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The Study of Equilibrium Strategies of the Unobservable Markovian Queue with Redundant Server for Balking and Delayed Repair

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Abstract *- Senlin and Zaiming (2016) [21] studied the equilibrium strategies for the fully unobservable and almost unobservable single-server queues with breakdowns and delayed repairs. The present paper aims to study the customers behavior of the system in markovian single server queue with presence of redundant server. Redundant server is an extra server which is used in our model so that the system provides a reliable working facility to the customer. In unobservable case an arriving customer does not know length of queue. The model under consideration can be observed as an $M / M /1$ queue in a casual environment. Equilibrium balking strategies in single server markovian queue with redundant server are calculated for the almost unobservable and fully unobservable queues. Finally, we demonstrate the effect of several system parameters on the equilibrium behavior.*

Keyword: *Equilibrium, Markovian queue, Queueing Theory, Redundant, Unobservable queue.*

I. INTRODUCTION

During the last decades, in economic point of view there are an emerging tendency to study queueing systems. In a queueing system, the analysis of strategic behavior of customer is based on some reward-cost structure which is compulsory on the system and reflects the customers' desire for service and their unwillingness to wait. An every arriving customer wants to maximize their expected benefit, considering that the other customers have the same objective so this situation can be seen as a game among them. In this type of studies, the fundamental problem is to identify individual and social optimal strategies.

In the queueing literature most of papers assume that the server is always available, although this assumption is not a realistic. Actually, perfectly reliable servers are virtually nonexistent. In Senlin, Zaiming, Wu [21] we studied the equilibrium strategies for the almost unobservable and fully unobservable m/m/1 queues with server breakdowns and delayed repairs. In this paper we consider equilibrium strategy of customer in unobservable markovian single server queue with redundant (extra) server. In many realistic situations, due to non-availability of the repair facility, the repair process may not be started immediately. This work compensates the game theoretic analysis in [8] by studying the corresponding unobservable cases.

In our paper we consider equilibrium strategy of customer in unobservable markovian single server queue with redundant (extra) server. In our model we minimize the waiting time of customer with help of the redundant server in M/M/1. If main server goes in breakdown state then working process is not affected because of the redundant server. Customers moves the redundant server and customer is served without any delay. In the case, when each server fails, service facility stops and system enters in repair state. Since repair process also takes some time, it also constitutes some delay. In this situation customers face delay in service. But with the help of redundant server overall reliability of the system increases so customers do not balk from the system. Redundant server is an extra server used in our model so that the system provides a reliable working facility to the customer.

II. BRIEF REVIEW

There are many researches works for economic analysis of customer behavior on the performance of a queueing system. Burnetas and Economou [1] first described the theory of several Markovian queues with setup times and four precision levels of system information and analyzed the customers' equilibrium strategies. Economou and Kanta [2] studied the equilibrium balking strategies in the observable $M / M /1$ queueing system with an unreliable server and repairs. There are some research works to described the server vacation policies, such as Guo and Hassin [17] and Sun et al. [22]. Moreover, there are many

papers in which described the economic analysis of the balking behavior of customers of the $M / M /1$ queue in variations. in [3], Economou and Manou discussed a Markovian clearing queueing system that operates in an alternating environment.
Liu , Yu [24] studied an $M / M / C$ queueing system in a random environment. there are some differences between vacations and breakdowns. A vacation must occur after completing the service of the customers in the system, but the break- downs can occur at the any point of time no matter the server is busy or idle. The economic analysis of customer behavior on the performance of queueing systems has been described by several authors. In the $M / M /1$ queueing system with an unreliable server and repairs, the equilibrium strategies under various levels of information have been studied by Economou, Li and Jagannathan. Li[12] studied the equilibrium analysis of a single-server Markovian queueing system with working breakdowns. In the case of working breakdown, when the system is defective, instead of stopping service completely, the service continues at a slower rate then normal woking rate .

In this paper, we provide supplement to the investigation in Senlin, Zaiming, Wu [21] by discussing the corresponding unobservable cases with redundant server in which the queue length is unknown to arriving customers. Due to the redundant server, the balking behavior of customers under unobservable case should be considered to obtain a reliable representation of the system. From a operational point of view, the almost unobservable case is interesting to study. The model under consideration is viewed as an M / M /1 queue with redundant server in a random environment and thus we can interpretate of the stability condition.
This type of model has wide applications in many fields, as argued in [2],[9],[8],[12]. For example, due to failures of machines or job-related problems , the machine may break down at any state . Such systems that require repairs after server's break- downs are very common in practice. But if there are redundant machine already present in the system then after breakdown redundant machine serve customers without any delay. If all machines fail at the same time or redundant machine breakdown before repair of main machine. Then customer faces delay. The repair delay time is introduced as the time interval between the epoch of server breakdown and the beginning of repair process to reflect the fact that the service may delay due to the repair process.

The un- observable case with redundant server in real-life situation can be illustrated by the decision making of customers in the ITES service provider working in government or private sector for providing support/services in such cases generally two data centers are created. One of which works at time and other work as backup in case of natural disasters/ accidents. Backup server provide services If main server fails. In this case backup server becomes main server and main server goes in repair process and when that server is repaired then it acts as backup server. the servers does not provide any information related to the number of customers just waiting in the system prior to the customer's arrival. In the wireless communication technology field , assume there is only one channel which is unreliable. Once the channel is breakdown , it goes a delayed time before recovery. In order to make full use of this channel, we should design some admission control policies and with redundant channel to allow an arriving customer to reliable service

The rest of this paper is organized as follows. In Section III, we describe the dynamics of the model and the reward- cost structure. In Section IV, we consider the equilibrium mixed strategies for the almost unobservable case. Section V is devoted to studying the fully unobservable case. Some conclusion and future research.

III. MODAL DESCRIPTION

We investigate the same model discussed in Wang and Zhang [8] but there is some difference in our model here we use redundant sever. We consider the M / M /1 queueing system with an infinite waiting queue in which customers arrive according to a Poisson process with intensity λ and customers are served at a rate of μ. The server has an exponential lifetime with failure rate 2ξ when he is working. Once the server fails it will not experience an exponential delayed time to activate the repair process. Because all customers load of system is transfer on the redundant server. Working process is continuing. In this situation fail server is going to repair process. But in this interval of time redundant server fails. Then delayed time is exponentially distributed with parameter δ. In the delay state, the server doesn't provide any type of service to arriving customers and to begin the repair process ,server waits for repair facility. The repair time is exponentially distributed with repair rate θ. In other words, when the server fails, then the repair process doesn't start immediately due to non-availability of the repair facility. The repair delayed time is define as the time interval between the period of server breakdown and the beginning of repair process. We realize that the repair delayed time has two stages and hence it is not memoryless. We describe the state of the system at time t by a pair (L (t), I (t)), where L (t) records the number of customers in the system and I (t) denotes the state of the server (3: working state 2: working state (redundant server); 1: delayed period; 0: under repair). The stochastic process $\{(L(t), I(t)), t \geq 0\}$ is a two-dimensional continuous-time Markov chain

$$q_{(n,i)(n+1,i)} = \lambda \quad , n \geq 0 \ ; i = 0, 1, 2, 3;-$$

$$q_{(n,i)(n-1,i)} = \mu \quad ,n \geq 1 \ ; i = 2,3;$$
$$q_{(n,i)(n+1,i)} = \lambda \quad ,n \geq 0 \ ; i = 0,1,2,3;$$
$$q_{(n,3)(n,2)} = 2\xi \quad n \geq 0;$$
$$q_{(n,2)(n,1)} = \xi \quad ,n \geq 0 \ ;$$
$$q_{(n,1)(n,0)} = \delta \quad ,n \geq 0 \ ;$$
$$q_{(n,0)(n,3)} = \theta \quad ,n \geq 0 \ ;$$

