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On Radio Heronian D-distance Mean Number of Degree Splitting Graphs

K. John Bosco[1], Dinesh M[2]

Assistant Professor[1], Research Scholar[2] (Reg.No.20213232091001)

Department of mathematics, St. Jude's College, Thoothoor, Tamilnadu-629176

Abstract

A Radio Heronian Mean D-distance Labeling of a connected graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $d^D(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^D(G)$ where $d^D(u,v)$ denotes the D-distance between u and v and $diam^D(G)$ denotes the D-diameter of G. The radio heronian D-distance number of f, $rhmn^D(f)$ is the maximum label assigned to any vertex of G. The radio heronian D-distance number of G, $rhmn^D(G)$ is the minimum value of $rhm\,n^D(f)$ taken over all radio heronian D-distance labeling f of G.

Keywords*: D-distance, Radio D-distance number, Radio heronian D-distance, Radio heronian D-distance number.*

I. INTRODUCTION

A graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Graph labeling was introduced by Alexander Rosa in 1967. Radio mean labeling was introduced by S. Somasundaram and R. Ponraj in 2004. Harmonic mean labeling was introduced by S. Somasundaram and S S Sandhya in 2012.

The concept of D-distance was introduced by D. Reddy Babu et al. The concept of radio D-distance was introduced by T. Nicholas and K. John Bosco in 2017. The concept of radio mean D-distance was introduced by T. Nicholas and K. John Bosco in 2017. The concept of heronian mean labeling was introduced by S S Sandhya in 2017. [13] we introduced the concept of radio heronian D-distance mean labeling of some basic graphs in 2021.

We are introduce the concept of radio heronian D-distance mean number of degree splitting graphs. A Radio Heronian Mean D-distance Labeling of a connected graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $d^D(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^D(G)$ where $d^D(u,v)$ denotes the D-distance between u and v and $diam^D(G)$ denotes the D-diameter of G. The radio heronian D-distance number of f, $rhmn^D(f)$ is the maximum label assigned to any vertex of G. The radio heronian D-distance number of G, $rhmn^D(G)$ is the minimum value of $rhm\,n^D(f)$ taken overf all radio heronian D-distance labeling f of G.

II. MAIN RESULT

Theorem 2.1

The radio heronian mean D-distance number of a splitting graph of star,

$$rhmn^D\left(spl(K_{1,n})\right) = \begin{cases} 10, n = 2 \\ 12, n = 3 \\ 2n + 7, n \geq 4 \end{cases}.$$

Proof:

Let $V\big(spl(K_{1,n})\big) = \{v_i, u_{i,} x_j / i = 1,2,3 \dots \dots \dots n\}$ be the vertex set and

$E = \{x_i v_i, x_j u_i / i = 1,2 \dots . n, j = 1,2 .. n$ It is $diam^D\big(spl(K_{1,n})\big) = 3n + 6$.

Since $spl(K_{1,n})$ has $2n + 2$ vertices it requires $2n + 2$ labels.

The D-distance is $d^D\big(v_i, v_j\big) = 2n + 4$, $f(v_1) = 6$

Then $rhmn^D\big(spl(K_{1,n})\big) \geq 2n + 7$

Let $f(x_1) = 2n + 7, f(x_2) = 2n + 6, f(v_i) = 5 + i, 1 \leq i \leq n, f(u_i) = n + 5 + i, 1 \leq i \leq n.$

We shall check the radio heronian mean D-distance condition

$d^D(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^D\big(spl(K_{1,n})\big) = 3n + 7$ for every pair of vertices (u,v) where $u \neq v$.

