

An M/M/s Queuing Model for Selected Petroleum Tank Farms in Oghara, Delta State, Nigeria

Tsetimi J.¹ and Orighoyegha, M. G.²

^{1,2}Department of Mathematics, Delta State University, Abraka, Nigeria

Abstract - In this paper we apply the M/M/S queuing model to Rain oil and Nepal tank farms, Oghara Delta State, Nigeria. The queuing patterns of the tank farms were analyzed using an M/M/S queuing model. The average waiting time of trucks in the system and in the queue and utilization rate of the supply system were computed with a view to determining how best the queuing system can be optimized in both tank farms. Comparison was also carried out between both tank farms. Findings revealed that the utilization rate was very high at both tank farms This means that the system is busy most of the time. We considered also, the waiting time of trucks on queue and the time spent in the system, and provided scientific evidence that the excessive amount of time spent in the system and on the queue can be greatly reduced by increasing the number of service channels. The study concluded by re-iterating the relevance of queuing theory to the effective service delivery of the tank farm industry in Nigeria and recommends the expansion of existing service channels as a way of optimizing efficiency and quality service delivery to truck owners.

Keywords — Multiple Servers, Petroleum Tank Farms, Queuing Model, Utilization Rate, Waiting Line

I. INTRODUCTION

The queuing theory or waiting line theory development is often credited to [1], who in his research work responded to the challenges of congestion of telephone traffic which resulted to delay calls. He began his research by concentrating initially on why the resulting delay for one operator and later on the findings and results were further expanded to accommodate more or several operators.

Further developments in queuing theory were applied to processes which customers have to wait in a facility to be served, including processes in commerce, telephone traffic, transportation, industries e.t.c. ([2]; [3]). Even though there are differences in operations in various systems queuing theory highlights their common properties in the queuing process, because the governing laws are similar [4]. Engineering, telecommunication and industries are known to have used queuing theory widely in modeling processes that relates to waiting lines [5].

Queues are formed when customer (humans and objects) demanding for service need to wait because their number exceeds the number of services available, or the inefficient working of available facilities which increase the time prescribed to serve a customer[6]. The study of how waiting lines are formed and managed is called *Queuing Theory*, hence, it is defined as a mathematical modeling and analysis of system that provides service to a random demand which deals with one of the most unpleasant experience of life i.e. waiting [7].

Queuing theory enable managers to determine the optional supply of fixed resources required to meet various demand [8]. It is also used to determine capacity requirements ([9];[5]). Schwarte and Freedman [10] defined Queuing theory as the mathematical study of waiting lines or the act of joining a line. Nosek and Wilson [11] considered Queuing theory as the usage of mathematical approach in analyzing waiting lines within the field of operation management. In Queuing theory, a model is constructed so that queue length and waiting time can be predictable. It is applicable to any wide range of life situations ranging from arrival of car at gas station, customers arriving at a bank for various services and list of trucks waiting to berth at a tank farm etc [12]. Queues are direct consequences of waiting patterns and are formed because of the variable nature of arrival and service patterns which may cause systems and subsystems to be temporarily overloaded [13].

The objective of this study is to determine how best the queuing system can be optimized in Rain oil and Nepal tank farms both in Oghara, Delta State, Nigeria. In order to achieve this, we shall determine the

- i. average time a truck spends in the system and in the queue,
- ii. utilization rate of the supply system, and
- iii. compare the utilization rates of the two tank farms as the number of service channel increases.

This study covers the Rain oil and Nepal tank farms in Oghara, Delta State, Nigeria. With the Oghara tank farm facility as a case study, we shall be using the M/M/S queuing model to resolve the problems of congestion and the long waiting times and service times in tank farm industry in Nigeria



II. DESIGN AND SYSTEM DESCRIPTION

The system is modeled as an M/M/S queuing model. Relevant data was collected from Rain oil and Nepal tank farms. The process was observed and secondary data was collected for a period of one week from each tank farm. Using the M/M/S queuing model. The collected data were used to analyze the congestion at the tank farms, two tank farms with the primary objective of determining the average time a truck spends in the system and in the queue, utilization rate of the supply system, and comparing the utilization rate of both tank farms when the service channels are increased.