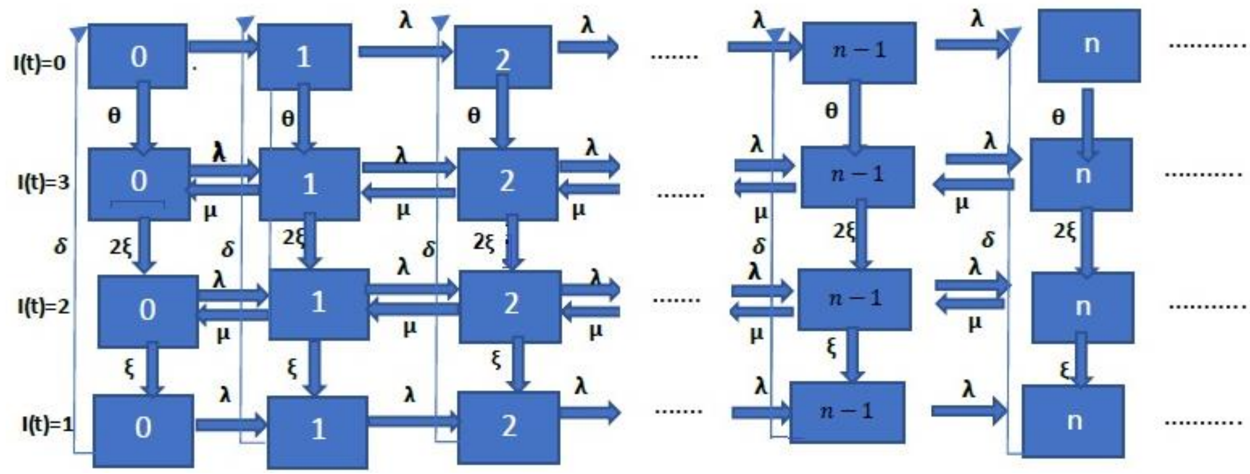

Fig.1.transition rate diagram for equilibrium strategy in unobservable queue with redundant server with breakdown and delayed repair

In fact, the model under consideration can be viewed as an M / M /1 queueing system in a random environment. More specifically, the external environment is an irreducible continuous-time Markov chain on a finite state space {3, 2, 1, 0}. That is, when the external environment I (t) is in state I, I = 0, 1, 2 ,3, the system behaves as an M (λ)/ M (μ)/1 queue with arrival intensity λ and service rate μ_i, where $\mu_0 = \mu_1 = 0$ and $\mu_2 = \mu_3 = \mu$. The infinitesimal generator (i.e., q -matrix) of the external environment I(t) is given by

$$Q=\begin{pmatrix} -2\xi & 2\xi & 0 & 0 \\ 0 & -\xi & \xi & 0 \\ 0 & 0 & -\delta & \delta \\ \theta & 0 & 0 & -\theta \end{pmatrix}$$

Let ($\pi_3, \ \pi_2, \pi_1 \ , \pi_0$) be the stationary distribution of the external environment, by solving ($\pi_3, \ \pi_2, \pi_1 \ , \pi_0$) Q = 0, we have Let $\pi_3 = \frac{\delta\theta}{3\theta\delta+2\theta\xi+3\xi\delta}$, $\pi_2 = \frac{2\delta\theta}{3\theta\delta+2\theta\xi+3\xi\delta}$, $\pi_1 = \frac{2\xi\theta}{3\theta\delta+2\theta\xi+3\xi\delta}$, $\pi_0 = \frac{2\xi\delta}{3\theta\delta+2\theta\xi+3\xi\delta}$

The system is said to be in state (n, I) if there are n customers in the system and the server is found at state I. Let p (n, I) be the limiting probability of the system in state (n, I). That is, p (n, I) = lime t→∞ P (L (t) = n, I(t) = I), n ≥0, if I= 0, 1, 2,3.

Theorem 1. For the M / M /1 queue with redundant server with breakdowns and delayed repairs, the stability holds if and only if

μ ($\pi_3, \ \pi_2$) > λ, i.e., μ(3δθ)> λ (3δθ+ 2ξθ+ 2ξδ). (1)

Proof. The steady-state balance equations are given below,

(λ+ θ) p (0, 0) = μ p (0, 1), (2)
(λ+ θ) p (n, 0) = λ p (n −1, 0) + δ p (n, 1), n ≥1, (3)
(λ+ δ) p (0, 1) = ξp (0, 2), (4)
(λ+ δ) p(n, 1) = λ p(n −1 , 1) + ξ p(n, 2) , n ≥1 , (5)
(λ+ ξ) p(0 , 2) = μ p(1 , 2) + 2ξp(0 , 3) , (6)
(λ+ μ+ ξ) p(n, 2) = λ p(n −1 , 2) + μ p(n + 1 , 2) + 2ξp(n, 3) , n ≥1 . (7)

(λ+ 2ξ) p(0 , 3) = μ p(1 , 3) + θp(0 , 0) , (8)
(λ+ μ+2 ξ) p(n, 3) = λ p(n −1 , 3) + μ p(n + 1 , 3) + θp(n, 0) , n ≥1 . (9)

Starting with n = 0 and summing each of these balance equations over i, i = 0, 1, 2 ,3, Then
μ p (n + 1, 3) + μ p (n + 1, 2) = λ p (n, 3) + λ p (n, 2) + λ p (n, 1) + λ p (n, 0), n ≥0. (10)

Clearly, $\sum_{n=0}^{\infty} p(n, i) = \pi_i$, i = 0, 1, 2. By summing (10) overall n, we arrive at
μ (π_2 −p (0, 2)) + μ (π_3 −p (0, 3)) = λ ($\pi_3+\pi_2+\pi_1+\pi_0$) = λ, that is,
μ (p (0, 2) + p (0, 3)) = μ ($\pi_2+\pi_3$) −λ. (11)

Since all states are communicating, from the theory of recurrent events it can be deduced that the probabilities p (n, i) (n ≥0, i = 0, 1, 3) are either all positive or, alternatively, all equal to zero. This property is crucial for our analysis.

