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L-Vague Rings , L-Vague Ideals And L-Vague Cossets

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Abstract: *In this paper we study some generalized properties of L-vague ring.In this direction the concept of image and inverse image of vague L-set under ring homorphisim are discussed. Vague ideal on vague L-ring and studied their properties. Further we investigate the development of some important results and theorems about L-vague ring, L-vague ideal, L-vague costs on a vague L-ring.*

Keywords: *Vague set, L-Vague ring , L-Vague ideal , L-vague cosset*

Mathematics Subject Classification (2000): 08A72

I. INTRODUCTION

The concept of fuzzy lattice was introduced by Ajmal. N & Thomas[1]. "Is there a common abstraction which includes Boolean algebra, Boolean rings and lattice ordered group or L-group is an algebraic structure connecting lattice and group.To answer this problem many common abstractions, namely dually residuated lattice ordered semigroups,commutative lattice ordered groups. Also the concept proposed by Zadeh.L.A.[12] defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A by means of a membership function $\mu_A(x)$ defined from X in to [0 1] has revolutionized the theory of Mathematical modeling. Decision making etc.,in handling the imprecise real life situations mathematically. Now several branches of fuzzy mathematics like fuzzy algebra,fuzzy topology ,fuzzy control theory ,fuzzy measure theory etc.,have emerged.But in the decision making, the fuzzy theory takes care of membership of an element x only, that is the evidence against x belonging to A .It is felt by several decision makers and researchers that in proper decision making, the evidence belongs to A and evidence not belongs to A are both necessary .and how much X belongs to A or how much x does not belongs to A are necessary.Several generalizations of Zadeh's fuzzy set theory have been proposed, such as L-fuzzy sets [5]. Interval valued fuzzy sets ,Intuitionistic fuzzy sets by Atanassov.K.T [1] ,Vague sets [7] are mathematically equivalent. Any such set A of a given Universe X can be charactrierized by means of a pair of function (t_A , f_A) where $t_A : X \to [0\ \ 1]$
and $f_A : X \to [0\ \ 1]$ such that $0 \le t_A(x) + f_A(x) \le 1$ for all x in X.The set $t_A(x)$ is called the truth functionand the set $f_A(x)$ is called false function or non membership function and $t_A(x)$ gives the evidence of how much x belongs to A $f_A(x)$ gives the evidence of how much x does not A.These concepts are being applied in several areas like decision-making, fuzzy control, knowledge discovery and fault diagonsis etc.It is believed the vague sets (or equivalently instuitionistic fuzzy sets) will more useful in decision making, and other areas of Mathematical modeling. Through Atanassov's instuitionistic fuzzysets,Gau and Buehrer and some other areas of Mathematical modeling.Since then the theory fuzzy sets developed extensively and embraced almost all subjects like engineering science and technology. But the membership function $\mu_{A\ (x)}$ gives only a approximation belong to A .To avid this and obtain a better estimation and analysis of data decision making.Gau.W.L and Bueher D.J. [7] have initiated the study ofvague sets with the hope that they form a better tool to understand, interpret and solve real life problems which are in general vague,than the theory of vague sets do. Ranjit Biswas[9]initiated the study of vague groups by Ramakrishna.N, Eswarlal.T[9][10] ,[11] and Nageswara Rao.B,Bullibabu.R,Eswarlal.T[10]are extended the some of the concepts in study of vague algebra. The objective of this paper is to contribute further to the study of vague algebra by

introducing the concepts of the development of some important results and theorems about the notation vague L-Ring and L-vague Ideal of a L-Ring ,L-vague osets and studied some of their properties.

II. PRELIMINARIES

In this section ,some definitions and results of congruence relation and rough sets are discussed. A ring R is a non-empty set (R,+, .) consisting of a non-empty set R together with two binary operations ' +' and ' . ' (called addition and multiplication) such that (R,+) is an abelian group, (R, .) is a semi –group and a.(b+c) = (a.b) + (a.c), (b+c)a = (b.a) + (c.a)for all a,b,c in R

If the multiplication is commutative ,then R is said to be commutative ring and R is said to be commutative ring and R said to have an identity say 1 if a.1=1.a=a for all a in R

Let r be the equivalence relation on a ring R.For $x \in R$, the equivalence class or coset of x modulo r is the set $[x]_r = \{ y \in R : (x,y) \in r \}$. In this paper R is commutative ring with identity.

