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Radio Square Difference Dd-Distance Number of Some Basic Graphs

K. John Bosco[1], G. Vishma George[2]

[1]*Assistant Proffessor, Department of Mathematics, St.Jude's college, Thoothoor, Tamil Nadu, India.*
[2]*Research Scholar(Reg.No.20211072092002), Department of Mathematics, St.Jude's college, Thoothoor, Tamil Nadu, India.*

ABSTRACT

A Radio square difference Dd-distance labeling of a connected graph G is an injective function f from the vertex set $V(G)$ to N such that for two distinct vertices u and v of $G, D^{Dd}(u,v) + |[f(u)]^2 - [f(v)]^2| \geq 1 + diam^{Dd}(G)$, where $D^{Dd}(u,v)$ denote the Dd-distance between u and v and also $diam^{Dd}(G)$denotes the Dd-diameter of G.The radio square difference number of $f, rsdn^{Dd}(f)$ is the maximum label assigned to any vertex of G. The radio square difference number of G, $rsdn^{Dd}(G)$ is the maximum value of f of G. In this paper we find the radio square difference number of some basic graph.

I. INTRODUCTION

First introduced the idea of graph theory by Euler. By a graph $G = (V(G), E(G))$ we mean a finite undirected graph without loops or multiple edges. Let $V(G)$ and $E(G)$ denotes the vertex set and edge set of G. The order and size of G are denoted by p and q respectively. In 2001,Chatrant et al.[1] defined the concept of radio labelling of G. Radio labelling of graphs is motivated by restrictions inherent in assigning channel frequencies for radio transmitters.

The Dd-distance was introduced by A. Anto Kinsely and P. Siva Ananthi[8]. For a connected graph G, the Dd-length of a connected $u - v$ is defined as $D^{Dd}(u,v) = D(u,v) + \deg(u) + \deg(v)$. The Dd-radius denoted by $r^{Dd}(G)$ is the minimum Dd-eccentricity among all vertices of u and v of G. That is $r^{Dd}(G) = \min\{e^{Dd}(G): v \in V(G)\}$. Similarly the Dd-diameter $D^{Dd}(G)$ is the maximum D^{Dd} eccentricity among all vertices of G. We observe that for any two vertices u, v of G. We have $d(u,v) \leq D^{Dd}(u,v)$. The equality holds if and only if u and v are identical. If G is any connected graph then the Dd-distance is metric on the set of vertices of G. We can check easily $r^{Dd}(G) \leq D^{Dd}(G) \leq 2r^{Dd}(G)$. The concept of square difference labelling was introduced by J.Shiama in 2012 [7].

The Radio Dd-distance was introduced by K.John Bosco and T.Nicholas in 2017 [9,10]. We introduced the concept of radio square difference Dd-distance labeling of a connected graph G is an injective function f from the vertex set $V(G)$ to N such that for two distinct vertices u and v of $G, D^{Dd}(u,v) + |[f(u)]^2 - [f(v)]^2| \geq 1 + diam^{Dd}(G)$, where $D^{Dd}(u,v)$ denote the Dd-distance between u , v and also $diam^{Dd}(G)$denotes the Dd-diameter of G.The radio square difference number of $f, rsdn^{Dd}(f)$ is the maximum label assigned to any vertex of G[11,12]. The radio square difference number of G, $rsdn^{Dd}(G)$ is the maximum value of f of G

Definition 1.1. The concept of radio square difference Dd-distance coloring is a function $f: V(G) \to N$ such that $D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) + 1$ where $diam^{Dd}(G)$ is the maximum color assigned to any vertex of G. It is denoted by $rsdn^{Dd}(G)$.

II. MAIN RESULT

Theorem 1.2

The radio square difference Dd-distance number of a complete graph K_n, $rsdn^{Dd}(K_n) = n$.

Proof:

Let $\{v_{1,} v_{2,}, \ldots, v_n\}$ be the vertex set. Then $D^{Dd}(v_i, v_j) = 3(n-1), 1 \leq i, j \leq n, i \neq j$,

So $diam^{Dd}(K_n) = 3(n-1)$.

