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Bipolar Pythagorean Fuzzy Regular α Generalized Closed Sets

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Abstract - *In this paper, The concept of Regular α Generalized Closed sets in Bipolar Pythagorean Fuzzy Topological Spaces are studied. We introduce the concept of Bipolar Pythagorean Fuzzy Regular α Generalized Open Sets. Some interesting properties are investigated with some examples.*

Keywords : *Bipolar Pythagorean Fuzzy Sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagoren Fuzzy Regular α Generalized Closed Sets, Bipolar Pythagorean Fuzzy Regular α Generalized Open Sets.*

I. INTRODUCTION

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by L.A. Zadeh [12]. Fuzzy sets have appilications in many field such as information and control. The theory of fuzzy topological space was introduced and developed by C.L. Chang[6] and since then various notions in classical topology have been extended to fuzzy topological space. The idea of intuitionistic fuzzy set was first published by Atanassov [2] and many works by the same author and his colleagues appeared in the literature. Yager [3] proposed another class of nonstandard fuzzy sets, called Pythagorean fuzzy sets and Murat Olgun, Mehmet Ünver, Seyhmus Yardimci[16] introduced the notion of Pythagorean fuzzy topological spaces. Zhang [14] introduced the extension of fuzzy set with bipolarity, called Bipolar value fuzzy sets. Bosc and Pivert[10] said that Bipolarity refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. In bipolar valued fuzzy set, the interval of membership value is [-1,1]. The positive membership degrees represents the possibilities of something to be happened whereas the negative membership degrees represents the impossibilities. Azhzgappan and Kamaraj[15] investigated Bipolar Fuzzy Topological Spaces. Kim et al[9]constructed bipolar fuzzy set and preserving mappings between them and studied it in the sense of a topological universe. Mohana[13] has introduced the bipolar pythagorean fuzzy π generalized pre-closed sets. In this paper we introduce bipolar pythagorean fuzzy α generalized closed set and bipolar pythagorean fuzzy α generalized open set in bipolar pythagorean fuzzy topological spaces.

II. PRELIMINARIES

Definition 2.1: Let X be the non empty universe of discourse. A fuzzy set A in X, $A = \{(x, \mu_A(x)): x \in X\}$ where $\mu_A: X \to [0,1]$ is the membership function of the fuzzy set A; $\mu_A(x) \in [0,1]$ is the membership of $x \in X$.

Definition 2.2: Let X be the non empty universe of discourse. An Intuitionistic fuzzy set(IFS) A in X is given by A=$\{x, \mu_A(x), \nu_A(x): x \in X\}$ where the functions $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$ for each $x \in X$. The degree of indeterminacy $I_A = 1 - (\mu_A(x) + \nu_A(x))$ for each $x \in X$.

Definition 2.3: Let X be the non empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by P=$\{\langle x, \mu_P(x), \nu_P(x)\rangle: x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set P, respectively, and 0$\leqslant \mu_P^2(x) + \nu_P^2(x) \leqslant$1 for each $x \in X$. The degree of indeterminacy $I_P = \sqrt{1 - (\mu_P^2(x) + \nu_P^2(x))}$ for each $x \in X$.

Definition 2.4: Let X be a non empty set. A Bipolar Pythagorean Fuzzy Set $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-): x \in X\}$ where $\mu_A^+: X \to [0,1], \nu_A^+: X \to [0,1], \mu_A^-: X \to [-1,0], \nu_A^-: X \to [-1,0]$ are the mappings such that 0$\leqslant (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leqslant$1 and -1$\leqslant (\mu_A^-(x))^2 + (\nu_A^-(x)^2 \leqslant$0 where

$\mu_A^+(x)$ denote the positive membership degree.
$\nu_A^+(x)$ denote the positive non membership degree.
$\mu_A^-(x)$ denote the negative membership degree.
$\nu_A^-(x)$ denote the negative non membership degree.

