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Radiating Heat-Mass Transport over a Leaning Surface using Group of Scaling Operations

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Abstract – *Impacts of radiation on free-convection heat-mass transfering over a leaning surface is probed using Lie's group. PDEs that model the liquid motion are reduced to ODEs together with the corresponding conditions on the boundary by symmetries. Approximate solution got by applying IV order R-K algorithm with trajectory shoot method exhibits that both thermal&concentration boundary-layer thicknesses are downsized while rising Gr(thermal) number and Sc number. The opposite phenomena takes place whenever Gr(solutal) number rises. Further, it is witnessed that velocity&temperature rise whereas concentration reduces as radiation intensifies.*

Keywords — *Free convection, Inclined surface, Lie's group, Mass transfer.*

I. INTRODUCTION

Many scholars have shown utmost interest on heat-mass transport with radiation on an incompressible liquid which flows along a heated platform on considering the fact that this field had find numerous uses in engineering and industrial situations such as crude oil industry, boundary control in aerodynamics, geothermal applications and nuclear reactor cooling etc. Our analysis in this article depends on certain symmetries applied to a case of natural convection boundary-layer. By applying symmetry, the independent variables reduce in number and hence these solution methods have become trendy nowadays.

Chen [1] made a deliberation on natural-convection flowing upon a leaning permeable surface for which wall temperature & concentration vary. He recorded a rise in velocity when magnetic field is present. Force of buoyancy has declined as angle of leaning lowers. Impacts of radiative and magnetic effects on free-convection mass-transfer flowing upon a flat-plate are considered by Ibrahim *et al:*[2]. Further study on free-convective boundary-layer problem using Lie's theory was done by Kalpakides&Balassas [3]. Similarity operations with their vast applications to PDEs were elaborately demonstrated by Yurusoy and Pakdemirli [4-6]. They had further discussed spiral group of transformations to obtain similarity solutions. These men have brought exact analytic solutions of boundary-layer equations of a special non-Newtonian liquid over a stretch-sheet.

MHD mixed-convection stagnating-point towards upright plate kept in penetrable surrounding with the transport of mass-energy influenced by Dufour-Soret parameters constrained with convective boundary condition is probed by Karthikeyan *et al*:[7]. Sivasankaran *et al*:[8] cast their effort on laminar, buoyant induced convection flowing and heat transport of Casson fluid in a square shaped porous box by simulation. Free convective flowing upon a plate embedded in a penetrating environment by symmetry groups was analyzed by Bhuvaneswari and Karthikeyan [9].

Until now, no investigation upon radiation heat-mass transfering on a leaned surface using Lie's group is attempted. Hence this study is initiated for dealing the changes in velocity-concentration-temperature on the above mentioned flow by incorporating scaling operations.

II. MATHEMATICAL ANALYSIS

The physical context we consider here is that the heat-mass transfer by natural convection in boundary-layer streamline flowing of an incompressible viscous liquid along a leaning plate having an angle α to the vertical where $\alpha < 90°$. The warmth of the surface (T_w) and that of the surrounding fluid (T_∞) are constant such that $T_w > T_\infty$. The surface concentration (C_w) and that of the surrounding fluid (C_∞) are constant such that $C_w > C_\infty$. The characteristics of the liquid are supposed to be constant. The mathematical formulation of this boundary-layer model is provided below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)cos\alpha - g\beta^*(C - C_\infty)cos\alpha, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{\lambda}{k}\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

under the conditions

$$\begin{aligned} &u=v=0,\ \ T=T_w,\ \ C=C_w \quad at\ y=0,\\ &u=0,\ \ T=T_\infty,\ \ T=T_\infty \quad as\ y\to\infty, \end{aligned} \tag{5}$$

where u & v: velocity compoments; x & y: space-coordinates; T:temperature; C:concentration; ν:liquid kinematic viscosity; g:acceleration of gravity; β:co-efficient of thermal-expansion; β^*:co-efficient of mass-expansion; λ:thermal diffusivity; q_r: local radiative heat flux; k:thermal conductivity of liquid; ρ:density of the liquid; c_p:specific heat of the liquid; D: diffusion-coefficient and α: leaning angle.

We use Rosseland estimate for q_r: $$q_r=\frac{4\sigma_0}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

(σ_0: Stefan-Boltzman constant & k^*: mean-absorption co-efficient.