The Oghara depot consists of seven petroleum tank farms, two tank farms were randomly selected using simple random sampling. This was achieved practically by a balloting process, in which the names of the tank farms were written down and pick from an urn. In the process Rain oil and Nepal were selected for the study.

The queuing system was virtually the same in both tank farms, where there is more than one service point. There is a single queue with three service channels. These service points or channels are parallel to one another and have equal capacity. Each channel can only serve one truck at a time. An arriving truck normally joins the existing queue. As soon as any service channel becomes free, it accepts the next truck on the queue for service. Figure 1 below shows a multiple channel queue queuing system as observed in the tank farm facility.

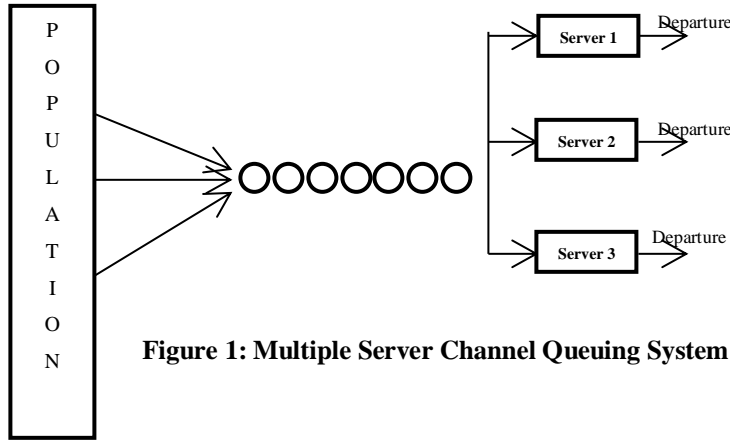


Figure 1: Multiple Server Channel Queuing System

Our case study tank farms have more than one petroleum pump (service channels) which suggests there are multiple servers attending to trucks at any given time. When the trucks arrive at the facility, they form a single line depending on the tank farm they are booked for, wait in the queue if the system is busy, receive service and eventually leave.

III. THE M/M/s MODEL

In the M/M/s Model inter-arrival and service times are independently and identically distributed according to an exponential distribution. The number of servers is indicated s . This model in a case of the arrival and departure process where the mean arrival rate and the mean service rate per busy server are constant in the queuing system. If the system has just a single server ($s = 1$), the parameters for the arrival and departure process are $\lambda_n = \lambda$ and $\mu_n = \mu$, but when the system has multiple server ($S > 1$), μ_n denotes the mean service rate for the overall queuing system when the system has n customers. Where μ is the service rate per busy server, the overall mean service rate for n busy servers must then be $n\mu$, therefore $\mu_n = n\mu$ when $n < s$, and $\mu_n = s\mu$ when $n \geq s$. When the maximum mean service rate $s\mu$ exceeds the mean arrival rate λ , that is when

$$\rho = \frac{\lambda}{s\mu} \quad (1)$$

at this stage the queuing system will eventually attain a steady state condition.

The M/M/s model adapted for this study has the following assumptions:

- (i) The models used in this study assume that customers are patient; they do not balk, renege, or jockey and that customers come from an infinite population.
- (ii) The queue discipline is first come, first served (FCFS) and the permissible length of the queue is infinite.
- (iii) The service time follows an exponential distribution

According to [14] the step by step method of determining probability P_n of n customers in the queuing system at time t and the performance measures is summarized below:

Step 1: Obtain the system of differential-difference equations.

Given,

$$P_n(t + \Delta t) = P_n(t)\{1 - \lambda\Delta t\}\{1 - n\mu\Delta t\} + P_{n+1}(t)\{1 - \lambda\Delta t\}\{(n + 1)\mu\Delta t\} + P_{n-1}(t)\{\lambda\Delta t\}\{1 - (n - 1)\mu\Delta t\} \quad (2)$$

$$= -(\lambda + n\mu)P_n(t)\Delta t + (n + 1)\mu P_{n+1}(t)\Delta t + \lambda P_{n-1}(t)\Delta t + P_n(t) + \text{terms involving } (\Delta t)^2; 1 \leq n < s$$