If the Markov chain {(L (t), I (t)), t ≥0} is ergodic (positive recurrent), then all the probabilities p (n, i) (n ≥0, i = 0, 1, 3) are positive.
Thus p (0, 2) > 0 and p (0, 3) > 0 we have μ ($\pi_2+\pi_3$) > λ from (11).
Conversely, if μ ($\pi_2+\pi_3$) > λ, then p (0, 2) > 0 and p (0, 3) > 0 from (11).

We can conclude that all the probabilities p (n, i) (n ≥0, i = 0, 1, 2 ,3) are positive from the ergodicity theory for continuous-time Markov chains. Thus, the stochastic process {(L (t), I (t)), t ≥0} is stable.
In a word, for the stochastic process {(L (t), I (t)), t ≥0}, the steady-state regime exists if and only if p (0, 2) > 0 and p (0, 3) > 0. The necessary and sufficient condition for its existence is μ ($\pi_2+\pi_3$) > λ.

The intuitive interpretation of the theorem is straightforward: note that μ ($\pi_2+\pi_3$) is the average capacity of the system to render service and λ is the arrival intensity. For steady-state conditions, the average service capacity of the system must exceed the arrival rate. The customers are allowed to decide whether to join or balk the system at their arrival instants. Every customer receives a reward of R units in the system after completing their service . This may reflect his satisfaction or the added value of being served. On the other hand, customers have a waiting cost of C units per time when they remain in the system including the waiting time in queue and being served. Every Customer want to maximize their expected net benefit of service. Their decisions are unchangeable that retrials of balking customers and reneging of entering customers are not allowed. Each arriving customer can observe the number of customers. We distinguish two cases depending on the information available to the customers at their arrival instants: (1) almost unobservable case: customers observe the server state I (t), but not the queue length L (t); (2) fully unobservable case: customers are not informed about the queue length L (t) or the server state I (t).

IV. THE ALMOST UNOBSERVABLE QUEUE

We now proceed to the almost unobservable case with redundant server where the arriving customers observe the state of the server upon arrival, but not the queue size. From a methodological point of view, the almost unobservable case is interesting. There are eight pure strategies for the customers. In the almost unobservable case, a mixed strategy is specified by a vector of joining probabilities (q_0 , q_1 , q_2 ,q_3), q q_i ∈ [0, 1], where q i denotes the joining probability of a customer if the server is found at state i upon arrival, i = 0, 1, 2 ,3.
Clearly, the new queue is equivalent to the original queue except that the arrival intensity λ should be replaced by λq_i when the server is found at state i. The mixed strategy has the form 'while arriving at time t, observe I (t), enter with probability q_i when I(t) = i'.
Suppose that all customers follow the same strategy and enter the system with probability q_i when the server is found at state i, the steady-state equations governing the almost unobservable queue are similar with
Eqs. (2) – (10).

Define the partial generating functions as G_0 (z) = $\sum_{n=0}^{\infty} p(n, i)\ z^n$, | z | ≤1, i = 0, 1, 2 ,3.
For Eqs. (2) – (5) and (8), multiplying both sides by z^n and summing overall n for state i, note that λ should be replaced by λq_i for state i, then
(λq_0 + θ) G_0 (z) = δG_1 (z) + λq_0 zG_0 (z), (12)
(λq_1 + δ) G_1 (z) = ξG_2 (z) + λq_1 zG_1 (z), (13)
For Eqs. (9) and (10), multiplying both sides by z^{n+1} and summing overall n for state i, note that λ should be replaced by λq_i for state i, then
(λq_3 +2ξ + μ) G_3 (z) = $\frac{\mu}{z}$ G_3 (z) + λq_3 zG_3 (z) + θG_0 (z) + μ(1- $\frac{1}{z}$)P(0,3) (14)

$$\mu\,(G_2(z) - p(0,2)) + \mu\,(G_3(z) - p(0,3)) = \lambda q_3\, z\, G_3(z) + \lambda q_2\, z\, G_2(z) + \lambda q_1\, z G_1(z) + \lambda q_0\, z G_0(z). \quad (15)$$

Clearly, $G_i(1) = \pi_i \sum_{n=0}^{\infty} p(n,i)$, i = 0, 1, 2,3.
By differentiating (12) – (15) and substituting z = 1, we find
$\theta G_0'(1) = \delta G_1'(1) + \lambda q_0 \pi_0$,
$\delta G_1'(1) = \xi G_2'(1) + \lambda q_1 \pi_1$,
$2\xi G_3'(1) = \theta G_0'(1) - \mu\pi_3 + \lambda q_3 \pi_3 + \mu P(0,3)$,
$\mu(G_2'(1) + G_3'(1)) = \lambda q_0 \pi_0 + \lambda q_1 \pi_1 + \lambda q_2 \pi_2 + \lambda q_3 \pi_3 + \lambda q_0 G_0'(1) + \lambda q_1 G_1'(1) + \lambda q_2 G_2'(1) + \lambda q_3 G_3'(1)$
It immediately follows that

$$G_0'(1) = \frac{(2\xi+2\mu)\lambda q_0\pi_0\theta\delta+(2\xi+2\mu)\lambda q_1\pi_1\delta\theta+2\lambda q_2\pi_2\xi\theta\delta+(2\xi-2\mu)\lambda q_3\pi_3\theta\delta+\mu^2\pi_3\theta\delta+2\lambda^2 {q_3}^2\pi_3\theta\delta-2\lambda^2 q_1q_2\pi_1\theta\delta -2\lambda^2 q_0q_1\pi_0\theta\xi-2\lambda^2 q_0q_3\pi_0\theta\delta+(\lambda q_3-\mu)\mu\,\delta\theta P(0,3)}{\theta(3\mu\theta\delta-2\lambda q_0\xi\delta-2\lambda q_1\xi\theta-2\lambda q_2\theta\delta-\lambda q_3\theta\delta)} \quad (16)$$

$$G_1'(1) = \frac{(2\xi-\mu)\lambda q_0\pi_0\theta\delta+(2\xi+2\mu)\lambda q_1\pi_1\delta\theta+2\lambda q_2\pi_2\xi\theta\delta+(2\xi-2\mu)\lambda q_3\pi_3\theta\delta+\mu^2\pi_3\theta\delta-2\lambda^2 q_1q_2\pi_1\theta\delta+2\lambda^2 {q_3}^2\pi_3\theta\delta +2\lambda^2 q_0^2\pi_0\xi\delta+\lambda^2 q_0q_3\pi_0\theta\delta+(\lambda q_3-\mu)\mu\,\delta\theta P(0,3)}{\delta(3\mu\theta\delta-2\lambda q_0\xi\delta-2\lambda q_1\xi\theta-2\lambda q_2\theta\delta-\lambda q_3\theta\delta)} \quad (17)$$