$$d^D(v_i,u_i) + \left\lceil \frac{f(v_i) + \sqrt{f(v_i)f(u_i)} + f(u_i)}{3} \right\rceil \geq 2n + 5 + \left\lceil \frac{5 + i + \sqrt{(5+i)(n+5+i)} + n + 5 + i}{3} \right\rceil \geq 3n + 7$$

$$d^D(v_i,x_1) + \left\lceil \frac{f(v_i) + \sqrt{f(v_i)f(x_1)} + f(x_1)}{3} \right\rceil \geq 2n + 2 + \left\lceil \frac{5 + i + \sqrt{(5+i)(2n+7)} + 2n + 7}{3} \right\rceil \geq 3n + 7$$

$$d^D(v_i,x_2) + \left\lceil \frac{f(v_i) + \sqrt{f(v_i)f(x_2)} + f(x_2)}{3} \right\rceil \geq 3n + 6 + \left\lceil \frac{5 + i + \sqrt{(5+i)(2n+6)} + 2n + 6}{3} \right\rceil \geq 3n + 7$$

$$d^D(x_1,x_2) + \left\lceil \frac{f(x_1) + \sqrt{f(x_1)f(x_2)} + f(x_2)}{3} \right\rceil \geq 3n + 4 + \left\lceil \frac{2n + 7 + \sqrt{(2n+7)(2n+6)} + 2n + 6}{3} \right\rceil \geq 3n + 7$$

$$d^D(u_i,x_1) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(x_1)} + f(x_1)}{3} \right\rceil \geq 2n + 3 + \left\lceil \frac{n + 5 + i + \sqrt{(n+5+i)(2n+7)} + 2n + 7}{3} \right\rceil \geq 3n + 7$$

$$d^D(u_i,x_2) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(x_2)} + f(x_2)}{3} \right\rceil \geq n + 3 + \left\lceil \frac{n + 5 + i + \sqrt{(n+5+i)(2n+6)} + 2n + 6}{3} \right\rceil \geq 3n + 7$$

$$d^D(u_i,u_j) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(u_j)} + f(u_j)}{3} \right\rceil \geq n + 6 + \left\lceil \frac{n + 5 + i + \sqrt{(n+5+i)(n+5+j)} + n + 5 + j}{3} \right\rceil \geq 3n + 7$$

$$d^D\big(v_i,v_j\big) + \left\lceil \frac{f(v_i) + \sqrt{f(v_i)f(v_j)} + f(v_j)}{3} \right\rceil \geq 2n + 4 + \left\lceil \frac{5 + i + \sqrt{(5+i)(5+j)} + 5 + j}{3} \right\rceil \geq 3n + 7$$

Therefore $f(x_1) = 2n + 7$ is the largest label.

Hence $rhmn^D\big(spl(K_{1,n})\big) = \begin{cases} 10, n = 2 \\ 12, n = 3 \\ 2n + 7, n \geq 4 \end{cases}$.

Theorem: 2.2

The radio heronian mean D-distance number of a degree splitting graph of wheel,

$rhmn^D(DS(W_n)) = 2n,\ n \geq 3.$

Proof:

Let $V(DS(W_n)) = \{v\} \cup \{u\} \cup \{u_i / i = 1,2,3 \ldots \ldots \ldots n\}$ and $E = \{u_i v,\ u_i u / i = 1,2 \ldots . n\}$.

It is $diam^D(DS(W_n)) = 2n + 4$

Since $DS(W_n)$ has $n + 2$ vertices it requires $n + 2$ labels.

The D-distance is $d^D(v_i, v_j) = n + 5$, $f(v) = n$ then the label n is forbidden.

$rhmn^D(DS(W_n)) \geq n + n$

$\geq 2n$

Let $f(v) = n,\ f(u_i) = n + i, 1 \leq i \leq n, f(u) = 2n.$

We shall check the radio heronian mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^D(DS(W_n)) = 2n + 5$ for every pair of vertices (u, v) where $u \neq v$.

If v_i and v_j are adjacent,

$$d^D(u_i, v) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(v)} + f(v)}{3} \right\rceil \geq n + 4 + \left\lceil \frac{n + i + \sqrt{(n+i)(n)} + n}{3} \right\rceil \geq 2n + 5$$

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(u_j)} + f(u_j)}{3} \right\rceil \geq 11 + \left\lceil \frac{n + i + \sqrt{(n+i)(n+j)} + n + j}{3} \right\rceil \geq 2n + 5$$

$$d^D(u_i, u) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(u)} + f(u)}{3} \right\rceil \geq n + 4 + \left\lceil \frac{n + i + \sqrt{(n+i)(2n)} + 2n}{3} \right\rceil \geq 2n + 5$$

$$d^D(u, v) + \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil \geq 2n + 4 + \left\lceil \frac{n + i + \sqrt{(n+i)(n)} + n}{3} \right\rceil \geq 2n + 5$$

Therefore $f(u) = 2n$ is the largest label.