$$\begin{aligned} P_n(t + \Delta t) &= P_n(t)\{1 - \lambda\Delta t\}\{1 - \mu\Delta t\} + P_{n+1}(t)\{1 - \lambda\Delta t\}\{s\mu\Delta t\} \\ &\quad + P_{n-1}(t)\lambda\Delta t\{1 - s\mu\Delta t\} \\ &= -(\lambda + s\mu) P_n(t)\Delta t + s\mu P_{n+1}(t) + \lambda P_{n-1}(t)\Delta t + P_n(t) \\ &\quad + \text{terms involving } (\Delta t)^2; n \geq s \end{aligned} \tag{3}$$

and

$$P_0(t + \Delta t) = P_0(t)(1 - \lambda\Delta t) + P_1(t)\mu\Delta t; n = 0 \tag{4}$$

By dividing these equations by Δt and taking limit as $\Delta t \rightarrow 0$, we get

$$\begin{cases} P'_n(t) = -(\lambda + n\mu)P_n(t) + (n + 1)\mu P_{n+1}(t) + \lambda P_{n-1}(t), & 1 \leq n < s \\ P'_n(t) = -(\lambda + n\mu)P_n(t) + s\mu P_{n+1}(t) + \lambda P_{n-1}(t), & n \geq s \\ P'_0(t) = -\lambda P_0(t) + \mu P_1(t), & n = 0 \end{cases}$$

Step 2: Obtain the system of steady state equations in the steady state condition, the differential-difference equations obtained from the above equations as $t \rightarrow \infty$, are:

$$\begin{cases} -\lambda P_0 + \mu P_1 = 0, & n = 0 \\ -(\lambda + n\mu)P_n + (n + 1)\mu P_{n+1} + \lambda P_{n-1} = 0, & 0 < n < s \\ -(\lambda - s\mu)P_n + s\mu P_{n+1} + \lambda P_{n-1} = 0, & n \geq s \end{cases}$$

Step 3: Solve the system of difference equations, applying the iterative method, the probability of n customers in the system is given by:

$$\begin{cases} \frac{\rho^n}{n!} P_0, & n \geq s \\ \frac{\rho^n}{s! s^{n-s}} P_0, & n > s; \rho = \frac{\lambda}{s\mu} \end{cases}$$

To find the value of P_0 , we use the following condition:

$$\begin{aligned} 1 &= \sum_{n=0}^{\infty} P_n = \sum_{n=0}^{s-1} P_n + \sum_{n=0}^{\infty} P_n = \sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n P_0 + \sum_{n=s}^{\infty} \frac{1}{s! s^{n-s}} \left(\frac{\lambda}{s\mu}\right)^n P_0 \\ &= P_0 \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{\infty} \frac{s^n}{s! s^{n-s}} \left(\frac{\lambda}{s\mu}\right)^n \right] \\ &= P_0 \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{s}{s!} \sum_{n=0}^{\infty} \rho^n \right] = P_0 \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{s^s}{s!} \frac{\rho}{1 - \rho} \right]; \rho = \frac{\lambda}{s\mu} \end{aligned} \tag{5}$$

[Since $\sum_{n=0}^{\infty} \rho^n = \rho^s = \rho^{s+1} + \dots = \frac{\rho^s}{(1-\rho)}$, sum of infinite G.P.; $P < 1$].

Thus, the probability that the system shall be idle is

$$\begin{aligned} &= P_0 \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{1}{s!} \frac{(s\rho)^s}{1-\rho} \sum_{n=0}^{\infty} \rho^n \right]^{-1}; \rho = \frac{\lambda}{s\mu} \\ &= \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} \right]^{-1} \end{aligned} \tag{6}$$

Where

s = Number of service channels

λ = Mean arrival rate

μ = Mean service rate

ρ = Utilization rate

P_0 = Probability of having no customers/trucks in the system

Others parameters are

L_q = The expected number of customers/trucks waiting in the queue (Length of queue).