Definition 2.5: Let $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B^+(x), \nu_B^+(x), \mu_B^-(x), \nu_B^-(x)\rangle : x \in X\}$ be two Bipolar Pythagorean Fuzzy sets over X. Then,

(i) The Bipolar Pythagorean fuzzy Complement of A is defined by
$A^c = \{\langle x, \nu_A^+(x), \mu_A^+(x), \nu_A^-(x), \mu_A^-(x)\rangle : x \in X\}$

(ii) The Bipolar Pythagorean fuzzy intersection of A and B is defined by
$A \cap B = \{\langle x, \min\{\mu_A^+(x), \mu_B^+(x)\}, \max\{\nu_A^+(x), \nu_B^+(x)\}, \max\{\mu_A^-(x), \mu_B^-(x)\}, \min\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$

(iii) The Bipolar Pythagorean fuzzy union of A and B is defined by
$A \cup B = \{\langle x, max\{\mu_A^+(x), \mu_B^+(x)\}, min\{\nu_A^+(x), \nu_B^+(x)\}, min\{\mu_A^-(x), \mu_B^-(x)\}, max\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$

(iv) A is a Bipolar Pythagorean subset of B and write $A \subseteq B$ if $\mu_A^+(x) \leqslant \mu_B^+(x), \nu_A^+(x) \geqslant \nu_B^+(x), \mu_A^-(x) \geqslant \mu_B^-(x), \nu_A^-(x)\ \nu_B^-(x)\rangle$ for each $x \in X$.

(v) $0_X = \{\langle x, 0,1,0,-1\rangle : x \in X\}$ and $1_X = \{\langle x, 1,0,-1,0\rangle : x \in X\}$.

Definition 2.6: Bipolar Pythagorean Fuzzy Topological Spaces: Let $X \neq \emptyset$ be a set and τ_p be a family of Bipolar Pythagorean fuzzy subsets of X. If

T_1 $0_X, 1_X \in \tau_p$.
T_2 For any $P_1, P_2 \in \tau_p$, we have $P_1 \cap P_2 \in \tau_p$.
T_3 $\cup P_i \in \tau_p$ for arbitrary family $\{P_i$ such that $i \in J\} \subseteq \tau_p$.

Then τ_p is called Bipolar Pythagorean Fuzzy Topology on X and the pair (X, τ_p) is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of τ_p is called Bipolar Pythagorean fuzzy open set(BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set(BPFCS).

Definition2.7: Let (X, τ_p) be a BPFTS and $P = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ be a BPFS over X. Then the Bipolar Pythagorean Fuzzy Interior, Bipolar Pythagorean Fuzzy Closure of P are defined by:

(i) BPFint(P) = $\cup\{G\ /G\ is\ a\ BPFOS\ in\ (X, \tau_p)\ and\ G \subseteq P\}$.
(ii) BPFcl(P) = $\cap\{K\ /K\ is\ a\ BPFCS\ in\ (X, \tau_p)\ and\ P \subseteq K\}$.

It is clear that
a. BPFint(P) is the biggest Bipolar Pythagorean Fuzzy Open set contained in P.
b. BPFcl(P) is the smallest Bipolar Pythagorean Fuzzy Closed set containing P.

Definition 2.8: Let (X, τ_p) be a BPFTS and A, B be two Bipolar Pythagorean Fuzzy sets in X. Then Bipolar Pythagorean Fuzzy Interior holds the following properties:

a) $int(A) \subseteq A$
b) $A \subseteq B \Longrightarrow int(A) \subseteq int(B)$
c) $int(int(A)) = int(A)$
d) $int(A \cap B) = int(A) \cap int(B)$
e) $int(0_A) = 0_A$
f) $int(1_A) = 1_A$
g) $int(A \cup B) \supseteq int(A) \cup int(B)$

Definition 2.9: Let (X, τ_p) be a BPFTS and A, B be two Bipolar Pythagorean Fuzzy sets in X. Then Bipolar Pythagorean Fuzzy Interior holds the following properties:

a) $A \subseteq cl(A)$
b) $A \subseteq B \Longrightarrow cl(A) \subseteq cl(B)$
c) $cl(cl(A)) = cl(A)$
d) $cl(A \cup B) = cl(A) \cup cl(B)$

e) $cl(0_A) = 0_A$
f) $cl(1_A) = 1_A$
g) $cl(A \cap B) \subseteq cl(A) \cap cl(B)$

