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A Comparative Study of Pointwise Convergence and Uniform Convergence

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Abstract *- The objective of the present paper is to compare the pointwise and uniform convergence of a sequence of functions.*

Keywords - *Functions, Convergence, Divergence, Sequence of functions.*

I. INTRODUCTION

It is well known that in mathematics, pointwise convergence is one of the various senses in which a sequence of functions can converge to a particular function; it is weaker than uniform convergence to which it is often compared. In the mathematical field of analysis uniform convergence is a mode of convergence of functions stronger than pointwise convergence. The term uniform convergence was probably first used by Christoph Gudermann, in an 1838 paper on elliptical functions where he employed the phrase "Convergence in a uniform way", when the "mode of convergence" of a series $\sum_{n=1}^{\infty} f\, n(x, \phi, \psi)$ is independent of the variables ϕ and ψ, while he thought it was a remarkable fact when a series converges in this way. He did not give a formal definition. Latter Karl Weierstrass coined the term Konvergent which he used in his paper published in 1894. Gorge Stokes and G.H. Hardy compare the three definitions in their paper "Sir Gorge Stokes and the concept of uniform convergence." Now we discuss the difference between pointwise and uniform convergence.

II. POINTWISE AND UNIFORM CONVERGENCE

We consider the sequence whose terms are functions (real or complex valued) defined on any set A. Let A be any set, consider a function f_n: A→ (R or C), where C is the set of Complex numbers. $\{f_n\}$ is a sequence of functions on A. Now for each x∈A, such a sequence gives another sequence $\{f_n (x)\}$ of complex or real numbers. This sequence may converge for certain values of x∈A and may diverge for other values of x∈A. Suppose that the sequence $\{f_n (x)\}$ converges for each x∈A. Then $\lim_{n\to\infty} fn (x)$ exists for each x∈A. So $f(x) = \lim_{n\to\infty} fn (x)$ for each x∈A, when such a function f (whose domain is A) exists, we say that the sequence $\{f_n\}$ converges pointwise on A and this f is called the pointwise limit of the sequence $\{f_n\}$ of functions. An equivalent definition of pointwise convergence of $\{f_n\}$ is as follows: $\{f_n\}$ converges pointwise to f on A ⟺for every $\varepsilon >$ o and for each x∈A ∃ an integer N depends upon ε and x s.t. $|f_n (x) – f(x)| \leq \varepsilon \ \forall n \geq N$

In this definition, the value of N, in general depends upon on both ε (any positive number) and x∈A.
In general, the pointwise convergence is not enough to transform any of the properties such as continuity, differentiability or integrability of the individual terms f_n to the limit function f. Now we need the stronger method of convergence that preserves these properties and here we need uniform convergence.
A sequence of functions $\{f_n\}$ is said to converge uniformly to a function f on a set A, if for every ε>0, there exists an integer N depends upon ∈ (not on x) s.t $|f_n (x) – f(x)| \leq \varepsilon \ \forall$ x∈A and write $f_n\to$ f uniformly on A where f is the limit of the sequence $\{f_n\}$

Hear we easily seen that the uniform converge ⇒pointwise convergence but not vice-versa, so every pointwise convergence need not be uniformly convergent. Uniform convergence assures the interchange of limits. A sequence may converge to a continuous limit function, although the convergence is not uniform. For uniform convergence the Cauchy criterion play very important role for theorems. A Sequence of functions $\{f_n\}$ defined on a set A converges uniformly on A if and only if for every ∈ > 0 there is an integer N s.t. $|f_n (x) – f_m(x)| \leq \in \ \forall$ n,m≥N, $\forall x \in A$

III. CONCLUSION

Uniform convergence is stronger than pointwise convergence. By uniform convergence, we easily satisfy the continuity, differentiability and integrability properties in a simple way. Uniform convergence always implies pointwise convergence but does not guarantee uniform convergence.

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