$$G_2'(1) = \frac{(2\xi-\mu)\lambda q_0\pi_0\theta\delta+(2\xi-\mu)\lambda q_1\pi_1\delta\theta+2\lambda q_2\pi_2\xi\theta\delta+(2\xi-2\mu)\lambda q_3\pi_3\theta\delta+\mu^2\pi_3\theta\delta+\lambda^2 q_1q_3\pi_1\theta\delta+2\lambda^2 q_0q_1\pi_1\delta\xi+ 2\lambda^2 {q_3}^2\pi_3\theta\delta+2\lambda^2 q_0^2\pi_0\xi\delta+2\lambda^2 q_1^2\pi_1\xi\theta+\lambda^2 q_0q_3\pi_0\theta\delta+(\lambda q_3-\mu)\mu\,\delta\theta P(0,3)}{\xi(3\mu\theta\delta-2\lambda q_0\xi\delta-2\lambda q_1\xi\theta-2\lambda q_2\theta\delta-\lambda q_3\theta\delta)} \quad (18)$$

$$G_3'(1) = \frac{(2\xi+2\mu)\lambda q_0\pi_0\theta\delta+(2\xi+2\mu)\lambda q_1\pi_1\delta\theta+2\lambda q_2\pi_2\xi\theta\delta+(2\xi+2\mu)\lambda q_3\pi_3\theta\delta-2\mu^2\pi_3\theta\delta+\lambda^2 {q_3}^2\pi_3\theta\delta-2\lambda^2 q_1q_2\pi_1\theta\delta -2\lambda^2 q_0q_1\pi_0\theta\xi-2\lambda^2 q_0q_3\pi_0\delta\theta-2\lambda^2 q_0q_3\pi_3\xi\delta-2\lambda^2 q_1q_3\pi_3\xi\theta-2\lambda^2 q_2q_3\pi_3\theta\delta+2\lambda\mu q_0\pi_3\xi\delta+2\lambda\mu q_1\pi_3\xi\theta+2\lambda\mu q_2\pi_3\theta\delta +\{(\lambda q_3-\mu)\delta\theta+2\xi\}\mu P(0,3)}{2\xi(3\mu\theta\delta-2\lambda q_0\xi\delta-2\lambda q_1\xi\theta-2\lambda q_2\theta\delta-\lambda q_3\theta\delta)} \quad (19)$$

The quantity $G_i(1)$ can be considered as the contribution of state i to the mean queue length. Intuitively, since all the three states are communicating, the accumulation of customers in one state will influence all the three states.

By using PASTA property, the probability that there are n customers in the system given that the server is found at state i is

$$P(n|i) = \frac{p(n,i)}{\sum_{n=0}^{\infty} p(n,i)} = \frac{p(n,i)}{\pi_i}, \quad n \geq 0,\ i = 0, 1, 2, 3.$$

Then the mean number of customers in the system found by an arriving customer under the condition that the server is found at state i is given by

$$E(L \mid I = i) = \sum_{n=0}^{\infty} np(n|i) = \frac{\sum_{n=0}^{\infty} np(n|i)}{\pi_i} = \frac{G_i'(1)}{\pi_i}, \quad i = 0, 1, 2, 3.$$

With known $G_i'(1)$ and π_i , we have

$$E(L \mid I=0) = \frac{2(2\xi+2\mu)\lambda q_0\xi\theta\delta^2+2(2\xi+2\mu)\lambda q_1\xi\delta\theta^2+4\lambda q_2\xi\theta^2\delta^2+(2\xi-2\mu)\lambda q_3\theta^2\delta^2+\mu^2\theta^2\delta^2+2\lambda^2 {q_3}^2\theta^2\delta^2-4\lambda^2 q_1q_2\xi\theta^2\delta -4\lambda^2 q_0q_1\xi^2\delta\theta-4\lambda^2 q_0q_3\xi\theta\delta^2+(\lambda q_3-\mu)\mu\,\delta\theta P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)}{2\xi\theta\delta(3\mu\theta\delta-2\lambda q_0\xi\delta-2\lambda q_1\xi\theta-2\lambda q_2\theta\delta-\lambda q_3\theta\delta)} \quad (20)$$

$$E(L \mid I=1) = \frac{2(2\xi-\mu)\lambda q_0\xi\theta\delta^2+2(2\xi+2\mu)\lambda q_1\xi\theta^2\delta+4\lambda q_2\xi\theta^2\delta^2+(2\xi-2\mu)\lambda q_3\theta^2\delta^2+\mu^2\theta^2\delta^2+2\lambda^2 {q_3}^2\theta^2\delta^2-4\lambda^2 q_1q_2\xi\theta^2\delta +4\lambda^2 q_0^2\xi^2\delta^2+2\lambda^2 q_0q_3\xi\delta^2\theta+(\lambda q_3-\mu)\mu\,\delta\theta P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)}{2\xi\theta\delta(3\mu\theta\delta-2\lambda q_0\xi\delta-2\lambda q_1\xi\theta-2\lambda q_2\theta\delta-\lambda q_3\theta\delta)} \quad (21)$$

$$E(L \mid I=2) = \frac{2(2\xi-\mu)\lambda q_0\xi\theta\delta^2+2(2\xi-\mu)\lambda q_1\xi\theta^2\delta+4\lambda q_2\xi\theta^2\delta^2+(2\xi-2\mu)\lambda q_3\theta^2\delta^2+\mu^2\theta^2\delta^2+2\lambda^2 {q_3}^2\theta^2\delta^2+4\lambda^2 q_0q_1\xi^2\delta\theta +4\lambda^2 {q_0}^2\xi^2\delta^2+4\lambda^2 {q_1}^2\xi^2\theta^2+2\lambda^2 q_0q_3\xi\theta\delta^2+2\lambda^2 q_1q_3\xi\delta\theta^2+(\lambda q_3-\mu)\mu\,\delta\theta P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)}{2\xi\theta\delta(3\mu\theta\delta-2\lambda q_0\xi\delta-2\lambda q_1\xi\theta-2\lambda q_2\theta\delta-\lambda q_3\theta\delta)} \quad (22)$$

$$E(L \mid I=3) = \frac{2(2\xi+3\mu)\lambda q_0\xi\theta\delta^2+2(2\xi+3\mu)\lambda q_1\xi\delta\theta^2+2(2\xi+\mu)\lambda q_2\theta^2\delta^2+(2\xi-2\mu)\lambda q_3\theta^2\delta^2-2\mu^2\theta^2\delta^2+2\lambda^2 {q_3}^2\theta^2\delta^2-4\lambda^2 q_1q_2\xi\theta^2\delta -4\lambda^2 q_0q_1\xi^2\delta\theta-6\lambda^2 q_0q_3\xi\theta\delta^2-2\lambda^2 q_1q_3\xi\delta\theta^2-2\lambda^2 q_2q_3\theta^2\delta^2++\{(\lambda q_3-\mu)\delta\theta+2\xi\}\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)}{2\xi\theta\delta(3\mu\theta\delta-2\lambda q_0\xi\delta-2\lambda q_1\xi\theta-2\lambda q_2\theta\delta-\lambda q_3\theta\delta)} \quad (23)$$