Hence $rhmn^D(DS(W_n)) = 2n,\ n \geq 3$.

Theorem 2.3

The radio heronian mean D-distance number of a degree splitting path,

$rhmn^D(DS(P_n)) = 2n + 4,\ n \geq 2.$

Proof:

Let $V(DS(W_n)) = \{v\} \cup \{u\} \cup \{u_i / i = 1,2,3 \ldots \ldots \ldots n\}$ and $E = \{u_i v,\ u_i u, u_i u_j\ / i = 1,2 \ldots . n\}$.

It is $diam^D(DS(W_n)) = 2n + 4$

Since $DS(P_n)$ has $n + 2 (n \geq 3)$ vertices it requires $n + 2$ labels.

The D-distance is $d^D(v_2, u_{i+1}) = n + 2$, $f(v_2) = 6$ then the label $n + 2$ is forbidden.

Then $rhmn^D(spl(K_{1,n})) \geq n + 2 + 2 + 2$

$\geq 2n+4$

Let $f(v_1)=2n+4, f(v_2)=6, f(u)=n,\ f(u_i)=n+i-1, 1\leq i\leq n$.

We shall check the radio heronian mean D-distance condition

$d^{D}(u,v)+\left\lceil\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rceil\geq 1+diam^{D}(DS(W_n))=n+10$ for every pair of vertices (u,v) where $u\neq v$.

If $f(v_i)$ and $f(v_j)$ are adjacent,

$$d^D(u_i,\ v_2)+\left\lceil\frac{f(u_i)+\sqrt{f(u_i)f(v_2)}+f(v_2)}{3}\right\rceil\geq n+2+\left\lceil\frac{n+i-1+\sqrt{(n+i-1)(6)}+6}{3}\right\rceil\geq n+10$$

$$d^D(u_i,u_j)+\left\lceil\frac{f(u_i)+\sqrt{f(u_i)f(u_j)}+f(u_j)}{3}\right\rceil\geq 6+\left\lceil\frac{n+i-1+\sqrt{(n+i-1)(n+j-1)}+n+j-1}{3}\right\rceil\geq n+10$$

$$d^D(u_i,u_j)+\left\lceil\frac{f(u_i)+\sqrt{f(u_i)f(u_j)}+f(u_j)}{3}\right\rceil\geq 7+\left\lceil\frac{n+i-1+\sqrt{(n+i-1)(n+j-1)}+n+j-1}{3}\right\rceil\geq n+10$$

$$d^D(u_i,v_1)+\left\lceil\frac{f(u_i)+\sqrt{f(u_i)f(v_1)}+f(v_1)}{3}\right\rceil\geq 5+\left\lceil\frac{n+i-1+\sqrt{(n+i-1)(2n+4)}+2n+4}{3}\right\rceil\geq n+10$$

If $f(v_i)$ and $f(v_j)$ are non adjacent,

$$d^D(u_i,\ v_2)+\left\lceil\frac{f(u_i)+\sqrt{f(u_i)f(v_2)}+f(v_2)}{3}\right\rceil\geq n+3+\left\lceil\frac{n+i-1+\sqrt{(n+i-1)(6)}+6}{3}\right\rceil\geq n+10$$

$$d^D(u_i,v_1)+\left\lceil\frac{f(u_i)+\sqrt{f(u_i)f(v_1)}+f(v_1)}{3}\right\rceil\geq n+3+\left\lceil\frac{n+i-1+\sqrt{(n+i-1)(2n+4)}+2n+4}{3}\right\rceil\geq n+10$$

$$d^D(u_i,u_j)+\left\lceil\frac{f(u_i)+\sqrt{f(u_i)f(u_j)}+f(u_j)}{3}\right\rceil\geq n+7+\left\lceil\frac{n+i-1+\sqrt{(n+i-1)(n+j-1)}+n+j-1}{3}\right\rceil\geq n+10$$

$$d^D(u_i,u_j)+\left\lceil\frac{f(u_i)+\sqrt{f(u_i)f(u_j)}+f(u_j)}{3}\right\rceil\geq n+10+\left\lceil\frac{n+i-1+\sqrt{(n+i-1)(n+j-1)}+n+j-1}{3}\right\rceil\geq n+10$$

$$d^D(v_1,\ v_2)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil\geq n+6+\left\lceil\frac{2n+4+\sqrt{(2n+4)(6)}+6}{3}\right\rceil\geq n+10$$

Therefore $f(v_1)=2n+4$ is the largest label.