Definition 2.1[5] :-A vague set A in the universe of discourse U is a pair (t_A , f_A) where $t_A : X \to [0,1]$, $f_A: X \to [0,1]$ with $t_A(x) + f_A(x) \leq 1$ for all x in X. Here t_A is called the membership function and f_A is called non-membership and also called true membership function, false membership function respectively.

Definition 2.2 [5]The interval $[t_A (x) , 1- f_A (x)]$ is called the vague value of x in A, and it is denoted by $V_A(x)$.

ie $V_A (x)= [t_A (x) , 1- f_A (x)]$

Definition 2.3 Let (G,.) Be a group. A vague set A of G is called a vague group of G if for $a \in G$,

$\forall$ x ,y in G. $V_A (xy) \geq \min \{ V_A (x) , V_A (y)\}$ and $V_A (x^{-1}) \geq V_A (x)$

i.e $t_A(xy) \geq \min \{ t_A (x) ,t_A (y) \}$ and $t_A(x^{-1}) \geq t_A (x)$,

$f_A(xy) \leq \max\{ f_A (x) , f_A (y) \}$ and $f_A(x^{-1}) \leq f_A (x)$.

Definition 2.4 Let A be a vague set of universe G with true- membership function t_A and false membership function f_A . For $\alpha ,\beta \in [0,1]$ with $\alpha \leq \beta$,the (α β) cut or vague cut of vague set A is the crisp subset G of X given by $A_{(\alpha\ \beta)} = \{ x/ x \in G, V_A (x) \geq [\alpha\ \beta] \}$

.i.e. $A_{(\alpha\ \beta)} = \{ x/ x \in G, t_A (x) \geq \alpha , 1- f_A (x) \geq \beta \}$.

The α-cut,A_α of vague set A is the (α α)-cut of A, and hence given by $A_\alpha = \{ x|x \in G, t_A (x) \geq \alpha \}$.

Definition 2.5 [5] Let A be a vague set of a group G. Then the true α-cut of A is the crisp subset

$T_{A\alpha} = \{x/x \in G , t_A (x) \geq \alpha \}$ of G ,where $\alpha \in [0 , 1]$.

Definition 2.6 [1] Let A be a vague set of a group G. Then the false α-cut of A is the crisp subset $F_{A\alpha} = \{x/x \in G , f_A (x) \leq \alpha \}$ of G ,where $\alpha \in [0 , 1]$.

III. L-Vague Ring and L-Vague Ideals

In this section we define vague L-ring, vague ideal on L-ring and also studied certain properties.

Definition 3.1: An L-Vague set $A = \{ (x, t_A^{(X)}, f_A^{(x)}) : x \in R\}$ of R is said to be an L- vague ring (LVR) if $\forall$ x,y ε R, satisfies the following conditions.

(i) $t_A^{(x-y)} \geq \min \{ t_A^{(x)}, t_A^{(y)} \}$

(ii) $t_A^{(x y)} \geq \min \{ t_A^{(x)}, t_A^{(y)} \}$

(iii) $f_A^{(x-y)} \leq \max \{ f_A^{(x)}, f_A^{(y)} \}$

(iv) $f_A^{(x y)} \leq \max \{ f_A^{(x)}, f_A^{(y)} \}$

Definition 3.2: Let A = { (x, $t_A^{(X)}$, $f_A^{(x)}$) : x ∈ R} be an L- vague ring of R. Then A is called an L-vague ideal of R if it satisfies the following conditions

(i) $t_A^{(x-y)} \geq \min \{ t_A^{(x)}, t_A^{(y)} \}$
(ii) $t_A^{(x\,y)} \geq t_A^{(x)}$
(iii) $f_A^{(x-y)} \leq \max \{ f_A^{(x)}, f_A^{(y)} \}$
(iv) $f_A^{(x\,y)} \leq f_A^{(x)}, \ \forall x, y \in R$

Definition 3.3: Let A = { (x, $t_A^{(X)}$, $f_A^{(x)}$) : x ∈ R} be an L-vague ideal of R, then we define

(i) $(t_A^{(x)})_* = \{ x \,\varepsilon\, R : t_A^{(x)} = t_A^{(0)} \}$ and
(ii) $(f_A^{(x)})_* = \{ x \,\varepsilon\, R : f_A^{(x)} > f_A^{(0)} \}$
(v) Definition: Let A = { (x, $t_A^{(X)}$, $f_A^{(x)}$) : x ∈ R} be an L- vague ring of R. Let x ∈ R then

