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The Radio Dd-Distance in Harmonic Mean Number of Some New Graphs

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Abstract:

A radio Dd-distance in harmonic mean labelling of a connected graph G is an injective map f from the vertex set $V(G)$ to the $\mathbb{N}$ such that for two distinct vertices u and v of G, $D^{Dd}(u,v)+\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil \geq diam^{Dd}(G)+1$. where $D^{Dd}(u,v)$ denote Dd-distance between u and v $diam^{Dd}(G)$ denotes the diameter of G. The radio Dd-distance in harmonic mean number of f, $rh^{Dd}n(f)$ is the maximum label assigned to any vertex of G. The radio Dd-distance in harmonic mean number of G, $rh^{Dd}n(G)$ is the minimum value of of G. In the paper we find the radio Dd-distance in harmonic mean number of some standard graphs.

Keywords: *Dd-distance, Radio harmonic mean number, Radio Dd-distance in harmonic mean number.*

INTRODUCTION:

By a graph $G=(V(G),E(G))$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

The Dd-distance was introduced by A. Anto Kingsley and P. Siva Ananthi [1]. For a connected graph G, the Dd-length of a connected $u-v$ path is defined as $D^{Dd}(u,v)=D(u,v)+\deg(u)+\deg(v)$.. The Dd-radius, denoted by $r^{Dd}(G)$ is the minimum Dd-eccentricity among all vertices of G. That is $r^{Dd}(G)=min\{e^{Dd}(G): v \in V(G)\}$ .Similarly the Dd-diameter, $D^{Dd}(G)$ is the maximum Dd-eccentricity among all vertices of G. We observe that for any two vertices u, of G,we have $d(u,v), D^{Dd}(u,v)$.The equality holds if and only if u,v are identical. If G is any connected graph then the Dd-distance is a metric on the set of vertices of G. We can check easily $r^{Dd}(G) \leq D^{Dd}(G) \leq 2r^{Dd}(G)$. The lower bound is clean from the definition and the upper bound follows from the triangular inequality.

We introduce the concept of radio Dd-distance in harmonic mean colouring of a graph G. Radio Dd-distance in harmonic mean colouring is a function $f: V(G) \to \mathbb{N}$ such that $D^{Dd}(u,v)+\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil \geq diam^{dd})G)+1$, where $diam^{Dd}(G)$ is the Dd-distance radio diameter of G. A Dd-distance radio colouring number of G is the maximum color assigned to any vertex of G. It is denoted by $rh^{Dd}n(G)$.

Radio labelling (multi-level distance labelling)can be regarded as an extention of distance-two labelling whi8ch is motivated by the channel assignment problem introduced by Hale[6]. Chartrand et al. [2] introduced the concept of radio labelling of graph. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [10]. However Chartrand et al.[2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of cycle. Liu [9] gave the lower bound for the radio number of tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. M. M. Rivera et al. [20]. Gave the radio number of $C_n \times C_n$ the Cartesian product of C_n. In [4] C. Fernandez et al. [20] gave the radio number for star graph, wheel graph, helm graph. M. T. Rahim and I. Tomescu [17] investigated the radio number of Helm graph. The radio number for the gerneralized prism graphs were presented by Paul Martinez et al. In [11]. In this paper, we fined the radio Dd-distance coloring of some basic graphs. We are introduced Radio Dd-distance number of some standard graphs.

Theorem:1.1

The radio Dd-distance in harmonic mean number of a gear graph, $rh^{Dd}n(G_n) = \begin{cases} 2n+1, & if\ n=3 \\ 3(n-1), & If\ n \geq 4 \end{cases}$.

Proof:

Let $\{v_0, v_1, \ldots, v_n\}$ and $\{u_1, u_2, \ldots, u_n\}$ are the vertex set, where v_0 is the central vertex and $E(G_n) = \{v_0 v_i, v_i u_i / i = 1, \ldots, n\}$ be the edge set. The $diam^{Dd}(G_n) = 3n + 2$.

Hence $rh^{Dd}(G_n) = \begin{cases} 2n+1, & if\ n=3 \\ 3(n-1), & If\ n \geq 4 \end{cases}$.

Theorem:1.2

The radio Dd-distance in harmonic mean number of a fan graph, $rh^{Dd}n(F_n) \leq \begin{cases} n+1, & if\ 3 \leq n \leq 6 \\ 2n+6, & If\ n \geq 7 \end{cases}$.

Proof:

Let $V(F_n) = \{v_0, v_1, \ldots, v_n\}$ be the vertex set, where v_0 is the central vertex and $E(F_n) = \{v_0 v_i, v_i v_{i+1} / i = 1, 2 \ldots, n-1\}$ be the edge set. The $diam^{Dd}(F_n) = 2n + 2$.

Hence $rh^{Dd}(F_n) \leq \begin{cases} n+1, & if\ 3 \leq n \leq 6 \\ 2n+6, & If\ n \geq 7 \end{cases}$.

Theorem:1.3

The radio Dd-distance in harmonic mean number of a double fan graph, $rh^{Dd}n(D(F_n)) \leq 2(n-1)$.

Proof:

Let $V(D(F_n)) = \left\{v_i, u_j \ \ / i = 1,2, \ldots, and\ j = 1,2, \ldots, n\right\}$ be the vertex set, and $E(D(F_n)) = \left\{v_i u_j, \frac{u_j u_{j+1}}{i} = 1, 2. and\ j = 1,2, \ldots, n\right\}$ be the edge set. The $diam^{Dd}(D(F_n)) = 3n + 1$..

Hence $rh^{Dd}(D(F_n)) \leq 2(n-1). n \geq 3$.

Theorem:1.4

The radio Dd-distance in harmonic mean number of a bistar graph, $rh^{Dd}n\left(B_{(n,n)}\right) \leq \begin{cases} \frac{5n-1}{2} + 3 & if\ n\ is\ odd \\ \frac{5n}{2} + 2 & if\ n\ is\ even \end{cases}. n \geq 3$.

Proof:

Let $V(B_{(n,n)}) = \{x_j \ \ / j = 1,2.\} \cup \{v_i, u_i \ \ / i = 1,2, \ldots, n\}$ be the vertex set, and $E\left(B_{(n,n)}\right) = \left\{x_j x_{j+1}, x_j v_i, x_{j+1} u_i \ \ i = 1, 2, \ldots, n-1. and\ j = 1\right\}$ be the edge set. The $diam^{Dd}\left(B_{(n,n)}\right) = 2n + 3$.

Hence $rh^{Dd}n\left(B_{(n,n)}\right) \leq \begin{cases} \frac{5n-1}{2} + 3 & if\ n\ is\ odd \\ \frac{5n}{2} + 2 & if\ n\ is\ even \end{cases}. n \geq 3$.

Theorem:1.5

The radio Dd-distance in harmonic mean number of a crown graph, $rh^{Dd}n(C_n \odot K_1) \leq 3n-2, n \geq 3$.

Proof:

Let $V(C_n \odot K_1) = \{v_i, u_i \ / \ i = 1,2, \dots, n\}$ be the vertex set and $E = \left\{v_i, v_j,\ v_i, u_i \ / \ i, j = 1,2, \dots, n\right\}$ be the edge set.

The $diam^{Dd}(C_n \odot K_1) = n+5$

Hence $rh^{Dd}n(C_n \odot K_1) \leq 3n-2, n \geq 3$.

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