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On Radio Geometric Mean D- Distance Number of Some Basic Graphs

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Abstract

A Radio geometric mean D-distance labeling of a connect graph G is an injective function f from the vertex set $V(G)$ to the $\mathbb{N}$ such that for two distinct vertices u and v of G, $d^D(u,v) + \left\lceil\sqrt{f(u)f(v)}\right\rceil \geq 1 + diam^D(G)$, where $d^D(u,v)$ denotes the D-distance between u and v $daim^D(G)$ denotes the D-diameter of G. The radio geometric mean D-distance number of f, $rgmn^D(f)$ is the maximum label assigned to any vertex of G . The radio geometric mean D-distance number of, $rgmn^D(G)$is the minimum value of G, $rgmn^D(G)$ is the minimum value of $rgmn^D(f)$ taken over all radio geometric mean D-distance labeling f of G. In this paper we find the radio geometric mean D-distance number of some basic graphs.

Keywords: *D-distance, Radio geometric mean D-distance, Radio geometric mean D-distance number.*

I. INTRODUCTION

By a graph $G = (V(G), E(G))$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Radio labeling (multi-level distance labeling) can be regarded as an extension of distance two labeling which is motivated by the channel assignment problem introduced by Hale [3]. Chartrand et al [2] introduced the concept of radio labeling of graph. We are introduce the concept of radio geometric mean $Dd-distance$ number of some basic graphs[6]

The concept of D-distance was introduced by D. Reddy Babu et.al [11]. For a connected graph G, the D-length of a connected $u - v$ path is defined as
$l^D(s) = l(s)$+ $deg(v) + deg(u) + \sum \deg(w)$ where the sum runs over all intermediate vertices w of s and $l(s)$ is length of the path. The $D - distance$, $d^D(u,v)$ between u and v of connected graph G is defined a$d^D(u,v) = min\{l^D(s)\}$ where the minimum is taken over all u-v paths s in G. In other words, $d^D(u,v) = min\{l(s) + deg(v) + deg(u) + \sum \deg(w))\}$ where the sum runs over all intermediate vertices w in s and minimum is taken over all u-v paths s in G. The lower bound is clear from the definition and the upper bound follows from the triangular inequality.

In [9], Radio geometric mean labeling was introduced by V. Hemalatha et al. A radio geometric mean labeling is a one to one mapping f from $V(G)$ to $\mathbb{N}$ satisfying the condition
$d(u,v) + \left\lceil\sqrt{f(u)f(v)}\right\rceil \geq 1 + diam(G)$, for every $u, v \in G$. The span of labeling f is the maximum integer that f maps to a vertex of G. The radio geometric mean number of G, $rgmn(G)$ is the lowest span taken over all radio geometric mean labeling of the graph G. The above condition is called radio geometric mean condition.

Further we are introduced the concept of radio geometric mean D-distance. The radio geometric labeling is a function $f: V(G) \to \mathbb{N}$ such that $d^D(u,v) + \left\lceil\sqrt{f(u)f(v)}\right\rceil \geq 1 + diam^D(G)$. It is denoted by $rgmn^D(G)$. where $rgmn^D(G)$ is called the radio geometric mean D- distance number. The radio geometric mean D-distance number of f, $rgmn^D(f)$ is the maximum label assigned to any vertex of G . The radio geometric mean D-distance number of, $rgmn^D(G)$is the minimum value of G, $rgmn^D(G)$ is the minimum value of $rgmn^D(f)$ taken over all radio geometric mean D-distance labeling f of G. In this paper we find the radio geometric mean D-distance number of some basic graphs

II. MAIN RESULTES

Theorem2.1

The Radio geometric mean D-distance number of a complete graph $K_n, rgmn^D\ (K_n)\ = n$.

Proof.

Let $V(K_n) = \{v_1\ , v_2\ , v_3, v_4, \dots, v_n\}$ be the vertex set and $E(K_n) = \{v_i\ v_j, 1 \le i, j \le n, i \ne j\}$ be the edge set. Its $diam^D(K_n) = 2n - 1$. we define the vertex label $f\ (v_i) =\ i, 1\ \le i \le n$.

