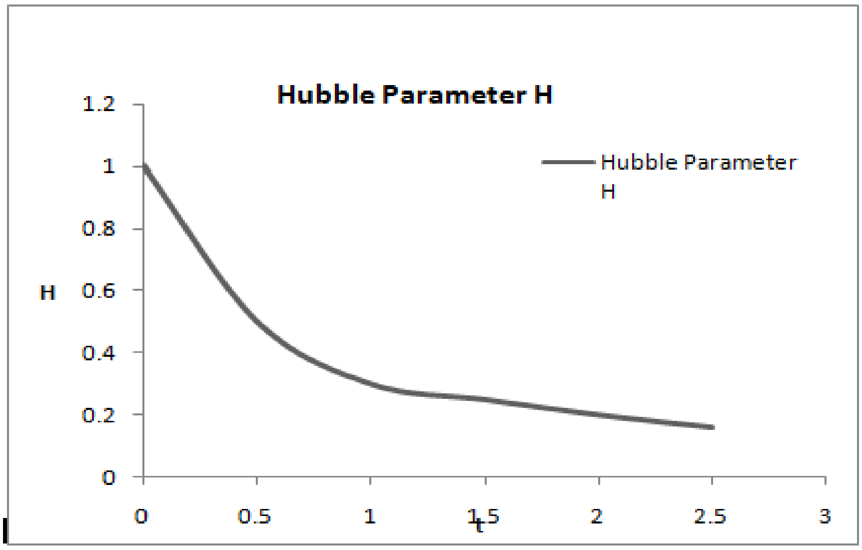

Figure 2: Hubble parameter as a function of time *t* with *q* = 1

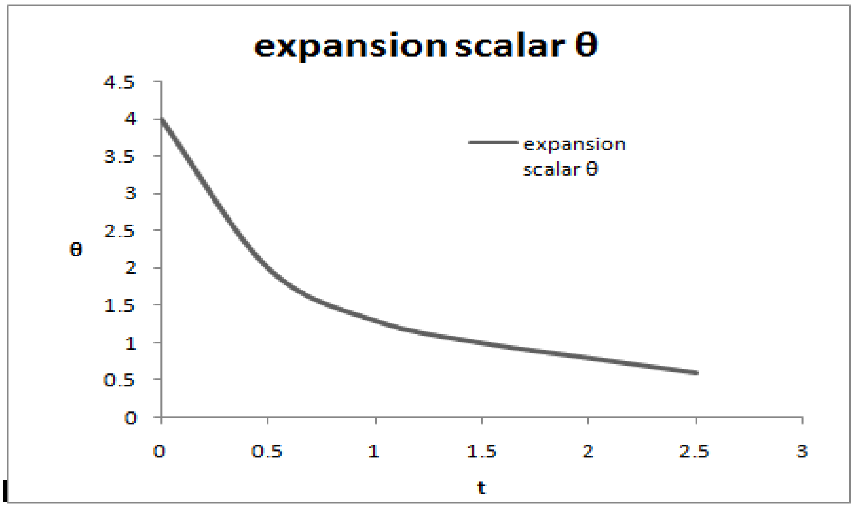

Figure 3: Expansion scalar as a function of time *t* with *q* = 1

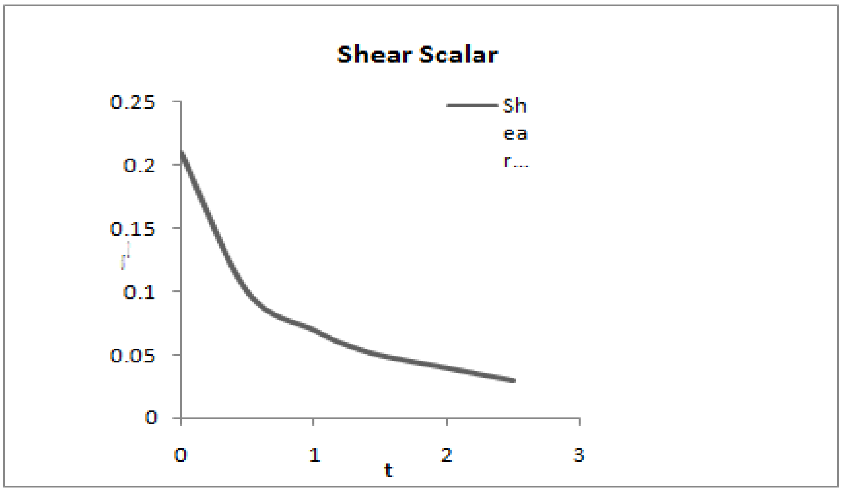

Figure 4: Shear scalar as a function of time *t* with *q* = 1 and *n* = 0.5

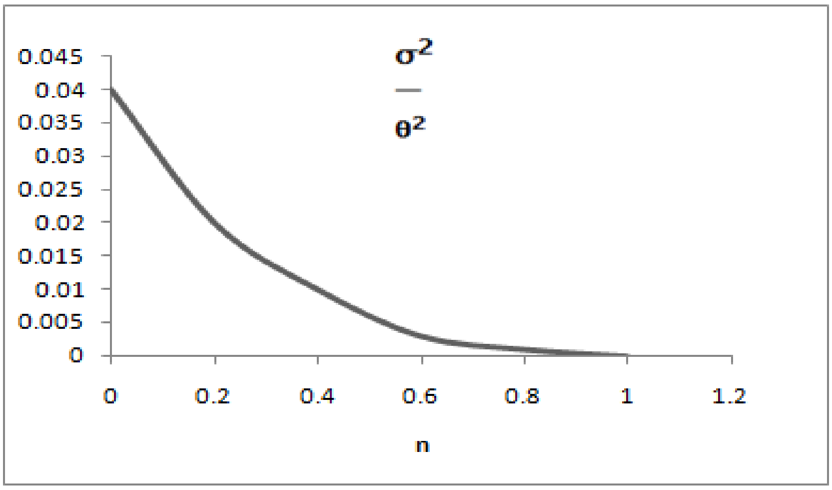

Figure (5) $\frac{\sigma^2}{\theta^2}$ as a function of constant n

Since $\frac{\sigma^2}{\theta^2} \neq 0$ (constant), it emphasizes that the model is highly anisotropic for large values of cosmic time t. Furthermore, at $n = 1$, the universe becomes shear free and isotropic as $\sigma^2 = 0$ and $\frac{\sigma^2}{\theta^2} = 0$ repectively (see figure (5)).

Moreover, we get $\lambda = 0$, hence strings do not survive in the universe. The metric functions A, B, C get vanish at this singular point. The scalar field also vanishes at $t = -\frac{d}{c}$ and it becomes constant as limit $t \to \infty$. The existence of particle horizon is found because,

$$\int_{t_0}^{t} \frac{dt}{v(t)} = \left[\frac{1+q}{q}(ct+d)^{\frac{q}{1+q}}\right]_{t_0}^{t}, \ q \neq 0, \tag{41}$$

it is a convergent integral.

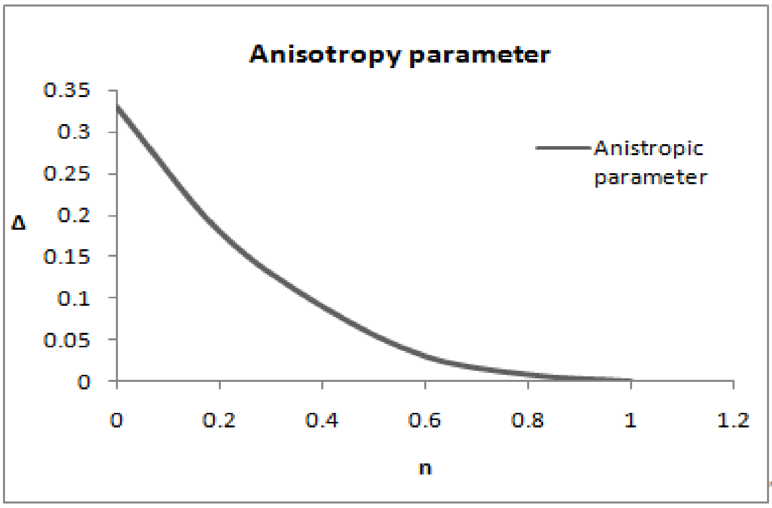

Figure 6: Anisotropic parameter as a function of constant n

Also, the anisotropic parameter A_h becomes zero at $n = 1$, the model represents late time acceleration (see figure (6)).

V. Conclusions

A five dimensional LRS Bianchi type-V string cosmological model has been investigated within the framework of Brans-Dicke scalar-tensor theory of gravitation. Determinate solutions of field equations have been found using trace free energy momentum tensor, Berman's (1983) special law of variation and relation between shear scalar and scalar expansion. The physical and kinematical parameters of the cosmological model have been explained. It is concluded that the model represents expanding, non-rotating and anisotropic nature at the early stage of the universe. The Cosmological parameters of the model diverge at singular point and become zero as t. The average anisotropic parameter and shear scalar seem to have vanished at some point so that model cannot stay anisotropic throughout the progression of universe and become shear free. It is observed that cosmic strings do not exist in this model. The consequences obtained in this cosmological model can be useful for us to give a better explanation of evolution of the universe

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