Definition 2.10: If BPFS $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ in a BPTS (X, τ_p) is said to be
(a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if $int(cl(A)) \subseteq A$
(b) Bipolar Pythagorean Fuzzy Semi open set (BPFSOS) if $A \subseteq cl(int(A))$
(c) Bipolar Pythagorean Fuzzy Pre-closed set(BPFPCS) if $cl(int(A)) \subseteq A$
(d) Bipolar Pythagorean Fuzzy Pre-open set(BPFPOS) if $A \subseteq int(cl(A))$
(e) Bipolar Pythagorean Fuzzy α closed set (BPFRαCS) if $cl(int(cl(A)) \subseteq A$
(f) Bipolar Pythagorean Fuzzy α open set (BPFαOS) if $A \subseteq int(cl(int(A))$
(g) Bipolar Pythagorean Fuzzy γ closed set (BPFγCS) if $A \subseteq int(cl(A) \cup cl(int(A))$
(h) Bipolar Pythagorean Fuzzy γ open set (BPFγOS) if $cl(int(A) \cup int(cl(A)) \subseteq A$
(i) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if $A = cl(int(A))$
(j) Bipolar Pythagorean Fuzzy regular open set (BPFROS) if $A = int(cl(A))$
(k) If BPF set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Generalized closed set(BPFGCS), if $cl(A) \subseteq U$whenever $A \subseteq U$ and U is BPFOS in X.
(l) If BPF set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Generalized open set(BPFGOS), if A^c is a BPFGCS in X.
(m) If BPF set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Regular Generalized closed set(BPFRGCS), if $cl(A) \subseteq U$whenever $A \subseteq U$ and U is BPFROS in X.
(n) If BPF set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Regular Generalized open set(BPFRGOS), if A^c is a BPFRGCS in X.

III. BIPOLAR PYTHAGOREAN FUZZY REGULAR α GENERALIZED CLOSED SETS

In this paper we define,
(i) Bipolar Pythagorean Fuzzy Regular α Generalized Closed sets shortly as BPFRαGCS and
(ii) Bipolar Pythagorean Fuzzy Regular α Generalized Open sets as BPFRαGOS.

Definition 3.1: A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space (X, τ_p) is called Bipolar Pythagorean Regular α Generalized closed set (BPFRαGCS in short), if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPF regular open set in X.

Example 3.2: Let X={a, b} and τ_p={$0_p, T, 1_p$} be a BPFT on X, where T=(x, (0.3, 0.2), (0.5, 0.6), (-0.4, -0.3), (-0.7, -0.4)). Then the BPFS A=(x, (0.5, 0.3), (0.3, 0.3),(-0.7, -0.2), (-0.5, -0.7)) is a BPFRαGCS in (X, τ_p).

Proposition 3.3: Every BPFCS is BPFRαGCS in X but not conversely.
Proof: Let U be a BPF regular open set in X such that $A \subseteq U$. Since A is BPFCS, cl(A)=A. $\alpha cl(A) = A \cup cl(int(cl(A))) = A \cup cl(int(A)) \subseteq A \cup cl(A) = A \cup A = A \subseteq U$. Thus A is BPFR$\alpha$GCS in X.

Example 3.4: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x,(0.5, 0.8), (0.4, 0.3), (-0.6, -0.5),(-0.5, -0.4)) and T_2 = (x, (0.3, 0.2), (0.7, 0.9), (-0.4, -0.3),(-0.6, -0.8)). Here A is the BPFS then A = (x, (0.2, 0.3), (0.6, 0.9), (-0.3, -0.4), (-0.6, -0.6)) is a BPFRαGCS, but A is not a BPFCS in X.

Proposition 3.5: Every BPFRCS is a BPFRαGCS, but not conversely.
Proof: Let U be a BPF regular open set in X such that $A \subseteq U$. Since every BPF regular closed set is BPFCS, cl(A)=A. By hypothesis, $\alpha cl(A) = A \cup cl(int(cl(A))) = A \cup cl(int(A)) \subseteq A \cup cl(A) = A \cup A = A \subseteq U$. Hence $\alpha cl(A) \subseteq U$. Thus, A is BPFRαGCS in X.

Example 3.6: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 =(x, (0.5 ,0.8), (0.4, 0.3), (-0.6, -0.5), (-0.5, -0.4)) and T_2 = (x, (0.3, 0.2), (0.7, 0.9), (-0.4, -0.3), (-0.6, -0.8)). Here A is the BPFS then A = (x, (0.2, 0.3), (0.6, 0.9), (-0.3, -0.4), (-0.6, -0.6)) is a BPFRαGCS. But cl(int(A))=$0_\sim \neq A$. Therefore, A is not a BPFRCS in X.