According to our guess that differences in temperature are small, we can approximate T^4 in terms of T_∞ by shortened Taylor's expansion as

$$T^4=4T_\infty^3T-3T_\infty^4 \tag{7}$$

Non-dimensional variables are

$$\bar{x}=\frac{xU_\infty}{\nu},\bar{y}=\frac{yU_\infty}{\nu},\bar{u}=\frac{u}{U_\infty},\bar{v}=\frac{v}{U_\infty},\theta=\frac{T-T_\infty}{T_w-T_\infty},\varphi=\frac{C-C_\infty}{C_w-C_\infty} \tag{8}$$

Substituting (6)-(8) into equations (1)-(5) and dropping the over bars, we obtain,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+Gr\theta cos\alpha-Gc\varphi cos\alpha \tag{10}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}(1+4R)\frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

$$u\frac{\partial \varphi}{\partial x}+v\frac{\partial \varphi}{\partial y}=\frac{1}{Sc}\frac{\partial^2 \varphi}{\partial y^2} \tag{12}$$

under the conditions

$$\begin{aligned} &u=v=0, \quad \theta=1, \quad \varphi=1 \quad at\ y=0,\\ &u=0, \quad \theta=0, \quad \varphi=0 \quad as\ y\to\infty, \end{aligned} \tag{13}$$

where $Gr=\frac{g\beta(T_w-T_\infty)\nu}{U_\infty^3}$, $Gc=\frac{g\beta^*(C_w-C_\infty)\nu}{U_\infty^3}$, $Pr=\frac{\rho c_p\nu}{k}$, $S=\frac{\nu}{D}$ & $R=\frac{4\sigma_0T_\infty^3}{3kk^*}$.

III. SYMMETRY GROUPS EQUATIONS

Equalities (9)-(12) of symmetries group are formed in reference to Bluman and Kumei [1] by Lie group notion. Lie group of transformations with single parameter which leave (9)-(12) as same is listed as follows:

$$\begin{aligned} x^*&=x+\epsilon\xi_1(x,y,u,v,\theta,\varphi)\\ y^*&=y+\epsilon\xi_2(x,y,u,v,\theta,\varphi)\\ u^*&=u+\epsilon\eta_1(x,y,u,v,\theta,\varphi)\\ v^*&=v+\epsilon\eta_2(x,y,u,v,\theta,\varphi)\\ \theta^*&=\theta+\epsilon\eta_3(x,y,u,v,\theta,\varphi)\\ \varphi^*&=\varphi+\epsilon\eta_4(x,y,u,v,\theta,\varphi) \end{aligned} \tag{14}$$

By employing rigorous algebraic computations, the infinitesimals are obtained as

$$\begin{aligned} \xi_1&=2s_1x-s_2x-s_3\\ \xi_2&=\frac{1}{2}s_1y-\frac{1}{2}s_2y-\alpha(x)\\ \eta_1&=s_1u\\ \eta_2&=-u\alpha'(x)-\frac{1}{2}s_1v+\frac{1}{2}s_2v\\ \eta_3&=s_2\theta+\frac{Gc}{Gr}s_4 \end{aligned} \tag{15}$$

$$\eta_4 = c_2\varphi + s_4$$

When the constraints from boundaries are considered and boundary restrictions on infinitesimals are imposed, the system (15) takes the form

$$\begin{aligned} \xi_1 &= 2s_1 x - s_2 x - s_3 \\ \xi_2 &= \tfrac{1}{2}s_1 y - \tfrac{1}{2}s_2 y \\ \eta_1 &= s_1 u \\ \eta_2 &- \tfrac{1}{2}s_1 v + \tfrac{1}{2}s_2 v \\ \eta_3 &= s_2\theta + \tfrac{Gc}{Gr}s_4 \\ \eta_4 &= s_2\varphi + s_4 \end{aligned} \quad (16)$$

(s_1, s_2: scaling transformation; s_3, s_4: translation in x, y co-ordinates)

IV. REDUCTION TO ODE'S

By considering s_1 as arbitrary and remaining parameters as zero in (12), the resulting subsidiary equations are

$$\frac{dx}{2x} = \frac{dy}{\left(\frac{1}{2}\right)y} = \frac{du}{u} = \frac{dv}{\left(-\frac{1}{2}\right)v} = \frac{d\theta}{0} = \frac{d\varphi}{0} \quad (17)$$

from which we obtain

$$\eta = x^{-\frac{1}{4}}y,\ u = x^{\frac{1}{2}}F_1(\eta),\ v = x^{-\frac{1}{4}}F_2(\eta),\ \theta = F_3(\eta),\ \varphi = F_4(\eta) \quad (18)$$

Substituting (18) into equations (9)-(12), system of non-linear ODEs are obtained as

$$\begin{aligned} F_1'' &= \tfrac{1}{2}F_1{}^2 - \tfrac{1}{K}\eta F_1 F_1' + F_2 F_1' - GrF_3\cos\alpha + GcF_4\cos\alpha \\ F_2' &= \tfrac{1}{4}\eta F_1' - \tfrac{1}{2}F_1 \\ F_3'' &= \mathrm{Pr}/(1+4\mathrm{R})\left(F_2F_3' - \tfrac{1}{4}\eta F_1 F'_3\right) \\ F_4'' &= \mathrm{Sc}\left(F_2F_4' - \tfrac{1}{4}\eta F_1 F'_4\right) \end{aligned} \quad (19)$$

with

$$\begin{aligned} &F_1 = F_2 = 0,\ F_3 = 1,\ F_4 = 1 \quad at\ \eta = 0, \\ &F_1 = 0,\ F_3 = 0,\ F_4 = 0 \quad as\ \eta \to \infty. \end{aligned} \quad (20)$$