The radio geometric mean D-distance condition is

$d^D(u, v) + \lceil\sqrt{f(u)f(v)}\,\rceil\ \ge 1 + diam^D(G)$,

$Compute$ the $pair\ (v_i, v_j) = 2n - 1, 1 \le i, j \le n, i \ne j\ are\ adjacent$

$$d^D(v_i, v_j) + \left\lceil\sqrt{f(v_i)f(v_j)}\right\rceil \ge 1 + diam^D(K_n),$$

$$\lceil\sqrt{ij}\,\rceil\ \ge 1$$

$Hence, rgmn^D\ (K_n)\ =\ \ n.$

Theorem 2.2

The Radio geometric mean D-distance number of a star graph $K_{1,n}$,

$rgmn^D\ (K_{1,n}) =\ \ n + 2.$

Proof.

Let $V(K_{1,n}) =\ \{v_0\ , v_1\ , v_2, v_3, \dots, v_n\}$ be the vertex set, where v_0 is the central vertex and $E(K_{1,n})\ =\ \{v_0 v_i, 1 \le i \le n\}$ be the edge set. Its $diam^D(K_{1,n}) = n + 4$. we define the vertex label $f(v_0) = 2, f(v_i) = i + 2, 1 \le i \le n, f(v_j) = j + 2, 1 \le j \le n.$

By the radio geometric mean D-distance condition is

$$d^D(u, v) + \left\lceil\sqrt{f(u)f(v)}\right\rceil \ge 1 + diam^D(G),$$

For every pair of vertices$(u, v)$where $u \ne v$.

$\boldsymbol{Case(i)}: Compute$ the $pair\ (v_0, v_i) = n + 2{,}1 \le i \le n\ are\ adjacent$

$$d^D(v_0, v_i) + \left\lceil\sqrt{f(v_0)f(v_i)}\right\rceil \ge 1 + diam^D(K_{1,n}),$$

$$\lceil\sqrt{(2)(i + 2)}\,\rceil\ \ge 3$$

$\boldsymbol{Case(ii)}$: $Compute$ the $pair\ (v_i, v_j) = n + 4{,}1 \le i, j \le n\ , i \ne j\ are\ non\ adjacent$

$$d^D(v_i, v_j) + \left\lceil\sqrt{f(v_i)f(v_j)}\right\rceil \ge 1 + diam^D(K_{1,n}),$$

$$\lceil\sqrt{\ (i + 2)(j + 2)}\rceil\ \ge 1$$

Hence, $rgmn^D\ (K_{1,n}) =\ \ n + 2.$

Theorem 2.3

The Radio geometric mean D-distance number of a Fan graph F_n,

$rgmn^D\ (F_n) = 2n-2, n \geq 6.$

Proof.

Let $V(F_n) = \{v_0, v_1, v_2, v_3, \dots, v_n\}$ be the vertex set and $E(F_n) = \{v_0 v_j, v_i v_{i+1}, 1 \leq i \leq n-1, 1 \leq j \leq n\}$ be the edge set. Its $diam^D(F_n) = n+6$. we define the vertex label $f(v_0) = 2, f(v_i) = n+i-2, 1 \leq i \leq n.$

By the radio geometric mean D-distance condition is

$$d^D(u,v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 1 + diam^D(G),$$

For every pair of vertices$(u,v)$where $u \neq v$.

$\boldsymbol{Case(i)}$: $Compute$ the $pair\ (v_0, v_i) = n+3\ are\ adjacent. If\ v_i\ is\ end\ vertices,$

$$d^D(v_0, v_i) + \left\lceil \sqrt{f(v_0)f(v_i)} \right\rceil \geq 1 + diam^D(F_n),$$

$$\left\lceil \sqrt{(2)(n+i-2)} \right\rceil \geq 4$$

$\boldsymbol{Case(ii)}$: $Compute$ the $pair\ (v_0, v_i) = n+4\ are\ adjacent. If\ v_i\ is\ intermediate\ vertices,$