C = { x, ($t_{A\{x\}}^{(0)} + t_A$) (x), ($f_{A\{x\}}^{(0)} + f_A$) (x) : x ∈ R } is called an L-vague cosset of A and is defined as

C = { (x, ($t_{A\{x\}}^{(0)} + t_A$) (x), ($f_{A\{x\}}^{(0)} + f_A$) (x) : x ∈ R } is called an L-vague cosset of A and is denotes as

C = { x, (x + t_A) (x), (x+ f_A) (x) : x ∈ R }

Theorem 3.4: Let R/A = { x, (x + t_A) , (x+ f_A) , : x ∈ R } be an L-vague ideal and let

R/A = { x, (x + t_A) , (x+ f_A) , : x ∈ R }. Define + and . on R/A by

(i) $(x + t_A^{(x)}) + (y + t_A^{(y)}) = x+ y + t_A^{(x)}$
(ii) $(x + y_A^{(x)}) + (y + f_A^{(x)}) = x+ y + f_A^{(x)}$, ∀ x, y ∈ R and
(iii) $(x + t_A^{(x)}) . (y + t_A^{(x)}) = x. y + t_A^{(x)}$
(iv) $(x + t_A^{(x)}) . (y + f_A^{(x)}) = x. y + f_A^{(x)}$, ∀ x, y ∈ R

Then R/A is a ring w.r.t + and is called Quotient ring of R by $t_A^{(x)}$, $f_A^{(x)}$

Proof: Let φ : R → S be a ring homomorphism.

Let A = { (x, $t_A^{(x)}$, $f_A^{(x)}$) : x ∈ R } and

B = { (x, $t_B^{(x)}$, $f_B^{(x)}$) : x ∈ R } be any two vague rings of R then

C = { (y, $\varphi(t_A^{(y)})$, $\varphi(f_A^{(y)}$) : y ∈S } is called vague image of A.

Where $\varphi(t_A^{(x)})\,(y) = \{\max \{ t_A^{(x)} : x \in R, \varphi(x) = y \text{ if } \varphi^{-1}(y) \neq 0\}$

= 0, otherwise

$\varphi(f_A^{(x)})\,(y) = \min \{ f_A^{(x)} : x \in R, \varphi(x) = y \text{ if } \varphi^{-1}(y) \neq 0\}$

= 0, otherwise ∀ y ∈ S

and D = { (x, $\varphi^{-1}(t_B^{(x)})$, $\varphi^{-1}(f_B^{(x)}$) : x ∈ R } is called vague invertible image of B.

where $\varphi^{-1}(t_B^{(x)}) = t_B^{(x)}$

$\varphi^{-1}(f_B^{(x)}) = f_B^{(x)} \ \forall x \in R$

where $\varphi(t_A)$ and $\varphi(f_A$) are called the image of t_A and f_A under φ

Also $\varphi^{-1}(t_B)$ and $\varphi^{-1}(f_B$) are called the invertible image of t_B and f_B under φ

Theorem 3.5: Let φ : R → S be a ring homomorphism.

Let A = { (x, $t_A^{(x)}$, $f_A^{(x)}$) : x ∈ R } and B = { (x, $t_B^{(x)}$, $f_B^{(x)}$) : x ∈ R } be any two vague ideals of R and S respectively, then

(i) $\varphi(f_A)^{(x)} = f_A^{(0)}$ where 0^I is the zero element of S and 0 is the zero element of R
(i) $\Phi(f_A)(x) \le (\varphi(f_A)(x)$
(ii) If f_A has the infium property then $\Phi(f_A)(x) = (\varphi(f_A)(x)$
(ii) If f_A is constant on ker f, then $(\varphi(f_A))(\varphi(x) = f_A(x)\ \forall\ x \in R$

Proof: The proof is very clear

Theorem 3.6 Let φ : R → S be a ring homomorphism.