$$\begin{aligned} L_q &= \sum_{n=s}^{\infty} (n - s)P_n = \sum_{n=s}^{\infty} (n - s) \frac{\rho^n}{s^{n-s} s!} P_0 \\ &= \frac{\rho^s P_0}{s!} \sum_{n=s}^{\infty} (n - s) \rho^{n-s} = \frac{\rho^s P_0}{s!} \sum_{m=0}^{\infty} m \rho^m; n - s = m, \rho = \frac{\lambda}{\mu} \end{aligned} \tag{7}$$

$$\begin{aligned}
 &= \frac{\rho^s}{s!} \cdot p P_0 \sum_{m=0}^{\infty} m p^{m-1} = \frac{\rho^s}{s!} \cdot p P_0 \frac{d}{dp} \left[\sum_{m=1}^{\infty} \rho^m \right] \\
 &= \left[\frac{1}{(s-1)!} \left(\frac{\lambda}{\mu} \right)^s \frac{\lambda \mu}{(s\mu - \lambda)^2} \right] P_0
 \end{aligned}$$

L_s = the expected number of customers/trucks in the system.

$$L_s = L_q + \frac{\lambda}{\mu} \tag{8}$$

W_q = the expected waiting time of a customer/truck in the queue.

$$W_q = \left[\frac{1}{(s-1)!} \left(\frac{\lambda}{\mu} \right)^s \frac{\mu}{(s\mu - \lambda)^2} \right] P_0 = \frac{L_q}{\lambda} \tag{9}$$

W_s = the expected waiting time that a customer/truck spends in the system.

$$W_s = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu} \tag{10}$$

$P(n \geq s)$ = the probability that all servers are simultaneously busy (utilization factor)

$$\begin{aligned}
 P(n \geq s) &= \sum_{n=s}^{\infty} P_n = \sum_{n=s}^{\infty} \frac{1}{s!} \frac{\lambda^n}{s^{n-s}} \left(\frac{\lambda}{\mu} \right)^s P_0 \\
 &= \frac{1}{s!} \left(\frac{\lambda}{\mu} \right)^s P_0 \sum_{m=0}^{\infty} \frac{\lambda^m}{\mu^m} \\
 &= \frac{1}{s!} \left(\frac{\lambda}{\mu} \right)^s \frac{s\mu}{s\mu - \lambda} P_0
 \end{aligned} \tag{11}$$

$P(n < s)$ = the probability that a customer/truck enters the system without waiting.

$$P(n < s) = 1 - P(n \geq s). \tag{12}$$

IV. APPLICATION OF MODEL

A. Truck Arrival and Departure

In this section, we present the observed truck arrival and departure patterns for the two tanks farms together with the calculations for the relevant parameters, The model parameters discussed in section 3 are calculated for the two different tank farms. Table 1 presents the observed arrival pattern for Nepal tank farm, while Table 2 shows the arrival pattern for the same tank farm for a five-day period. The respective arrival and departure patterns for Rain Oil tanks farm are shown in Tables 3 and 4.

Table 1: Arrival pattern of trucks at Nepal tank farm

TIME	DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	TOTAL	ARR./ HOUR	ARR./ MIN.
8am – 9am	8	6	4	10	6	34	6.80	0.11
9am – 10am	12	6	6	8	9	41	8.20	0.14
10am – 11am	10	10	4	6	12	42	8.40	0.14
11am – 12pm	6	8	6	4	9	33	6.60	0.11
12pm – 1pm	7	6	10	6	6	35	7.00	0.12
1pm – 2pm	6	8	8	8	12	42	8.40	0.14
2pm – 3pm	6	4	4	6	9	29	5.80	0.10
3pm – 4pm	10	7	6	6	6	35	7.00	0.12
TOTAL	65	55	48	54	69	291	58.20	0.97

Table 2: Departure pattern of trucks at Nepal tank farm

TIME	DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	TOTAL	ARR./ HOUR	ARR./ MIN.
8am – 9am	3	2	3	2	3	13	2.60	0.043
9am – 10am	3	2	3	3	3	14	2.80	0.046
10am – 11am	3	2	3	2	3	13	2.60	0.043
11am – 12pm	2	2	2	2	3	11	2.20	0.036
12pm – 1pm	2	2	3	3	2	12	2.40	0.040
1pm – 2pm	2	2	2	2	3	11	2.20	0.036
2pm – 3pm	3	2	2	2	3	12	2.40	0.040
3pm – 4pm	3	2	2	2	3	12	2.40	0.040
TOTAL	21	16	20	18	23	98	19.60	0.326