Proposition 3.7: Every BPFαCS is a BPFRαGCS, but not conversely.
Proof: Let U be a BPF regular open set in X such that $A \subseteq U$. Since A is BPF α closed set, cl(int(cl(A))) $\subseteq A$. By hypothesis,

$\alpha cl(A) = A \cup cl(int(cl(A))) \subseteq A \cup A = A \subseteq U$. Hence $\alpha cl(A) \subseteq U$. Thus, A is BPFRαGCS in X.

Example 3.8: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.8), (0.4, 0.3), (-0.6, -0.5), (-0.5, -0.4)) and T_2 = (x, (0.3, 0.2), (0.7, 0.9), (-0.4, -0.3), (-0.6, -0.8)). Here A is the BPFS then A = (x, (0.2, 0.3), (0.6, 0.9), (-0.3, -0.4), (-0.6, -0.6)) is a BPFRαGCS. But cl(int((cl(A)) = $T_1^c \nsubseteq A$. Therefore, A is not a BPFαCS in X.

Proposition 3.9: The union of two BPFRαGCS is BPFRαGCS in X.
Proof: Let A and B be the BPFRαGCS in X. Let $A \cup B \subseteq U$, where U is BPF regular open in X. Then $A \subseteq U$ and $B \subseteq U$, where U is BPF regular open, which implies $\alpha cl(A) \subseteq U$ and $\alpha cl(B) \subseteq U$, this implies $\alpha cl(A \cup B) \subseteq U$, since $cl(A \cup B) = cl(A) \cup cl(B)$. Hence $(A \cup B)$ is also a BPFRαGCS in X.

Remark 3.10: The intersection of two BPFRαGCS is not BPFRαGCS in X as shown in the following Example.

Example 3.11: Let X={a, b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.5, 0.3), (0.6, 0.5), (-0.5, -0.3), (-0.7, -0.8)). Then the BPFS A = (x, (0.3, 0.1), (0.7, 0.4), (-0.5, -0.6), (-0.7, -0.4)) and B = (x, (0.6, 0.2), (0.3, 0.7), (-0.8, -0.3), (-0.3, -0.9)) is a BPFRαGCS, $A \cap B$ is not a BPFRαGCS in X.

Remark 3.12: Every BPFRαGCS and BPFPCS in X are independent to each other.

Example 3.13: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.6), (0.3, 0.4), (-0.6, -0.7), (-0.5, -0.7)) and T_2 = (x, (0.2, 0.3), (0.7, 0.7), (-0.3, -0.4), (-0.6, -0.8)). Here A is the BPFS then A = (x, (0.3, 0.3), (0.5, 0.4), (-0.4, -0.6), (-0.5, -0.3)) is a BPFRαGCS, but A is not a BPFPCS in X.

Example 3.14: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.3, 0.4), (0.8, 0.7), (-0.4, -0.5), (-0.7, -0.8)) and T_2 = (x, (0.8, 0.9), (0.3, 0.2), (-0.3, -0.3), (-0.6, -0.6)). Here A is the BPFS then A = (x, (0.1, 0.2), (0.8, 0.7), (-0.1, -0.2), (-0.8, -0.9)) is a BPFPCS, but A is not a BPFRαGCS in X.

Remark 3.15: Every BPFRαGCS and BPFSCS in X are independent to each other.

Example 3.16: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.6), (0.3, 0.4), (-0.6, -0.7), (-0.5, -0.7)) and T_2 = (x, (0.2, 0.3), (0.7, 0.7), (-0.3, -0.4), (-0.6, -0.8)). Here A is the BPFS then A = (x, (0.3, 0.3), (0.5, 0.4), (-0.4, -0.6), (-0.5, -0.3)) is a BPFRαGCS, but A is not a BPFSCS in X.

Example 3.17: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.4, 0.3), (0.5, 0.8), (-0.6, -0.3), (-0.6, -0.7)) and T_2 = (x, (0.3, 0.3), (0.8, 0.8), (-0.4, -0.2), (-0.7, -0.8)). Here A is the BPFS then A = (x, (0.4, 0.3), (0.5, 0.8), (-0.6, -0.3), (-0.6, -0.7)) is a BPFSCS, but A is not a BPFRαGS in X.