Then radio square difference Dd-distance condition becomes,

$D^{Dd}(v_i, v_j) + \left|f(v_i)^2 - f(v_j)^2\right| \geq diam^{Dd}(K_n) + 1$ for any $v_i, v_j \in V(K_n)$,

Now, $D^{Dd}(v_1, v_2) + |f(v_1)^2 - f(v_2)^2| \geq 3(n-1) + 1$

Therefore, $f(v_i) = i, \; 1 \leq i \leq n$,

Hence, $rsdn^{Dd}(K_n) = n$.

Theorem 1.3

The radio square difference Dd-distance number of a star graph $K_{1,n}$,

$$rsdn^{Dd}(K_{1,n}) = \begin{cases} n+1, if \;\; n < 7 \\ \frac{3}{2}(n-1), \quad if \; n \; is \; odd \;\; n \geq 7 \\ \frac{1}{2}(3n-4), if \;\; n \; is \; even \; n \geq 8 \end{cases}$$

Proof:

Let $V(K_{1,n}) = \{v_0, v_1, v_2, \ldots, v_n\}$ be the vertex set where v_0 is the central vertex and $E(K_{1,n}) = \{v_0 v_i / i = 1,2,3, \ldots, n\}$ be the edge set. $D^{Dd}(v_0, v_i) = n+2$, $D^{Dd}(v_i, v_{i+1}) = 4$, $1 \leq i \leq n$. so $diam^{Dd}(K_{1,n}) = n+2$. By radio square difference Dd-distance condition,

$D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) + 1$, for any pair of vertices (u,v) where $u \neq v$

Now, $D^{Dd}(u,v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(K_{1,n}) + 1$

Case (a) n is odd

For(v_0,v_1), $D^{Dd}(v_0,v_1)+|f(v_0)^2-f(v_1)^2|\geq diam^{Dd}\left(K_{1,n}\right)+1$.

$|f(v_0)^2-f(v_1)^2|\geq 1$, which implies $f(v_0)=1$ and $f(v_1)=\frac{n-1}{2}$

For (v_1,v_2), $D^{Dd}(v_1,v_2)+|f(v_1)^2-f(v_2)^2|\geq diam^{Dd}\left(K_{1,n}\right)+1$

which implies$f(v_1)=2$ and $f(v_2)=\frac{n+1}{2}$,

Therefore, $f(v_i)=\frac{n-1}{2}+i-1$, $1\leq i\leq n$

Case (b) n is even

For (v_0,v_1), $D^{Dd}(v_0,v_1)+|f(v_0)^2-f(v_1)^2|\geq diam^{Dd}\left(K_{1,n}\right)+1$

$|f(v_0)^2-f(v_1)^2|\geq 1$, which implies $f(v_0)=1$ and $f(v_1)=\frac{n}{2}$-1,

For (v_1,v_i), $D^{Dd}(v_i,v_i)+|f(v_1)^2-f(v_i)^2|\geq diam^{Dd}\left(K_{1,n}\right)+1$,

Which implies $f(v_1)=\frac{n}{2}$-1 and $f(v_3)=\frac{n}{2}+1$

Therefore, $f(v_i)=\frac{n}{2}+i-2, 1\leq i\leq n$

Hence $rsdn^{Dd}\left(K_{1,n}\right)=\begin{cases} n+1, if\ n<7 \\ \frac{3}{2}(n-1),\ if\ n\ is\ odd\ n\geq 7 \\ \frac{1}{2}(3n-4), if\ n\ is\ even\ n\geq 8 \end{cases}$

Theorem 1.4.

The radio square difference Dd-distance number of friendship graph $C_3^{(t)}$,

$rsdn^{Dd}(C_3^{(t)})\leq 4t-5$.

Proof:

Let V$(C_3^{(t)})=\{v_0,v_1,\dots,v_t,v_{t+1},\dots,v_{2t}\}$ and E $(C_3^{(t)})=\{v_iv_{t+1},v_0,v_i,1\leq i\leq n\}$

Then $D^{Dd}(v_0,v_1)$ = 2t + 4 , $D^{Dd}(v_1,v_2)$ = 8 , So $diam^{Dd}(C_3^{(t)})$ =2t + 4.

By radio square difference Dd-distance condition,

$D^{Dd}(u,v)+|f(u)^2-f(v)^2|\geq diam^{Dd}(G)+1$,

for any pair of vertices (u,v) where $u\neq v$

Now, $D^{Dd}(v_0, v_1) + |f(v_0)^2 - f(v_1)^2| \geq diam^{Dd}(C_3^{(t)}) +1$,

$|f(v_0)^2 - f(v_1)^2| \geq 1$, which implies $f(v_0)$ = 1 and $f(v_1)$ = 2t – 4.