$$d^D(v_0, v_i) + \left\lceil \sqrt{f(v_0)f(v_i)} \right\rceil \geq 1 + diam^D(F_n),$$

$$\left\lceil \sqrt{(2)(n+i-2)} \right\rceil \geq 3$$

$\boldsymbol{Case(iii)}$: $Compute$ the $pair\ (v_i, v_j) = n+6, are$ both end vertices$1 \leq i \leq n, i+1 \leq j \leq n,$

$$d^D(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \geq 1 + diam^D(F_n),$$

$$\left\lceil \sqrt{(n+i-2)(n+j-2)} \right\rceil \geq 1$$

$\boldsymbol{Case(iv)}$: $Compute$ the $pair\ (v_i, v_j) = 7, intermediate\ adjacent\ vertices$

$$d^D(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \geq 1 + diam^D(F_n),$$

$$\left\lceil \sqrt{(n+i-2)(n+j-2)} \right\rceil \geq n$$

Hence, $rgmn^D\ (F_n) = 2n-2, n \geq 6.$

Note: $rgmn^D\ (F_n) = n+4\ if\ n = 1,2,3,4,5.$

Theorem 2.4

The Radio geometric mean D-distance number of a Double Fan graph DF_n,

$rgmn^D\ (DF_n) = 2n, n \geq 5.$

Proof.

Let $V(DF_n) = \{ v_1 , v_2, v_3, \dots , v_n, w, u\}$ be the vertex set and let $v_1 , v_2, v_3, \dots , v_n$ be the path graph and u, w are two vertex are joined to the end vertex of the path graph. Its $diam^D(DF_n) = 2n + 5$. we define the vertex label as

$f(u) = 9, f(v_i) = n + i, 1 \leq i \leq n. f(w) = 10, f(v_j) = n + j, 1 \leq j \leq n.$

By the radio geometric mean D-distance condition is

$$d^D(u, v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 1 + diam^D(G),$$

For every pair of vertices$(u, v)$where $u \neq v$.

$\boldsymbol{Case(i)}$: $Compute$ the $pair$ $(u, v_i) = n + 4, i = 1, n$

$$d^D(u, v_i) + \left\lceil \sqrt{f(u)f(v_i)} \right\rceil \geq 1 + diam^D(DF_n)$$

$$\left\lceil \sqrt{(9)(n + i)} \right\rceil \geq n + 2$$

$\boldsymbol{Case(ii)}$: $Compute$ the $pair$ $(w, v_i) = n + 4, i = 1, n$

$$d^D(w, v_i) + \left\lceil \sqrt{f(w)f(v_i)} \right\rceil \geq 1 + diam^D(DF_n)$$

$$\left\lceil \sqrt{(10)(n + i)} \right\rceil \geq n + 2$$

$\boldsymbol{Case(iii)}$: $Compute$ the $pair$ $(v_i, v_j) = n + 8$, are both end vertices

$$d^D(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \geq 1 + diam^D(DF_n)$$

$$\left\lceil \sqrt{(n + i)(n + j)} \right\rceil \geq n - 2$$

$\boldsymbol{Case(iv)}$: $Compute$ the $pair$ $(u, w) = 2n + 5, are$ both end vertices

$$d^D(u, w) + \left\lceil \sqrt{f(u)f(w)} \right\rceil \geq 1 + diam^D(DF_n),$$

$$\left\lceil \sqrt{(9)(10)} \right\rceil \geq 1$$

$\boldsymbol{Case(v)}$: $Compute$ the $pair$ $(v_i, v_j) = 9$ both are intermediate adjacent vertices

$$d^D(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \geq 1 + diam^D(DF_n),$$

$$\left\lceil \sqrt{(n + i)(n + j)} \right\rceil \geq 2n - 3$$

$\boldsymbol{Case(vi)}$: $Compute$ the $pair$ $(u, v_i) = n + 5, 2 \leq i \leq n - 1$

$$d^D(u, v_i) + \left\lceil \sqrt{f(u)f(v_i)} \right\rceil \geq 1 + diam^D(DF_n),$$

$$\left\lceil \sqrt{(9)(n + i)} \right\rceil \geq n + 1$$

Hence, $rgmn^D (DF_n) = 2n, n \geq 5$.