Let A = { (x, $t_A^{(x)}$, $f_A^{(x)}$) : x ∈ R } and B = { (x, $t_B^{(x)}$, $f_B^{(x)}$) : x ∈ R } be L- vague ideal of R and S, then

(i) D = { (x, $\varphi^{-1}(t_B^{(x)})$, $\varphi^{-1}(f_B^{(x)}$) : x ∈ R } is on L- vague ideal of R which is a constant on ker φ
(ii) $\Phi^{-1}(f_B^{(x)})_* = (\varphi^{-1}(f_B^{(x)})_*$
(iii) If φ is onto then $(\varphi o \varphi^{-1})(f_B) = f_B$
(iv) If f_A is constant on ker f, then $(\varphi^{-1} o \varphi)(f_A) = f_A$

Proof: (i) Let x, y ε R, then

$$\Phi^{-1}(f_B)^{(x-y)} = f_B^{(x-y)} = f_B^{(\varphi(x)-\varphi(y)} \le \max\{f_B^{(\varphi(x)}, f_B^{(\varphi(y)}\}$$

$$= \max\{\varphi^{-1}(f_B)^{(x)}, \varphi^{-1}(f_B)^{(y)}\}$$

and $$\Phi^{-1}(f_B^{(xy)}) = f_B^{(x-y)} = f_B^{(\varphi(xy))} \le \min\{f_B^{(\varphi(x)}, f_B^{(\varphi(y)}\}$$

$$= \min\{\varphi^{-1}(f_B)^{(x)}\ \varphi^{-1}(f_B)^{(y)}\}$$

Hence 'D' is a L- vague ideal of R.

Let x ε ker φ, then $\varphi^{-1}(f_B)(x) = f_B^{\varphi(x)}$

$= f_B^{\varphi(0)}$

$= f_B^{(0')}$

Hence $\varphi^{-1}(f_B)$ is constant on ker φ.

(ii) Let x ∈ R, then x ε $\varphi^{-1}(f_B)_* \Leftrightarrow f_B^{(\varphi(x))} > f_B^{(0')} = f_B^{(f(0))}$

$\Leftrightarrow \varphi^{-1}(f_B)^{(x)} > \varphi^{-1}(f_B)^{(0)}$

$\Leftrightarrow x \in (\varphi^{-1}(f_B))_*$

Hence $\varphi^{-1}(f_B)_* = (\varphi^{-1}(f_B))_*$

(iii) Let y ε S, then y = φ(x) for same x ∈ R, so that
$(\varphi o \varphi^{-1})(f_B)^{(x)} = \varphi(\varphi^{-1})(f_B)^{(y)} = \varphi(\varphi^{-1})(f_B)^{(\varphi x)} = \varphi^{-1}(f_B)^{(x)} = f_B^{(\varphi(x))} = f_B^{(y)}$
Hence $(\varphi 0 \varphi^{-1})(f_B) = f_B$

(iv) Let x ε R, then $(\varphi^{-1} o \varphi)(f_A)^{(x)} = \varphi^{-1}(\varphi(f_A)(x) = \varphi(f_A)^{(\varphi(X))} = f_A^{(X)}$

Hence $(\varphi^{-1} o \varphi)(f_A) = f_A$

Theorem 3.7 : Let φ : R -> S be on to ring homomorphism. Let A = {(x, $t_A^{(x)}$, $f_A^{(x)}$) : x ∈ R } be an vague ideal of R, then c = { (y, $\varphi(t_A^{(y)})$, $\varphi(f_A^{(y)})$: y ∈ S } is an vague ideal of S. If f_A is a constant on Ker φ, then $\varphi(f_A^{(y)})_* = (\varphi(f_A^{(y)}))_*$

Proof: Let s_1,s_2 ∈ S then $s_1 = \varphi(r_1)$, $s_2 = \varphi(r_2)$ for same r_1,r_2 ∈ R.

Now $\varphi(f_A^{(s_1-s_2)}) = \min\{f_A^{(x)} : x \in R, \varphi(x) = s_1 - s_2\}$

$\leq \min\{ f_A^{(r_1-r_2)} : r_1, r_2 \in R, \varphi(r_1) = s_1, \varphi(r_2) = s_2\}$

$\leq \min\{ \max \{f_A^{(r_1)}, f_A^{(r_2)} : r_1, r_2 \in R, f_A(r_1) = s_1, f_A(r_2) = s_2\}\}$

$= \max[\min\{ f_A^{(r_1)}: r_1 \in R, f_A^{(r_1)} = s_1\}, \min \{ f_A^{(r_2)} : r_2 \in R, f_A^{(r_2)} = s_2 \}]$

$= \max \{ \varphi(f_A^{(s_1)}), \varphi(f_A^{(s_2)}) \}$

$\therefore \varphi(f_A^{(s_1-s_2)}) = \max \{ \varphi(f_A^{(s_1)}), \varphi(f_A^{(s_2)}) \}$