Table 3 Arrival pattern of trucks at Rain Oil tank farm

TIME	DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	TOTAL	ARR./ HOUR	ARR./ MIN.
8am – 9am	10	12	12	10	10	54	10.80	0.18
9am – 10am	8	10	9	9	8	44	8.80	0.15
10am – 11am	6	8	11	5	3	33	6.60	0.11
11am – 12pm	12	12	9	6	5	42	8.40	0.14
12pm – 1pm	9	10	7	3	11	40	8.00	0.13
1pm – 2pm	8	6	12	2	10	38	7.60	0.13
2pm – 3pm	10	8	12	9	12	51	10.0	0.17
3pm – 4pm	14	10	10	8	8	50	10.0	0.17
TOTAL	77	76	82	52	65	352	70.40	1.17

Table 4: Departure pattern of trucks at Rain Oil tank farm

TIME	DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	TOTAL	ARR./ HOUR	ARR./ MIN.
8am – 9am	3	3	3	3	3	15	3.0	0.050
9am – 10am	3	4	2	3	4	16	3.2	0.053
10am – 11am	3	3	3	3	2	14	2.8	0.046
11am – 12pm	3	4	3	2	3	15	3.0	0.050
12pm – 1pm	3	4	3	2	3	15	3.0	0.050
1pm – 2pm	3	2	3	3	3	14	2.8	0.046
2pm – 3pm	3	3	3	3	3	15	3.0	0.050
3pm – 4pm	3	2	4	2	3	14	2.8	0.046
TOTAL	24	25	24	21	24	118	23.6	0.393

B. Traffic Intensity and Measures of Effectiveness

The following are the formula for measuring traffic intensity and effectiveness in the tank farms (Rain Oil and Nepal).

(i) Probability of having no unit in the system (P_0)

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} \right]^{-1}$$

$\rho = \frac{\lambda}{s\mu}$ = Utilization rate

S = number of service channels

λ = mean arrival rate

μ =mean service rate

(ii) The expected number of unit on queue(L_q)

$$L_q = \left[\frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu-\lambda)^2} \right] P_0 \tag{14}$$

(iii) The expected number of unit in the system (L_s)

$$L_s = L_q + \frac{\lambda}{\mu} \tag{15}$$

Whereas also,

$$L_q = \left[\frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu-\lambda)^2} \right] P_0$$

(iv) The average waiting time on queue (W_q)

$$W_q = \frac{L_q}{\lambda} \tag{17}$$

(v) The average waiting time in the system (W_s)

$$W_s = \frac{L_q}{\lambda} + \frac{1}{\mu} \tag{18}$$

(vi) The probability that the truck has to wait($\rho(n \geq s)$)

$$\rho(n \geq s) = \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu-\lambda} P_0 \tag{19}$$

C. Calculated Parameters

Based on available data the calculated parameters are as follows:

(i) Rain Oil

Mean arrival rate $\lambda = \frac{1.17}{8} = 0.14625/min$

Mean service rate $\mu = \frac{0.393333}{8} = 0.04916/min$

Number of channel $S = 3$

(ii) Nepal

Mean arrival rate $\lambda = \frac{0.97}{8} = 0.12125/min$

Mean service rate $\mu = \frac{0.32666}{8} = 0.04083/min$

Number of channel $S = 3$

The expected figures for the various parameters when the service channels are increased are shown in Tables 5 and 6. Table 6 shows the values for Rain Oil while those for Nepal are shown in Table 6.

Table 5: Calculated Parameters for RAIN Oil Tank Farm

s	λ	μ	ρ	P_0	$P(n \geq s)$	$P(n < s)$	L_s	L_q	W_s	W_q
3	0.14625	0.04916	0.00165	0.00187	0.98428	0.01571	120.00899	117.03401	820.57434	800.23260
4	0.14625	0.04916	0.74374	0.03917	0.49898	0.50101	4.42320	1.44822	30.24412	9.90238
5	0.14625	0.04916	0.59499	0.04796	0.23000	0.76999	3.31287	0.33789	22.65214	2.31040
6	0.14625	0.04916	0.49582	0.05024	0.09595	0.90404	3.06934	0.09436	20.98696	0.64522
7	0.14625	0.04916	0.42499	0.05084	0.03618	0.9638	3.00172	0.02674	20.52460	0.18286
8	0.14625	0.04916	0.37187	0.05099	0.01235	0.98764	2.98229	0.00731	20.39175	0.05001