Hence $rhmn^D(DS(P_n))=2n+4,\ n\geq 2.$

Theorem: 2.4

The radio heronian mean D-distance number of a double fan,

$rhmn^D(DS(F_n))=2n+2, n\geq 2.$

Proof:

Let $V(DS(F_n))=\{v_i,u_{j,}/i=1,2,3\dots\dots\dots n\ and\ j=1,2,\dots.n\}$ be the vertex set and

$E = \{v_i u_j, u_i u_{j+1} / i = 1,2 \ldots . n, j = 1,2..n$ It is $diam^D(DS(F_n)) = 2n + 5$.

Since $DS(F_n)$ has $n + 2$ vertices it requires $n + 2$ labels.

The D-distance is $d^D(v_1, u_i) = n + 4$, $f(v_1) = n + 1$ then the label n is forbidden.

Then $rhmn^D(DS(F_n)) \geq 2n + 2$

Let $f(v_1) = n + 1, f(v_2) = n + 2,\ f(u_i) = n + i + 2, 1 \leq i \leq n$.

We shall check the radio heronian mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^D(DS(F_n)) = 2n + 6$ for every pair of vertices (u, v) where $u \neq v$.

$$d^D(u_i, v_1) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(v_1)} + f(v_1)}{3} \right\rceil \geq n + 4 + \left\lceil \frac{n + i + 2 + \sqrt{(n + i + 2)(n + 1)} + n + 1}{3} \right\rceil \geq 2n + 6$$

$$d^D(u_i,\ v_2) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(v_2)} + f(v_2)}{3} \right\rceil \geq n + 4 + \left\lceil \frac{n + i + 2 + \sqrt{(n + i + 2)(n + 2)} + n + 2}{3} \right\rceil \geq 2n + 6$$

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(u_j)} + f(u_j)}{3} \right\rceil \geq 7 + \left\lceil \frac{n + i + 2 + \sqrt{(n + i + 2)(n + j + 2)} + n + j + 2}{3} \right\rceil \geq 2n + 6$$

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(u_j)} + f(u_j)}{3} \right\rceil \geq n + 8 + \left\lceil \frac{n + i + 2 + \sqrt{(n + i + 2)(n + j + 2)} + n + j + 2}{3} \right\rceil \geq 2n + 6$$

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i) + \sqrt{f(u_i)f(u_j)} + f(u_j)}{3} \right\rceil \geq n + 9 + \left\lceil \frac{n + i + 2 + \sqrt{(n + i + 2)(n + j + 2)} + n + j + 2}{3} \right\rceil \geq 2n + 6$$

$$d^D(v_1,\ v_2) + \left\lceil \frac{f(v_1) + \sqrt{f(v_1)f(v_2)} + f(v_2)}{3} \right\rceil \geq 2n + 5 + \left\lceil \frac{n + 1 + \sqrt{(n + 1)(n + 2)} + n + 2}{3} \right\rceil \geq 2n + 6$$

Therefore $f(u_n) = 2n + 2$ is the largest label.

Hence $rhmn^D(DS(F_n)) = 2n + 2, n \geq 2$.

Theorem: 2.5

The radio heronian mean D-distance number of a total graph of path,

$$rhmn^D(T(P_n)) = \begin{cases} 7, n = 3 \\ 6n - 11, n \geq 4 \end{cases}$$

Proof:

Let $V(T(P_n)) = \{v_i, u_j / i = 1,2,3 \ldots \ldots \ldots n \text{ and } j = 1,2, \ldots n\}$ be the vertex set and

$E = \{v_i v_{i+1}, v_i u_j, u_j u_{j+1} / i = 1,2 \ldots . n, \text{and } j = 1,2..n\}$ It is $diam^D(T(P_n)) = 5n - 5$.