Also $\varphi(f_A^{(s_1 s_2)}) = \min \{ f_A^{(x)} : x \in R, f_A(x) = s_1 s_2\}$

$\leq \min\{ f_A^{(r_1 r_2)} : r_1, r_2 \in R, f_A(r_1) = s_1, f_A(r_2) = s_2\}$

$\leq \min\{ \min \{f_A^{(r_1)}, f_A^{(r_2)} : r_1, r_2 \in R, f(r_1) = s_1, f(r_2) = s_2\}\}$

$= \min[\min\{ f_A^{(r_1)}: r_1 \varepsilon R, f_A^{(r_1)} = s_1\}, \min \{ f_A^{(r_2)} : r_2 \in R, f_A^{(r_2)} = s_2 \}]$

$= \min \{ \varphi(f_A^{(s_1)}), \varphi(f_A^{(s_2)}) \}$

$\therefore \varphi(f_A^{(s_1 s_2)}) = \min \{ \varphi(f_A^{(s_1)}), \varphi(f_A^{(s_2)}) \}$

Hence $c = \{ (y, \varphi(t_A^{(y)}), \varphi(f_A^{(y)}): y \in S \}$ is an vague ideal of S

Next f_A is a constant on ker φ, and for $y \in \varphi(f_A^{(y)})$

We have $\varphi(f_A^{(y)}) > \varphi(f_A^{(0)}) = f_A^{(0)}$

Since φ is onto, $y = \varphi(x)$ for same $x \in R$

Hence $\varphi(f_A^{(x)}) = f_A^{(x)}) > f_A^{(0)}$

Thus $f_A^{(x)}) > f_A^{(0)}$ for $x \in f_A$

Theorem 3.8:- Let $A =\{ (x, t_A^{(x)}, f_A^{(x)}): x \in R \}$ be an vague ideal then A^* is an vague ideal of R/A.

Where $A^* = \{ (x, t_A^{*(x)}, f_A^{*(x)}): x \in R/A\}$ is defined by $t_A^*(x + t_A) = t_A^{(x)}$ and $f_A^*(x + f_A) = f_A^{(x)}\ \forall x \in R$.

Proof: Let x,y ε R then

(i) $t_A^* ((x+ t_A) + (y + t_A)) = t_A^*(x+ y+ t_A) = t_A^{(x+y)} \geq \max \{t_A^{(x)}, t_A^{(y)}\}$
$= \max \{ t_A^* (x+ t_A) , t_A^* (y + t_A)\}$
$\therefore t_A^* ((x+ t_A) + (y + t_A)) \geq \max \{ t_A^* (x+ t_A) , t_A^* (y + t_A)\}$

(ii) $f_A^* ((x+ f_A) (y + f_A)) = f_A^*(xy+ f_A) = f_A^{(xy)} \leq \min\{f_A^{(x)}, f_A^{(y)}\}$
$= \min \{ f_A^* (x+ f_A) , f_A^* (y + f_A)\}$
$\therefore f_A^* ((x+ f_A)(y + f_A)) \leq \min \{ f_A^* (x+ f_A) , f_A^* (y + f_A)\}$
Hence A^* is an vague ideal of R/A

Theorem 3.9:- Let $B = \{(x, t_B^{(x)}, f_B^{(x)}) : x \in R \}$ be an L-vague ring and c is any ideal of R. Let $D = \{[x], t_D^{[x]}, f_D^{[x]}) : [x] \in R/C \}$ be an L-vague ring of R/C where

$t_D^{[x]} = \max \{ t_B^{[z]} : z \in [z]\}$

$f_D^{[x]} = \min \{ f_B^{[z]} : z \in [z]\}$ for all $x\ \varepsilon$ R and $[x] = x + c$, then D is L- vague ring of R/C

Proof:- Let $x, y \in R$, then

$f_D^{([x]-[y])} = f_D^{(IX-YI)} = \min \{f_B^{(x-y+z)} : z \in c\} \leq \min \{f_B^{(x-y+a-b)} : a,b\ \varepsilon\ c\}$

$= \min \{ f_B^{((x+a)-(y+b))} : a,b \in c.$

$\leq \max \{(\min\{ f_B^{(x+a)} : a \in c\}) \vee (\min \{ f_B^{(y+b)} : b \in c\}\} = \max \{ f_D^{[x]}, f_D^{[y]}\}$