The equilibrium balking strategy in the fully observable case has been studied in [8]. Consider a customer who finds the system at state (n, i) upon arrival, we can get the expected mean sojourn time of such a customer that decides to enter the system from [8]:

T (n, 3) = T (n, 2) = (n + 1) $(1 + \frac{\xi}{\delta} + \frac{\xi}{\theta})\frac{1}{\mu}$,

T (n, 1) = (n + 1) $(1 + \frac{\xi}{\delta} + \frac{\xi}{\theta})\frac{1}{\mu} + \frac{1}{\theta} + \frac{1}{\delta}$,

T (n, 0) = (n + 1) $(1 + \frac{\xi}{\delta} + \frac{\xi}{\theta})\frac{1}{\mu} + \frac{1}{\theta}$,

If a customer decides to join the system given that the server is found at state i, then his expected net benefit will be

$$s_3(q_0, q_1, q_2, q_3) = R - C\,[(E(L \mid I = 3) + 1)(1 + \frac{\xi}{\delta} + \frac{\xi}{\theta})\frac{1}{\mu}], \quad (24)$$

$$s_2(q_0, q_1, q_2, q_3) = R - C\,[(E(L \mid I = 2) + 1)(1 + \frac{\xi}{\delta} + \frac{\xi}{\theta})\frac{1}{\mu}], \quad (25)$$

$$s_1(q_0, q_1, q_2, q_3) = R - C\,[(E(L \mid I = 1) + 1)(1 + \frac{\xi}{\delta} + \frac{\xi}{\theta})\frac{1}{\mu} + \frac{1}{\theta} + \frac{1}{\delta}], \quad (26)$$

$$s_0(q_0, q_1, q_2, q_3) = R - C\,[(E(L \mid I = 0) + 1)(1 + \frac{\xi}{\delta} + \frac{\xi}{\theta})\frac{1}{\mu} + \frac{1}{\theta}], \quad (27)$$

Theorem 1. In the almost unobservable M / M /1 queue with redundant server breakdowns and delayed repairs under the stability condition $\mu(3\delta\theta) > \lambda(3\delta\theta + 2\xi\theta + 2\xi\delta)$, there exists a mixed strategy($q_e(0)$, $q_e(1)$, $q_e(2)$, $q_e(3)$)and the strategy has the form 'while arriving at time t, observe I (t), enter with probability $q_e(I(t))$'. In addition, the vector ($q_e(0)$, $q_e(1)$, $q_e(2)$, $q_e(3)$) is specified by:

Case A: If $\frac{R}{C} < [(6\xi + \mu)\delta\theta - \mu P(0,3)(3\delta\theta + 2\xi\theta + 2\xi\delta)]\left(\frac{\delta\theta + \xi\theta + \xi\delta}{6\xi\delta^2\theta^2\mu}\right)$ then ($q_e(0)$, $q_e(1)$, $q_e(2)$, $q_e(3)$) = (0, 0, 0, 0);

Case B: If $\frac{R}{C} > \left(\frac{(6\xi+\mu)\delta\theta - \mu P(0,3)(3\delta\theta + 2\xi\theta + 2\xi\delta)}{3\mu - \lambda}\right)\left(\frac{\delta\theta + \xi\theta + \xi\delta}{2\xi\delta^2\theta^2}\right) + \frac{1}{\theta} + \frac{1}{\delta}$, then ($q_e(0)$, $q_e(1)$, $q_e(2)$, $q_e(3)$) = (0, 0, 1 ,0);

Case C: If $\frac{R}{C} > \left(\frac{(6\xi+\mu)\delta\theta + 4\lambda\xi\delta - \mu P(0,3)(3\delta\theta + 2\xi\theta + 2\xi\delta)}{[(3\mu - \lambda)\theta - 2\lambda\xi]}\right)\left(\frac{\delta\theta + \xi\theta + \xi\delta}{2\xi\delta^2\theta}\right) + \frac{1}{\theta} + \frac{1}{\delta}$, then

($q_e(0)$, $q_e(1)$, $q_e(2)$, $q_e(3)$) = (1, 0, 1, 0);

Case D: If $\frac{R}{C} > \left(\frac{\mu\theta\delta(6\xi+\mu) + 4\lambda\xi(\delta\mu + \mu\theta - \theta\lambda)(\theta + \xi) - \mu P(0,3)(3\delta\theta + 2\xi\theta + 2\xi\delta)}{\delta\theta(3\mu - 2\lambda) - 2\lambda\xi(\delta + \theta)}\right)\left(\frac{\delta\theta + \xi\theta + \xi\delta}{2\xi\theta\delta\mu}\right) + \frac{1}{\theta} + \frac{1}{\delta}$, then

($q_e(0)$, $q_e(1)$, $q_e(2)$, $q_e(3)$) = (1, 1, 1, 0);

Proof. The proof is fairly delicate and lengthy. It is intuitively clear that, q_e (1) ≤, q_e (0) ≤, q_e (2) because customers are willing to enter the system when the server is in working state than in breakdown state. We tag a customer at his arrival instant, the customer prefers to balk if $S_i(q_0, q_1, q_2, q_3) < 0$. if $S_i(q_0, q_1, q_2, q_3) = 0$, he is indifferent between joining and balking. Otherwise, he joins the queue. We will show that there are seven cases;

Case A: If $\frac{R}{C} < [(6\xi + \mu)\delta\theta - \mu P(0,3)(3\delta\theta + 2\xi\theta + 2\xi\delta)]\left(\frac{\delta\theta + \xi\theta + \xi\delta}{6\xi\delta^2\theta^2\mu}\right)$, then S_2 (0, 0, 0, 0) <0 and S_1 (0, 0, 0, 0) = S_2 (0, 0, 0, 0) - $\frac{C}{\theta}$ - $\frac{C}{\delta}$ and S_0 (0, 0, 0, 0) = S_2 (0, 0, 0, 0) - $\frac{C}{\theta}$. *if* S_3 (0, 0, 0, 0) <0 therefore, if all other customers use (0, 0, 0, 0) as their strategy, the tagged customer suffers a negative reward. Hence, the tagged customer's best choice would be to balk if he observes the server at state 3.

Furthermore S_2 (0, 0, 0, 0) < 0, if all other customers use (0, 0, 0, 0) as their strategy, the net reward is negative. Thus, the best choice is balking if he finds the server at state 2.

Similarly, S_1 (0, 0, 0, 0) <0 and S_0 (0, 0, 0, 0) < 0 so that, q_e (1) = 0, q_e (0) = 0.