Note: $rgmn^D (DF_n) = 2n - 2$ if $n = 1,2,3,4$.

Theorem 2.5

The Radio geometric mean D-distance number of a Subdivision of a star graph $S(K_{1,n})$,

$rgmn^D\left(S(K_{1,n})\right) = 3n+4, n \geq 3.$

Proof.

Let $V\left(S(K_{1,n})\right) = \{v_0, v_1, v_2, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_n\}$ be the vertex set, where v_0 is the central vertex and $E\left(S(K_{1,n})\right) = \{v_0 v_i, v_i u_i 1 \leq i \leq n\}$ be the edge set. Its $diam^D(S(K_{1,n})) = n+10$. we define the vertex label $f(v_0) = 7, f(v_i) = n+2i+3, 1 \leq i \leq n, f(u_i) = n+2i+4, 1 \leq i \leq n$.

By the radio geometric mean D-distance condition is

$$d^D(u,v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 1 + diam^D(G),$$

For every pair of vertices$(u,v)$where $u \neq v$.

$\boldsymbol{Case(i)}: Compute$ the $pair\ (v_0, v_i) = n+3, 1 \leq i \leq n\ are\ adjacent$

$$d^D(v_0, v_i) + \left\lceil \sqrt{f(v_0)f(v_i)} \right\rceil \geq 1 + diam^D(S(K_{1,n})),$$

$$\left\lceil \sqrt{(7)(n+2i+3)} \right\rceil \geq 8$$

$\boldsymbol{Case(ii)}: Compute$ the $pair\ (v_0, u_i) = n+5, 1 \leq i \leq n\ are\ non\ adjacent$

$$d^D(v_0, u_i) + \left\lceil \sqrt{f(v_0)f(u_i)} \right\rceil \geq 1 + diam^D(S(K_{1,n})),$$

$$\left\lceil \sqrt{(7)(n+2i+4)} \right\rceil \geq 6$$

$\boldsymbol{Case(iii)}: Compute$ the $pair\ (v_i, v_j) = n+6, 1 \leq i,j \leq n, i \neq j\ are\ non\ adjacent$

$$d^D(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \geq 1 + diam^D\left(S(K_{1,n})\right),$$

$$\left\lceil \sqrt{(n+2i+3)(n+2j+3)} \right\rceil \geq 5$$

$\boldsymbol{Case(iv)}: Compute$ the $pair\ (v_i, u_j) = 4, if\ |i-j| = 1\ are\ non\ adjacent$

$$d^D(v_i, u_j) + \left\lceil \sqrt{f(v_i)f(u_j)} \right\rceil \geq 1 + diam^D\left(S(K_{1,n})\right),$$

$$\left\lceil \sqrt{(n+2i+3)(n+2j+4)} \right\rceil \geq n+7$$

$\boldsymbol{Case(v)}: Compute$ the $pair\ (u_i, u_j) = n+10, 1 \leq i,j \leq n, i \neq j\ are\ non\ adjacent$

$$d^D(u_i, u_j) + \left\lceil \sqrt{f(u_i)f(u_j)} \right\rceil \geq 1 + diam^D(S(K_{1,n}))$$

$$\left\lceil \sqrt{(n+2i+4)(n+2j+4)} \right\rceil \geq 1$$

$\boldsymbol{Case(vi)}: Compute$ the $pair\ (v_i, u_j) = n+8, if\ |i-j| > 1$

$$d^D(v_i,u_j) + \left\lceil \sqrt{f(v_i)f(u_j)} \right\rceil \geq 1 + diam^D\left(S(K_{1,n})\right)$$

$$\left\lceil \sqrt{(n+2i+3)(n+2j+4)} \right\rceil \geq 3$$

Hence, $rgmn^D\left(S(K_{1,n})\right) = 3n+4, n \geq 3.$

Note: $rgmn^D\left(S(K_{1,n})\right) = 2n+7, if\ n = 1,2.$

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