Since $T(P_n)$ has $2n - 1$ vertices it requires $2n - 1$ labels.

The D-distance is $d^D(v_i, v_{i+1}) = 7$, $f(v_2) = 4n - 9$ then the label $4n - 10$ is forbidden.

$rhmn^D(T(P_n)) \geq 4n - 10 + 2n - 1$

$\geq 6n - 11$

Let $f(v_i) = 4n - 11 + i, 2 \leq i \leq n, f(v_1) = 6n - 11, f(u_i) = 5n + i - 11, 1 \leq i \leq n$.

We shall check the radio heronian mean D-distance condition

$d^D(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^D(T(P_n)) = 5n - 4$ for every pair of vertices (u,v) where $u \neq v$.

If v_i and v_j are adjacent,

$$d^D(v_i,v_1) + \left\lceil \frac{f(v_i)+\sqrt{f(v_i)f(v_1)}+f(v_1)}{3} \right\rceil \geq 7 + \left\lceil \frac{4n-11+i+\sqrt{(4n-11+i)(6n-11)}+6n-11}{3} \right\rceil \geq 5n-4$$

If v_i and v_j are non adjacent,

$$d^D(v_i,v_j) + \left\lceil \frac{f(v_i)+\sqrt{f(v_i)f(v_j)}+f(v_j)}{3} \right\rceil \geq 5n-5 + \left\lceil \frac{4n-11+i+\sqrt{(4n-11+i)(4n-11+j)}+4n-11+j}{3} \right\rceil \geq 5n-4$$

$$d^D(v_i,v_1) + \left\lceil \frac{f(v_i)+\sqrt{f(v_i)f(v_1)}+f(v_1)}{3} \right\rceil \geq 5n-8 + \left\lceil \frac{4n-11+i+\sqrt{(4n-11+i)(6n-11)}+6n-11}{3} \right\rceil \geq 5n-4$$

If u_i and u_j are adjacent,

$$d^D(u_i,u_j) + \left\lceil \frac{f(u_i)+\sqrt{f(u_i)f(u_j)}+f(u_j)}{3} \right\rceil \geq 8 + \left\lceil \frac{5ni-11+i\sqrt{(5n-11+i)(5n-11+j)}+5n-11+j}{3} \right\rceil \geq 5n-4$$

If u_i and u_j are non adjacent,

$$d^D(u_i,u_j) + \left\lceil \frac{f(u_i)+\sqrt{f(u_i)f(u_j)}+f(u_j)}{3} \right\rceil \geq 5n-8 + \left\lceil \frac{5ni-11+i\sqrt{(5n-11+i)(5n-11+j)}+5n-11+j}{3} \right\rceil \geq 5n-4$$

$$d^D(u_i,u_j) + \left\lceil \frac{f(u_i)+\sqrt{f(u_i)f(u_j)}+f(u_j)}{3} \right\rceil \geq 5n-8 + \left\lceil \frac{5n+i-11+i\sqrt{(5n-11+i)(5n-11+j)}+5n-11+j}{3} \right\rceil \geq 5n-4$$

If v_i and u_j are adjacent,

$$d^D(u_i,v_j) + \left\lceil \frac{f(u_i)+\sqrt{f(u_i)f(v_j)}+f(v_j)}{3} \right\rceil \geq 6 + \left\lceil \frac{5n+i-11+\sqrt{(5n+i-11)(4n+j-11)}+4n+j-11}{3} \right\rceil \geq 5n-4$$

If v_i and u_j are non adjacent,

$$d^D(u_i,v_j) + \left\lceil \frac{f(u_i)+\sqrt{f(u_i)f(v_j)}+f(v_j)}{3} \right\rceil \geq 5n-4 + \left\lceil \frac{5n+i-11+\sqrt{(5n+i-11)(4n+j-11)}+4n+j-11}{3} \right\rceil \geq 5n-4$$

Therefore $f(v_1) = 6n - 11$ is the largest label.

Hence $rhmn^D(T(P_n)) = \begin{cases} 7, n = 3 \\ 6n-11, n \geq 4 \end{cases}$

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