Proposition 3.18: Every BPFGCS is BPFRαGCS in X but not conversely.
Proof: Let A⊆ U and U be a BPF regular open set in X. By hypothesis, cl(A)⊆ U, whenever A⊆ U. This implies $\alpha cl(A) = A \cup cl(int(cl(A))) \subseteq A \cup cl(A)) \subseteq U$. Therefore, A is BPFR$\alpha$GCS in X.

Example 3.19: Let X={a, b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.6, 0.8), (0.3, 0.2), (-0.5, -0.7), (-0.3, -0.2)). Here A is the BPFS then A = (x, (0.5, 0.3), (0.6, 0.3), (-0.4, -0.5), (-0.6, -0.4)) is a BPFRαGCS, but A is not a BPFGCS in X.

Proposition 3.20: If A is both a BPF regular open set and BPFRαGCS in X, then A is a BPFRGCS in X.
Proof: Let $A \subseteq U$ and U be a BPFROS in X. By hypothesis, we have $\alpha cl(A) \subseteq U$ and cl(A)=cl(int(cl(A)))⊆ $A \cup cl(int(cl(A))) = \alpha cl(A) \subseteq U$. Hence A is BPFRGCS in X.

Proposition 3.21: If A is both a BPF pre-open set and BPFRαGCS in X, then A is a BPFRGCS in X.
Proof: Let $A \subseteq U$ and U be a BPFROS in X. By hypothesis, we have $\alpha cl(A) \subseteq U$ and cl(A)=cl(int(cl(A)))⊆ $A \cup cl(int(cl(A))) = \alpha cl(A) \subseteq U$. Hence A is BPFRGCS in X.

Proposition 3.22: If A is both BPFROS and BPFRαGCS in X, then A is BPFαCS in X.

Proof: As A ⊆ A, by hypothesis, $\alpha cl(A) \subseteq A$. But we have $A \subseteq \alpha cl(A)$. This implies α cl(A) =A. Hence A is BPFαCS in X.

Proposition 3.23: Let A be BPFRαGCS in X and $A \subseteq B \subseteq \alpha cl(A)$, then B is BPFR$\alpha$GCS in X.
Proof: Let B⊆ U and U is BPFROS in X. Then $A \subseteq U$ since $A \subseteq B$. As A is BPFRαCS in X, $\alpha cl(A) \subseteq U$ and by hypothesis B⊆ $\alpha cl(A)$. This implies $\alpha cl(B) \subseteq \alpha cl(A) \subseteq U$. Therefore, $\alpha cl(B) \subseteq U$ and hence B is BPFRαGCS in X.

Proposition 3.24: Let A be BPFRGCS in X and $A \subseteq B \subseteq cl(A)$, then B is BPFR$\alpha$GCS in X.
Proof: Let B⊆ U and U is BPFROS in X. Then $A \subseteq U$ since $A \subseteq B$. As A is BPFRCS in X, $cl(A) \subseteq U$ and by hypothesis B⊆ $cl(A)$. This implies $\alpha cl(B)(B) \subseteq cl(A) \subseteq U$. Therefore, $\alpha cl(B) \subseteq U$ and B is BPFRαGCS in X.

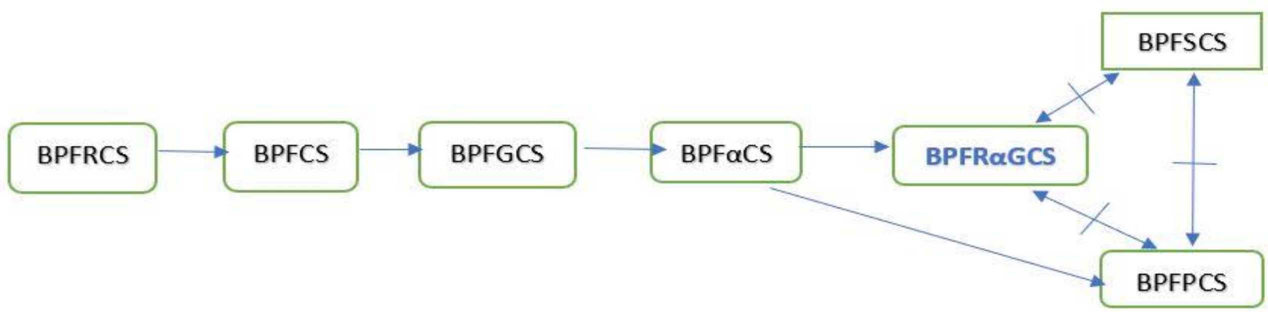

Figure 1: The relation between various types of BPFCSs are given in the following diagram