In a word, If $\frac{R}{C} < [(6\xi+\mu)\delta\theta - \mu P(0,3)(3\delta\theta + 2\xi\theta + 2\xi\delta)]\left(\frac{\delta\theta+\xi\theta+\xi\delta}{6\xi\delta^2\theta^2\mu}\right)$,then ($q_e$(0), q_e (1), q_e (2), q_e (3)) = (0, 0, 0, 0).

Case B: If $\frac{R}{C} > \left(\frac{(6\xi+\mu)\delta\theta-\mu P(0,3)(3\delta\theta+2\xi\theta+2\xi\delta)}{3\mu-\lambda}\right)\left(\frac{\delta\theta+\xi\theta+\xi\delta}{2\xi\delta^2\theta^2}\right)+\frac{1}{\theta}+\frac{1}{\delta}$, then S_1 (0, 0, 1, 0) > 0 If all other customers use (0, 0, 0 ,1) as their strategy, the tagged customer receives a positive reward. Therefore, the tagged customer enters the system with probability 1 if the server is found at state 1.

S_1 (0, 0,1, 0) = S_0 (0, 0, 1, 0) $-\frac{C}{\delta}$ >0 then S_0 (0, 0, 1, 0) > 0. If all other customers use (0, 0, 1, 0) as their strategy, then the net benefit for the tagged customer is positive and we have q_e(0) = 0. Finally, S_2 (0, 0, 1, 0) > 0 and q_e (2) = 1.

For the Case B, $if \frac{R}{C} > \left(\frac{(6\xi+\mu)+4\lambda}{3\mu-\lambda}\right)\left(\frac{\delta\theta+\xi\theta+\xi\delta}{2\xi\theta\delta}\right)+\frac{1}{\theta}+\frac{1}{\delta}$ then the equilibrium mixed strategy is (q_e(0), q_e (1), q_e (2), q_e (3)) = (0, 0, 1,0)

Case C: If $\frac{R}{C} > \left(\frac{(6\xi+\mu)\delta\theta+4\lambda\xi\delta-\mu P(0,3)(3\delta\theta+2\xi\theta+2\xi\delta)}{[(3\mu-\lambda)\theta-2\lambda\xi]}\right)\left(\frac{\delta\theta+\xi\theta+\xi\delta}{2\xi\delta^2\theta}\right)+\frac{1}{\theta}+\frac{1}{\delta}$, then S_1 (1, 0, 0, 1) > 0 If all other customers use (1, 0, 1 ,0) as their strategy, the tagged customer receives a positive reward. Therefore, the tagged customer enters the system with probability 1 if the server is found at state 1.

S_1 (1, 0, 1, 0) = S_2 (1, 0, 1, 0) $-\frac{C}{\theta}-\frac{C}{\delta}$ >0 then S_2 (1, 0, 1,0) > 0. If all other customers use (1, 0, 1, 0) as their strategy, then the net benefit for the tagged customer is positive and we have q_e (0) = 1. Finally, S_2 (1, 0, 1, 0) > 0 and q_e (2) = 1 .as a result the best response as the tagged customer who finds the server at state 0 and 2 or 3 upon arrival is to enter the system

To sum up, we have (q_e(0), q_e (1), q_e (2), q_e (3)) = (1, 0, 1, 0) in this case.

Case D: If $\frac{R}{C} > \left(\frac{\mu\theta\delta(6\xi+\mu)+4\lambda\xi(\delta\mu+\mu\theta-\theta\lambda)(\theta+\xi)-\mu P(0,3)(3\delta\theta+2\xi\theta+2\xi\delta)}{\delta\theta(3\mu-2\lambda)-2\lambda\xi(\delta+\theta)}\right)\left(\frac{\delta\theta+\xi\theta+\xi\delta}{2\xi\theta\delta\mu}\right)+\frac{1}{\theta}+\frac{1}{\delta}$ thenS_1 (1, 1, 1, 0) > 0. Substitution of $q_0 = q_1 = q_2$ = 1 into it follows that S_2 (1, 1, 1, 0) > S_0 (1 ,1, 1, 0) > S_1 (1,1, 1, 0) > 0. Evidently, S_1 (1, 1, 1, 0) > 0. That is, if all other customers use (1 ,1 ,1, 0) as their strategy, the best response of the tagged customer is entering if he finds the server at state 1. And we have q_e(1) = 1.

Moreover, S_0(1, 1, 1 ,0) > 0, if all other customers use (1, *1, 1, 0*) as their strategy, the tagged customer receives a positive reward. Thus, he joins the system with probability 1 if he finds the server at state 0. We have q_e (0) = 1

Similarly, S_2 (1, 1, 1, 0) >0 and q_e (2) = 1. In a word, we have *(*q_e(0), q_e (1), q_e (2), q_e (3)) = (1, 1, 1, 0) in this case.

V. THE FULLY UNOBSERVABLE QUEUE

Now we focus our attention on the fully unobservable queue with redundant server where arriving customers do not observe the number of customers in the system or the state of the server. The fully unobservable case in real-life situation can be illustrated by the decision making of customers in the call centers and data center, the servers may not provide the information with respect to the number of customers just waiting in the system prior to the customer's arrival. And there is one extra server is available the customer has to evaluate the net benefits of his decisions. In the fully unobservable case, a mixed strategy has the form 'while arriving at time t, do not observe (L (t), I (t)), enter with probability q'. Clearly, the new queue is equivalent to the original queue except that the arrival intensity λ should be replaced by λq. The formulas for the fully unobservable case are special cases of the formulas for the almost unobservable case. By taking $q_0 = q_1 = q_2 = q_3$ = q, we can get the expressions of $G_i'(1)$ in a way similar to that exhibited in Section IV, Then

$$G_0'(1) = \frac{\begin{array}{c}(2\xi+2\mu)\lambda q\pi_0\theta\delta+(2\xi+2\mu)\lambda q\pi_1\delta\theta+2\lambda q\pi_2\xi\theta\delta+(2\xi-2\mu)\lambda q\pi_3\theta\delta+\mu^2\pi_3\theta\delta+2\lambda^2q^2\pi_3\theta\delta-2\lambda^2q^2\pi_1\theta\delta\\ -2\lambda^2q^2\pi_0\theta(\xi+\delta)+(\lambda q-\mu)\mu\delta\theta P(0,3)\end{array}}{\theta(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)} \quad (28)$$

$$G_1'(1) = \frac{\begin{array}{c}(2\xi-\mu)\lambda q\pi_0\theta\delta+(2\xi+2\mu)\lambda q\pi_1\delta\theta+2\lambda q\pi_2\xi\theta\delta+(2\xi-2\mu)\lambda q\pi_3\theta\delta+\mu^2\pi_3\theta\delta-2\lambda^2q^2\pi_1\theta\delta+2\lambda^2q^2\pi_3\theta\delta\\ +\lambda^2q^2\pi_0\delta(2\xi+\theta)+(\lambda q-\mu)\mu\delta\theta P(0,3)\end{array}}{\delta(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)} \quad (29)$$