Table 6: Calculated Parameters for Nepal Tank Farm

s	λ	μ	ρ	P_0	$P(n \geq s)$	$P(n < s)$	L_s	L_q	W_s	W_q
3	0.12125	0.04083	0.98987	0.00227	0.98093	0.01906	98.8873	98.91820	815.56980	791.0780
4	0.12125	0.04083	0.74240	0.03948	0.49675	0.50324	4.40133	1.43170	36.29970	11.80791
5	0.12125	0.04083	0.59392	0.04825	0.22869	0.77130	3.30412	0.33449	27.25051	2.75871
6	0.12125	0.04083	0.49493	0.05051	0.09527	0.90472	3.06299	0.09336	25.26183	0.77004
7	0.12125	0.04083	0.42423	0.05116	0.03587	0.96412	2.99606	0.02643	24.70980	0.21801
8	0.12125	0.04083	0.37120	0.05127	0.01223	0.98776	2.97685	0.00722	24.55134	0.05955

V. DISCUSSION OF RESULTS

From Table 5, it can be seen that when the number of service channel (s) is 3, the utilization rate (ρ) for the Rain Oil system is about 99%, meaning at every point in time the system will be busy. This can be further justified by the probability that a customer has to wait ($P(n \geq s)$) which is 0.98. The probability of having no customer in the system ($P_0 = 0.001$), probability that a customer enters the system without waiting ($P(n < s) = 0.015$), expected number of customers in the system ($L_s = 120$), expected number of customers on queue ($L_q = 117$), expected number of minutes spent waiting in the system ($W_s = 820.57$); approximately 14 hours, and expected number of minutes spent waiting on the queue ($W_q = 800mins \equiv 13hours$).

We can also see from Table 6 that when the number of service channel (s) is 3, the utilization rate ρ for the NEPAL system is about 98.9%, meaning at every point in time the system will be busy. This can be further justified by the probability that a customer has to wait ($P(n \geq s)$) which is 0.98. The probability of having no customer in the system ($P_0 = 0.002$), probability that a customer enters the system without waiting ($P(n > s) = 0.019$), expected number of customers in the system ($L_s = 99$), expected number of customers on queue ($L_q = 96$), expected number of minutes spent waiting in the system ($W_s = 815.56mins \equiv 13.5hours$), and expected number of minutes spent waiting on the queue ($W_q = 791.07mins \equiv 13hours$). From the analysis carried out, the two systems are overloaded, but an increment in the number of service channels will greatly reduce the demand on the machines, reduce customer waiting time both in the system and on the queue, and improve the system efficiency.

Also, from Tables 5 and 6, with just an increment of the current number of service channels by 1 (i.e. 4) the average waiting time of trucks in the system and on queue reduced to 30 minutes and 10 minutes approximately for Rain oil tank farm while still maintaining a high utilization of 74.37%. Also, the average waiting time of trucks in the system and on queue reduced to

36 minutes and 12 minutes approximately for Nepal tank farm while still maintaining a high utilization of 74.24%. There is an observed improvement in the performance measures as the number of service channels increases. When the number of service channels is 8, we obtain the respective values $L_q = 0.00731$, $W_q = 0.05001$ for Rain Oil tank farm and $L_q = 0.00722$, $W_q = 0.05955$ for NEPAL tank farm.

VI. CONCLUSION

In this endeavor to apply Queuing Theory to the tanks farm industry in Nigeria we used the M/M/s queuing model. We analyzed the congestion as the two selected tank farms by considering the average time trucks spend in the system and on the queue, the utilization rates together with the waiting times of the trucks both in on the queue and in the system. The performance measures show that there is serious congestion in the present system with these high utilization rates

This study has shown that Queuing Theory can be applied to the operations of the tank farm industry in Nigeria with very good results. Its application using M/M/s queuing model is most relevant to reduce the long queues and congestion in many tank farm facilities. Excessive queues and time wasting in the tank farms would be significantly reduced in petroleum tanks farms in Negeria if the service facilities are given attention. Based on the presented scientific evidence, this study recommends an expansion of the service channels in the tank farms.

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