IV. BIPOLAR PYTHAGOREAN FUZZY REGULAR α GENERALIZED OPEN SETS

Definition 4.1: A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space (X, τ_p) is called a Bipolar Pythagorean Regular α Generalized Open set (BPFRGOS in short), if $\alpha int(A) \supseteq U$ whenever $A \supseteq U$ and U is BPFRCS in (X, τ_p). Alternatively, a BPF set A is said to be Bipolar Pythagorean Fuzzy Regular α Generalized Open Set(BPFRαGOS), if its complement A^c is BPFRαGCS in (X, τ_p).

Example 4.2: Let X={a,b} and τ_p={$0_p, T, 1_p$} be a BPFT on X, where T = (x, (0.3, 0.2), (0.5, 0.6), (-0.4, -0.3), (-0.7, -0.4)). Then the BPFS A = (x, (0.3, 0.3), (0.5, 0.3), (-0.5, -0.7), (-0.7, -0.2)) is a BPFRαGOS in X.

Proposition 4.3: Every BPFOS is BPFRαGOS in X but not conversely.
Proof: Let U be a BPF regular closed set in X such that $A \supseteq U$. Since A is BPFOS, int(A)= A. $\alpha int(A) = A \cap int(cl(int(A))) = A \cap int(cl(A)) \supseteq A \cap int(A) = A \cap A = A \supseteq U$. Thus, A is BPFR$\alpha$GOS in X.

Example 4.4: Let X= {a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.8), (0.4, 0.3), (-0.6, -0.5), (-0.5, -0.4)) and T_2 = (x, (0.3, 0.2), (0.7, 0.9), (-0.4, -0.3), (-0.6, -0.8)). Here A is the BPFS then A = (x, (0.6, 0.9), (0.2, 0.3), (-0.6, -0.6),(-0.3, -0.4)) is a BPFRαGOS, but A is not a BPFOS in X.

Proposition 4.5: Every BPFROS is a BPFRαGOS, but not conversely.
Proof: Let U be a BPF regular closed set in X such that $A \supseteq U$. Since every BPF regular open set is BPFOS, int(A)=A. By hypothesis, $\alpha int(A) = A \cap int(cl(int(A))) = A \cap int(cl(A)) \supseteq A \cap int(A) = A \cap A = A \supseteq U$. Hence $\alpha int(A) \supseteq U$. Thus, A is BPFR$\alpha$GOS in X.

Example 4.6: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.8), (0.4, 0.3), (-0.6, -0.5), (-

0.5, -0.4)) and T_2 = (x, (0.3, 0.2), (0.7, 0.9), (-0.4, -0.3), (-0.6, -0.8)). Here A is the BPFS then A = (x, (0.6, 0.9), (0.2, 0.3), (-0.6, -0.6),(0.3, -0.4))is a BPFRαGOS, but A is not a BPFROS in X.

Proposition 4.7: Every BPFαOS is a BPFRαGOS but not conversely.
***Proof*:** Let U be a BPF regular closed set in X such that $A \supseteq U$. Since A is BPF α open set, int(cl(int(A))) $\supseteq A$. By hypothesis, $\alpha int(A) = A \cap int(cl(int(A))) \supseteq A \cap A = A \supseteq U$. Hence $\alpha int(A) \supseteq U$. Thus, A is BPFR$\alpha$GOS in X.

Example 4.8: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.8), (0.4, 0.3), (-0.6, -0.5), (-0.5, -0.4)) and T_2 = (x, (0.3, 0.2), (0.7, 0.9), (-0.4, -0.3), (-0.6, -0.8)). Here A is the BPFS then A = (x, (0.6, 0.9), (0.2, 0.3), (-0.6, -0.6),(0.3, -0.4)) is a BPFRαGOS, but A is not a BPFαOS in X.

Theorem 4.9: If A and B are two BPFGRαOS in X, then the intersection of A and B is also BPFRαGOS in X.
Proof: Let A and B be two BPFRαGOS in X. Then A^c and B^c are BPFRαGCS in X. By Theorem 3.9, $A^c \cup B^c$ is BPFRαGCS in X. $(A \cap B)^c$ is BPFRαGCS in X. Therefore, $A \cup B$ is BPFRαGOS in X.