$$G_2'(1) = \frac{\begin{matrix}(2\xi-\mu)\lambda q\pi_0\theta\delta+(2\xi-\mu)\lambda q\pi_1\delta\theta+2\lambda q\pi_2\xi\theta\delta+(2\xi-2\mu)\lambda q\pi_3\theta\delta+\mu^2\pi_3\theta\delta+\lambda^2q^2\pi_1(\delta\theta+2\xi\theta+2\xi\delta)\\ +\lambda^2q^2\pi_0(2\xi\delta+\delta\theta)+2\lambda^2q^2\pi_3\theta\delta+(\lambda q-\mu)\mu\delta\theta P(0,3)\end{matrix}}{\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)} \quad (30)$$

$$G_3'(1) = \frac{\begin{matrix}(2\xi+2\mu)\lambda q\pi_0\theta\delta+(2\xi+2\mu)\lambda q\pi_1\delta\theta+2\lambda q\pi_2\xi\theta\delta+2\lambda q\pi_3\xi\theta\delta-2\mu^2\pi_3\theta\delta-\lambda^2q^2\pi_3\theta\delta-2\lambda^2q^2\pi_1\theta\delta\\ -2\lambda^2q^2\pi_0(\theta\xi+\delta\theta)-2\lambda^2q^2\pi_3\xi(\theta+\delta)+2\lambda\mu q\pi_3(\theta\xi+\xi\delta+2\delta\theta)+\{(\lambda q-\mu)\delta\theta+2\xi\}\mu P(0,3)\end{matrix}}{2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)} \quad (31)$$

Clearly $G_i'(1) = \sum_{n=0}^{\infty} np(n,i)$ i = 0, 1, 2,3.

thus, the mean queue length is given by

E (L) = $G_0'(1)$ + $G_1'(1)$ + $G_2'(1)$ + $G_3'(1)$ =

$$\frac{\begin{matrix}2\lambda q\xi[\pi_0(2\xi\delta+2\mu\delta+2\xi\theta+3\theta\delta-2\mu\theta)+\pi_1(2\xi\delta+2\mu\delta+2\mu\theta+2\xi\theta+3\theta\delta)+\pi_2(2\xi\delta+2\xi\theta+3\theta\delta)+\pi_3(2\xi\delta+2\xi\theta+3\theta\delta)]+2\mu^2\pi_3\xi(\delta+\theta)\\ +\lambda^2q^2\pi_3\theta\delta(2\xi\delta+2\xi\theta+3\theta\delta)+\{(\lambda q-\mu)(2\xi\delta+2\xi\theta+3\theta\delta)+2\xi\delta\theta\}\mu P(0,3)\end{matrix}}{2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)}$$

$$= \frac{2\lambda q\xi[\pi_0 2\mu\delta-\pi_0 2\mu\theta+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+2\mu^2\pi_3\xi(\delta+\theta)+\lambda^2q^2\theta^2\delta^2+\{(\lambda q-\mu)(2\xi\delta+2\xi\theta+3\theta\delta)+2\xi\delta\theta\}\mu P(0,3)}{2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)} \quad (32)$$

Therefore, the mean sojourn time of a customer who joins the system can be derived by using Little's law, then

E (W) = $\frac{E(L)}{\lambda q}$

$$= \frac{2\xi[\pi_0 2\mu\delta-\pi_0 2\mu\theta+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+\lambda q\theta^2\delta^2+\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)+\frac{2\mu^2\pi_3\xi(\delta+\theta)+\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}\mu P(0,3)}{\lambda q}}{2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)} \quad (33)$$

Clearly, E (W) is strictly increasing for q, q ∈ [0, 1]. This property is crucial for our analysis. A general balking strategy in the fully unobservable case is specified by a single joining probability q. The case q = 0 corresponds to the pure strategy 'to balk' whereas the case q = 1 corresponds to the pure strategy 'to join'. Any value of q ∈ (0, 1) corresponds to a mixed strategy 'to join with probability q'. We can describe the equilibrium behavior of the customers in the following theorem. The equilibrium strategies depend on the value of the ratio R /C. Customers have a greater incentive to enter the system if the value of R/ C is higher.

Theorem 2. In the fully unobservable M / M /1 queue with redundant server with breakdowns and delayed repairs, there exists a unique equilibrium strategy 'enter with probability q_e ', where q_e is specified

$$q_e = \begin{cases} 0 & , \frac{R}{C} < \mathrm{E(W)}|_{q=0}, \\ q_e^* & , \mathrm{E(W)}|_{q=0} \le \frac{R}{C} \le \mathrm{E(W)}|_{q=1} \\ 1 & , \frac{R}{C} > \mathrm{E(W)}|_{q=1} \end{cases}$$

$$\mathrm{E(W)}|_{q=0} > \frac{[\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+(2\xi\delta+2\xi\theta+3\theta\delta)(1+\mu P(0,3)]}{3\mu\theta\delta}$$

$$\mathrm{E(W)}|_{q=1} = \frac{\begin{matrix}2\xi[\pi_0 2\mu\delta-\pi_0 2\mu\theta+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+\lambda\theta^2\delta^2+\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)\\ +\frac{2\mu^2\pi_3\xi(\delta+\theta)+\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}\mu P(0,3)}{\lambda}\end{matrix}}{2\xi(3\mu\theta\delta-2\lambda\xi\delta-2\lambda\xi\theta-3\lambda\theta\delta)}$$

$$q_e^* = \frac{\begin{matrix}[6R\mu\xi\theta\delta-C\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)-C\xi\{\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta\}]\pm\\ \sqrt{\begin{matrix}\{6R\mu\xi\theta\delta-C\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)-C\xi\{\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta\}\}^2\\ -4C[2\mu^2\pi_3\xi(\delta+\theta)+\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}\mu P(0,3)]\{C\theta^2\delta^2+4R\xi^2\delta+4R\xi^2\theta+6R\xi\theta\delta\}\end{matrix}}\end{matrix}}{2\lambda\{C\theta^2\delta^2+4R\xi^2\delta+4R\xi^2\theta+6R\xi\theta\delta\}}$$

Proof. Based on the reward-cost structure, if a tagged customer decides to enter the system at his arrival instant, his expected net reward is

S (q) = R −C E(W)

$$=R-C\ \frac{2\xi[\pi_0 2\mu\delta-\pi_0 2\mu\theta+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+\lambda q\theta^2\delta^2+\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)+\frac{2\mu^2\pi_3\xi(\delta+\theta)+\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}\mu P(0,3)}{\lambda q}}{2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)} \quad (34)$$