Also, $f_D^{([x][y])} = f_D^{[XY]} = \min \{f_B^{(xy+z)} : z \in c\}$

$\leq \min \{f_B^{(xy+(xv+uy+uv)))} : u,v \in c\} = \min \{ f_B^{((y+v)(x+u))} : u,v \in c.$

$\leq \max \{(\min\{ f_B^{(y+v)}), (f_B^{(x+u)}):u,v \, \varepsilon \, c)\}$

$= \max (\min\{ f_B^{(x+u)} : u \in c), (\min(f_B^{(y+v)} : v \in c)\} = \min\{f_D^{[x]}, f_D^{[y]}\}$

$= f_D^{([x][y]) = f_D^{[xy]}}$

Hence D is an L-vague ring of D/C.

The L-vague subring, $D = \{[x], t_D^{[x]}, f_D^{[x]}) : [x] \in R/C \}$ is called the vague L- quotient ring of B relative to c and denoted as B/C and is noted as VLSR.

Theorem 3.10 : - Let $B = \{(x, t_B^{(x)}, f_B^{(x)}) : x \in R \}$ be an L-vague ring and $A = \{(x, t_A^{(x)}, f_A^{(x)}) : x \in R \}$ be an L-vague ideal of B. Suppose that L is regular, then $B/B^* \cong B/A$.

Proof: Let φ be the natural homomorphism from B^* onto B^*/A^*, then

$\varphi(t_B/t_B^*)([y]) = \min \{(t_B/t_B^*)(x) : x \in B^*, f(x) = [y]\}$

$= \min \{ t_B^{(z)} :z \in [y]$

$= (t_B/t_A)[y] \ \forall \ y \in B^*$ where $[y] = y+ A^*$ …… (3.10.1)

$\varphi(f_B/f_B^*)([y]) = \max \{(f_B/f_B^*)(x) : x \in B^*, f(x) = [y]\}$

$= \max \{ f_B^{(z)} :z \in [y]$

$= (f_B/f_A)[y] \ \forall \ y \in B^*$ where $[y] = y+ A^*$ …… (3.10.2)s

From (3.9.1) ,(3.9.2)$B/B^* \cong B/A$.

Theorem 3.11 : Ley $\varphi : R \to S$ be an onto homomorphism. Let $B = \{(x, t_B^{(x)}, f_B^{(x)}) : x \in S \}$ be an L-vague ring of S and $A = \{(x, t_A^{(x)}, f_A^{(x)}) : x \in S \}$ be an vague ideal of B, then $\varphi^{-1}(A)$ is an L-vague ideal of $\varphi^{-1}(B)$

Proof: Clearly $\varphi^{-1}(A)$ and $\varphi^{-1}(B)$ are L-vague ideal of a ring R and $\varphi^{-1}(A) \sqsubseteq \varphi^{-1}(B)$

Now $\varphi^{-1}(t_A^{(xy)}) = t_A^{(\varphi(xy))} = t_A^{(\varphi(x)\varphi(y))} \geq \min \{ t_A^{\varphi(x)}, t_A^{\varphi(y)}\}$

$= \min \{ t_B^{\varphi(x)}, t_B^{\varphi(y)}\} = \min \{ \varphi^{-1}(t_B)^{(x)}, \varphi^{-1}(t_B)^{(y)})$

$\therefore \varphi^{-1}(t_A^{(xy)}) \geq \min \{ \varphi^{-1}(t_B)^{(x)}, \varphi^{-1}(t_B)^{(y)})$ ………. (3.11.1)

Now $\varphi^{-1}(f_A^{(xy)}) = f_A^{(\varphi(xy))} = f_A^{(\varphi(x)\varphi(y))} \leq \max\{ f_A^{\varphi(x)}, f_A^{\varphi(y)}\}$

$= \max \{ f_B^{\varphi(x)}, f_B^{\varphi(y)}\} = \max \{ \varphi^{-1}(f_B)^{(x)}, \varphi^{-1}(f_B)^{(y)})$

$\therefore \varphi^{-1}(f_A^{(xy)}) \leq \max \{ \varphi^{-1}(f_B)^{(x)}, \varphi^{-1}(f_B)^{(y)}) \}$ ………. (3.11.2)

From (3.10.1), (3.10.2) , $\varphi^{-1}(A)$ is an L-vague ideal of $\varphi^{-1}(B)$.

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