Remark 4.10: The union of any two BPFRαGOS is not BPFRαGOS in X as shown in the following Example.

Example 4.11: Let X={a, b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T= (x, (0.5, 0.3), (0.4, 0.8), (-0.5, -0.4), (-0.6, -0.8)). Then the BPFS A =(x, (0.6, 0.1), (0.2, 0.8), (-0.8, -0.2), (-0.5, -0.9)) and B= (x, (0.4, 0.9),(0.6, 0.2), (-0.5, -0.9), (-0.9, -0.3)) is a BPFRαGOS in X, but $A \cup B$ is not a BPFRαGOS in X.

Remark 4.12: Every BPFRαGOS and BPFSOS in X are independent to each other.

Example 4.13: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.6), (0.3, 0.4), (-0.6, -0.7), (-0.5, -0.7)) and T_2 = (x, (0.2, 0.3), (0.7, 0.7), (-0.3, -0.4), (-0.6, -0.8)). Here A is the BPFS then A = (x, (0.5, 0.4), (0.3, 0.3), (-0.5, -0.3),(-0.4,-0.6)) is a BPFRαGOS, but A is not a BPFSOS in X.

Example 4.14: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.4, 0.3), (0.5, 0.8), (-0.6, -0.3), (-0.6, -0.7)) and T_2 = (x, (0.3, 0.3), (0.8, 0.8), (-0.4, -0.2), (-0.7, -0.8)). Here A is the BPFS then A = (x, (0.5, 0.8), (0.4, 0.3), (-0.6, -0.7), (-0.6, -0.3)) is a BPFSOS, but A is not a BPFRαGOS in X.

Remark 4.15: Every BPFRαGOS and BPFPOS in (X, τ_p) are independent to each other.

Example 4.16: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.6), (0.3, 0.4), (-0.6, -0.7), (-0.5, -0.7)) and T_2 = (x, (0.2, 0.3), (0.7, 0.7), (-0.3, -0.4), (-0.6, -0.8)). Here A is the BPFS then A = (x,(0.5,0.4)(0.3,0.3),(-0.5,-0.3),(-0.4,-0.6)) is a BPFRαGOS, but A is not a BPFPOS in X.

Example 4.17: Let X={a, b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.3, 0.4), (0.8, 0.7), (-0.4, -0.5), (-0.7, -0.8)) and T_2 = (x, (0.8, 0.9), (0.3, 0.2), (-0.3, -0.3), (-0.6, -0.6)). Here A is the BPFS then A = (x, (0.7, 0.8), (0.1, 0.2), (-0.8, -0.9), (-0.1, -0.2)) is a BPFPOS, but A is not a BPFRαGOS in X.

Proposition 4.18: Every BPFGOS is BPFαGOS in X but not conversely.
Proof: Let A$\supseteq$ U and U be a BPF regular closed set in X. By hypothesis, cl(A)$\supseteq$ U, whenever A$\supseteq$ U. This implies, $\alpha int(A) = A \cap int(cl(int(A))) \supseteq A \cap cl(A)) \supseteq U$. Therefore A is BPFR$\alpha$GOS in X.

***Example 4.19*:** Let X={a, b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.6, 0.8), (0.3, 0.2), (-0.5, -0.7), (-0.3, -0.2)). Here A is the BPFS then A = (x, (0.6, 0.3), (0.5, 0.3), (-0.6, -0.4), (-0.4, -0.5)) is a BPFRαGOS, But A is not a BPFGOS in X.

***Proposition 4.20*:** If A is both a BPF regular closed set and BPFRαGOS in X, then A is a BPFRGOS in X.
Proof: Let $A \supseteq U$ and U be a BPFRCS in X. By hypothesis, we have $\alpha int(A) \supseteq U$ and int(A) = int(cl(int(A)))$\supseteq A \cap int(cl(int(A))) = \alpha int(A) \supseteq U$. Hence A is BPFRGOS in X.

Proposition 4.21: If A is both a BPF pre-closed set and BPFRαGOS in X, then A is a BPFRGOS in X.