Clearly, S (q) is strictly decreasing for q, q ∈ [0, 1]. In addition,

$$E(W)=\frac{2\xi[\pi_0 2\mu\delta-\pi_0 2\mu\theta+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+\lambda q\theta^2\delta^2+\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)+\frac{2\mu^2\pi_3\xi(\delta+\theta)+\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}\mu P(0,3)}{\lambda q}}{2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)}$$

$$>\frac{2\xi[\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+\lambda q\theta^2\delta^2+\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)}{2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)}$$

$$E(W)|_{q=0}>\frac{[\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+(2\xi\delta+2\xi\theta+3\theta\delta)(1+\mu P(0,3)]}{3\mu\theta\delta} \quad (35)$$

$$E(W)|_{q=1}=\frac{2\xi[\pi_0 2\mu\delta-\pi_0 2\mu\theta+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+\lambda\theta^2\delta^2+\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)+\frac{2\mu^2\pi_3\xi(\delta+\theta)+\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}\mu P(0,3)}{\lambda}}{2\xi(3\mu\theta\delta-2\lambda\xi\delta-2\lambda\xi\theta-3\lambda\theta\delta)} \quad (36)$$

Now,from eq(34)

$$S(0)<R-C\ \frac{[\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+(2\xi\delta+2\xi\theta+3\theta\delta)(1+\mu P(0,3)]}{3\mu\theta\delta}$$

$$S(1)=R-C\ \frac{2\xi[\pi_0 2\mu\delta-\pi_0 2\mu\theta+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+\lambda\theta^2\delta^2+\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)+\frac{2\mu^2\pi_3\xi(\delta+\theta)+\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}\mu P(0,3)}{\lambda}}{2\xi(3\mu\theta\delta-2\lambda\xi\delta-2\lambda\xi\theta-3\lambda\theta\delta)}$$

When $\frac{R}{C}<E(W)|_{q=0}$, S (q) is negative for every q. Therefore, the tagged customer's best choice would be to balk. If $\frac{R}{C}>E(W)|_{q=1}$, then S (q) ≥S (1) > 0, the expected net benefit of the tagged customer is positive, thus he joins the system with probability 1.

When $E(W)|_{q=0}\le\frac{R}{C}\le E(W)|_{q=1}$, there exists a unique root q_e^* of the equation S (q) = 0 in the interval [0, 1].

$$q_e^*=\frac{[6R\mu\xi\theta\delta-C\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)-C\xi\{\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta\}]\pm\sqrt{\{6R\mu\xi\theta\delta-C\mu P(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)-C\xi\{\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta\}\}^2-4C[2\mu^2\pi_3\xi(\delta+\theta)+\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}\mu P(0,3)]\{C\theta^2\delta^2+4R\xi^2\delta+4R\xi^2\theta+6R\xi\theta\delta\}}}{2\lambda\{C\theta^2\delta^2+4R\xi^2\delta+4R\xi^2\theta+6R\xi\theta\delta\}}$$

The goal of a social planer is to maximize overall social welfare, that is, the sum of customer utility and the payoff of server. The expected net social benefit per time unit, given that the customers follow a mixed strategy with joining probability q is given by

$$SB(q)=\lambda qR-C\ \frac{2\lambda q\xi[\pi_0 2\mu\delta-\pi_0 2\mu\theta+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta]+2\mu^2\pi_3\xi(\delta+\theta)+\lambda^2q^2\theta^2\delta^2+\{(\lambda q-\mu)(2\xi\delta+2\xi\theta+3\theta\delta)+2\xi\delta\theta\}\mu P(0,3)}{2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)}$$

then

$$SB'(q)=\lambda R-C\ \frac{\begin{array}{c}2\xi(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)\left[2\xi\lambda\{\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta\}+\lambda^2 2q\theta^2\delta^2+\lambda\mu p(0,3)(2\xi\delta+2\xi\theta+3\theta\delta)\right]\\-\left[2\lambda q\xi\{\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta\}+\lambda^2q^2\theta^2\delta^2+2\mu^2\pi_3\xi(\delta+\theta)+\{(\lambda q-\mu)(2\xi\delta+2\xi\theta+3\theta\delta)+2\xi\delta\theta\}\mu P(0,3)\right]\\(-4\lambda\xi^2\delta-4\lambda\xi^2\theta-6\lambda\xi\theta\delta)\end{array}}{4\xi^2(3\mu\theta\delta-2\lambda q\xi\delta-2\lambda q\xi\theta-3\lambda q\theta\delta)^2}$$

Let $\hat{q}$ denote the root of equation SB'(q)= 0 and the optimal entrance probability for the system is denoted by q^*,then

$$\widehat{q} = \frac{3\mu\delta\theta}{\lambda(2\xi\theta+2\xi\delta+3\delta\theta)} \pm \frac{1}{\lambda(2\xi\theta+2\xi\delta+3\delta\theta)}\sqrt{\frac{\begin{array}{c}9\mu^2\delta^2\theta^2\{2R\xi(2\xi\theta+2\xi\delta+3\delta\theta)+C\delta^2\theta^2\} \\ -2\mu\xi(2\xi\theta+2\xi\delta+3\delta\theta)[9R\delta\mu\delta^2\theta^2-C(\theta+\delta)[2\mu\pi_3\xi(\delta+\theta)+p(0,3)\{2\xi\theta\delta-\mu(2\xi\delta+2\xi\theta+3\theta\delta)\}] \\ -3C\delta\theta\{\pi_0 2\mu(\delta-\theta)+\pi_1 2\mu(\delta+\theta)+2\xi\delta+2\xi\theta+3\theta\delta+\mu\pi_3(\delta+\theta)+\delta\theta p(0,3)\}]\end{array}}{\{2R\xi(2\xi\theta+2\xi\delta+3\delta\theta)+C\delta^2\theta^2\}}}$$

Based on the stability condition, we can conclude that SB''(q)< 0 for any probability q ∈ [0, 1]. Thus, the social welfare function SB (q) is strictly concave, and it attains a unique maximum at the point q = $\widehat{q}$ If 0 < $\widehat{q}$ < 1, then the optimal entrance probability for the system is $q^*=\widehat{q}$. If $\widehat{q}$ > 1, then q^*= 1. In a word, we have

q^*= min { $\widehat{q}$, 1}.

VI. CONCLUSION

Inspired by Wang and Zhang [8], we study the equilibrium strategies for the almost unobservable and fully unobservable single-server queues with redundant server with breakdowns and delayed repairs. In practical customer faces a big problem before enter the system he has to decide whether balk the system or enter the system in our model the balking capacity of customers is less and customer most probably enter the system. By the redundant server the system provides a reliable working facility to the customer the repair time is not memoryless since the repair time has two stages. This work can be generalized in the different directions. the case of general interarrival times is the simplest generalization of this paper . In addition, A multi-state queueing model is a direct extension to this study. In practice, a system is frequently subjected to breakdowns with different difficulties. we can also study the social benefit for different information levels.

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