$D^{Dd}(v_1, v_2) + |f(v_1)^2 - f(v_2)^2| \geq diam^{Dd}\left(C_3^{(t)}\right) + 1$,

Therefore , $f(v_{t+i})$ = 3t+ i -5, $1 \leq i \leq t$,

Hence, $rsdn^{Dd}(C_3^{(t)}) \leq 4t - 5$, $t \geq 3$.

Theorem 1.5.

The radio square difference number of a bistar $B_{n,n}$, $rsdn^{Dd}(B_{n,n}) = 3n$, $n \geq 2$.
Proof:

Let V ($B_{n,n}$) = $\{v_0, v_1, v_2, \dots, v_n , u_0, u_1, u_2, \dots, u_n, x_1 , x_2\}$ be the vertex set , x_1 , x_2 are the apex vertices.

Let E ($B_{n,n}$) = $\{x_1 v_i , x_1 x_2 , v_i u_i, 1 \leq i \leq n\}$ be the edge set.

Then $D^{Dd}(x_1, u_i) = D^{Dd}(x_1, v_i) = n + 3$, $1 \leq i \leq n$

$D^{Dd}(x_1, x_2)$ = 2n+ 3 , $D^{Dd}(v_1, u_1)$ = 2n – 1,

$D^{Dd}(v_1, v_i) = D^{Dd}(u_1, u_i) = 4$, $1 \leq i \leq n$.

So $diam^{Dd}(B_{n,n})$ = 2n +3.

The radio square difference Dd-distance condition,

$D^{Dd}(u, v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) +1$,

for any pair of vertices (u, v) where $u \neq v$.

Now , $D^{Dd}(x_1, x_2) + |f(x_1)^2 - f(x_2)^2| \geq diam^{Dd}(B_{n,n}) + 1$

$f(v_i) = n + 1$, $1 \leq i \leq n$ and $f(u_i) = 2n + i + 1$, $1 \leq i \leq n$

Hence , $rsdn^{Dd}(B_{n,n}) = 3n, n \geq 2$.

Theorem 1.6

The radio square difference $Dd -$ difference number of path P_n ,

$$rsdn^{Dd}(P_n) = \begin{cases} n , & 1 \leq n \leq 11 \\ \frac{3n-9}{2} , & if\ n\ isodd\ and\ n \geq 13 \\ \frac{3n-10}{2} , & if\ n\ is\ even\ and\ n \geq 12 \end{cases}$$

Proof:

Let $\{ v_{1,} v_{2,}, . . . , v_n \}$ be the vertex set. $E(C_n) = \{v_i v_{i+1}, v_1 v_n / i = 1, \ldots, n-1\}$.

be the edge set. Then $D^{Dd}(v_1, v_n)$ = $D^{Dd}(v_n, v_2)$ = n+1, $D^{Dd}(v_i, v_{i+1}) = 5$, $1 \leq i \leq n$

$diam^{Dd}(P_n)$ = $n + 1$.

The radio square difference Dd-distance condition,

$D^{Dd}(u, v) + |f(u)^2 - f(v)^2| \geq diam^{Dd}(G) + 1$,

for any pair of vertices (u, v) where $u \neq v$.

If n is odd then $f(v_i) = \frac{n-1}{2} + i - 3$, $2 \leq i \leq n-1$

If n is even then $f(v_i) = \frac{n}{2} + i - 4$, $2 \leq i \leq n-1$

Hence, $rsdn^{Dd}(P_n) = \begin{cases} n , & 1 \leq n \leq 11 \\ \frac{3n-9}{2} , & if\ n\ is\ odd\ and\ n \geq 13 \\ \frac{3n-10}{2} , & if\ n\ is\ even\ and\ n \geq 12 \end{cases}$

III. CONCLUSION

Though we have obtained the radio square difference Dd distance number of various different graphs with respect to the distance variants defined. The general results of radio numbers depend on the distance constraints, rather than structure of the graph. This certainly throws up more scope for further research. Moreover, equality can be tried for those cases ending up with sharp upper bounds.

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