Proof: Let $A \supseteq U$ and U be a BPFRCS in X. By hypothesis, we have $\alpha int(A) \supseteq U$ and int(A)=int(cl(int(A)))⊇ $A \cap cl(int(cl(A))) = \alpha int(A) \supseteq U$. Hence A is BPFRGOS in X.

Proposition 4.22: If A is both BPFRCS and BPFRαGOS in X, then A is BPFαOS in X.
Proof: As A⊇ A, by hypothesis, $\alpha int(A) \supseteq A$. But we have $A \supseteq \alpha int(A)$. This implies α int(A)=A. Hence A is BPFαOS in X.

Proposition 4.23: Let A be BPFRαGOS in X and $A \supseteq B \supseteq \alpha int(A)$, then B is BPFR$\alpha$GOS in X.
Proof: Let A be a BPFRαGOS in X and B be a BPF set in X. Let A ⊇ B ⊇ α int(A). Then A^c is a BPFRαGCS in X and $A^c \subseteq B^c \subseteq \alpha cl(A^c)$. Then B^c is a BPFRαGCS in X. Hence Bis BPFRαGOS in X.

Proposition 4.24: Let (X,τ_p) be a BPFTS and every B be a BPFRCS, B ⊇ A ⊇ int(cl(B). Then A is BPFRαGOS in X.
Proof: Let B be a BPFRCS in X. Then B = cl(int(B)). By hypothesis, A ⊇ int(cl(B) ⊇ int(cl(cl(int(B)))) ⊇ int(cl(int(B)))⊇ int(cl(int(A))). Therefore A is a BPF α open set. Since every BPF α open set is a BPFRαGOS. Hence A is a BPFRαGOS in X.

REFERENCES

[1] Coker, D., An introduction to intuitionistic topological spaces, Busefal, 81(2000), 5. Chen,S. Li,S. Ma and X. Wang, m-Polar Fuzzy Sets: An Extension of Bipolar Fuzzy Sets, The Scientific World Journal, 2014.
[2] K.T. Atanassov, Intuitionisticfuzzysets, FuzzySetsSyst20(1986), 87-96.
[3] Yager RR Pythagorean fuzzy subsets in Proceedings of the joint IFSA world congress NAFIPS annual meeting(2013), 57-61.
[4] T.Fukutake, R.K.Saraf, M.Caldas, M.Mishra, Mappings via Fgp BPFclosed Sets, Bull. of Fukuaka Univ. of Edu., 52 (2003),11-20.
[5] K.J.Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI- Algebras, Bull.Malays.
[6] C.L.Chang, Fuzzy Topological Spaces, J.Math.Anal.Appl., 24(1968),182-190.
[7] Xindong peng, Yong Yan, Some Results for Pythagorean Fuzzy Sets, PFinternational Journal of BPFintelligent Systems, 30(2015),1133-1160.
[8] K. Sakthivel, Intuitionistic Fuzzy Alpha Generalized Continuous Mappings and intuitionistic Fuzzy Generalized Irresolute Mappings, Applied Mathematical Sciences, 4 (2010),1831-1842.
[9] Kim, J. H., P. K. Lim, J. G. Lee, and K. Hur,Bipolar Fuzzy Intuitionistic topologicalspaces, Infinite Study, 2018.
[10] P. Bosc, O.Pivert, On a fuzzy bipolar relational algebra, InformationSciences,219 (2013) 1-16.
[11] J. Chen, S. Li, S. Ma and X. Wang. m-polar Fuzzy sets: An Extension of Bipolar Fuzzy Sets. The Scientific World Journal, 2014.
[12] L. A. Zadeh Fuzzy sets, Information and Control 8(1965), 338-353.
[13] K. Mohana, R. Jansi, Bipolar Phytagorean π generalized Pre-closed sets in Topological spaces, International journal of analytical and experimental model analysis, Jan 2020, 164-171.
[14] W-R. Zhang, Bipolar fuzzy sets, Proc. of IEEE(1998) 835-840.
[15] M. Azhagappan and M. Kamaraj, Notes on bipolar valued fuzzy RW-closed and bipolar valued fuzzy RW-open sets in bipolar valued fuzzy topological spaces, International Journal of Mathematical Archive 7(3) (2016) 30-36.
[16] Murat Olgun, Mehmet Unver, Seyhmus Yardimci, Pythagorean fuzzy topological spaces, complex and Intelligent Systems (2019) 5: 177-183.