V. NUMERICAL APPROACH

Because of the reason that the final ODEs derived in the last section are not linear, resorting to numerical treatment is appropriate. Equations listed in (19) caccompanied by boundary restrictions is solved by numerical approach, nnamely, IV order R-K algorithm and trajectory shoot-up method with initial guesses for $F_1'(0)$ and $F_3'(0)$. The iterations are continued upto a stage where an accuracy of 10^{-5} is realized. A code using MATHEMATICA is formulated and the outcomes are depicted with various sketches.

VI. RESULTS ANALYSIS

Solutions through numerical technique are evolved for the following range of values for Pr, Gr, Gc, Sc and R:

Parameter	Range
Pr	0.1-2.05
Gr	0.1-2.5
Gc	0.1-1.0
Sc	1-10
R	0-5
α	$0°, 30°, 45°$

We consider the cases for $\alpha = 0^o$, 30^o and 45^o. Velocity-temperature-concentration variations are sketched via graphs. The analysis is performed for $\alpha = 45^o$. Results related to $\alpha = 0°$ (vertical plate case) and $\alpha = 30°$ are also discussed.

Figures 1(a-c) show the velocity-temperature-concentration variations due to R for $Pr = 0.71$, $Gr = 0.1$, $Gc = 0.1$ & $Sc = 1$. Looking at these figures, it is evident that the speed rises, temperature becomes linear and the concentration boundary-layer reduces due to ascending R values.

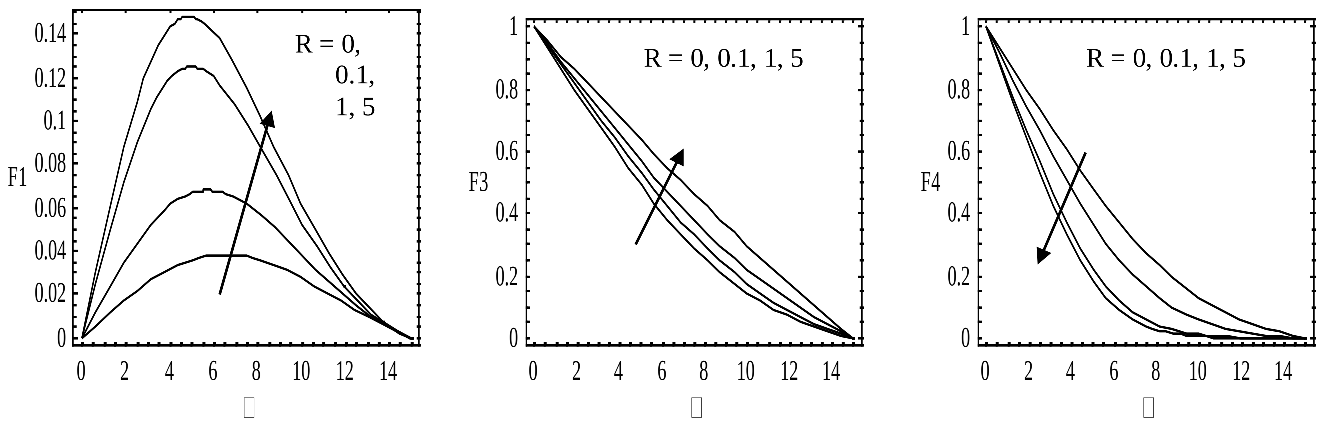
Figures 2(a-c) sketch the velocity-temperature-concentration with same values except for Gr (i.e., Gr =2.5). Evidently, because of the favouring buoyant force, the velocity is noted to be higher when compared to the last case. Thickness of the thermal boundary-layer rises with R. But the solutal boundary layer undergo only a small change due to R.

Profiles of velocity-temperature-concentration with respect to Pr values are sketched in 3(a-c). A rise in Pr impacts the velocity in the boundary-layer to decrease. Temperature is observed to be linear for a low Pr value. On the other hand, it diminishes for rising Pr. By rising Pr, the concentration tends to rise.

From the sketches 4(a-c), it is observed that, for the rising values of Sc, velocity rises but not the temperature and concentration. The velocity-temperature-concentration variations for rising Gr values are exhibited in Figures 5(a-c). Velocity rises with an increase in Gr in view of the presence of buoyant force. But, when temperature and concentration are concerned, this trend is opposite.

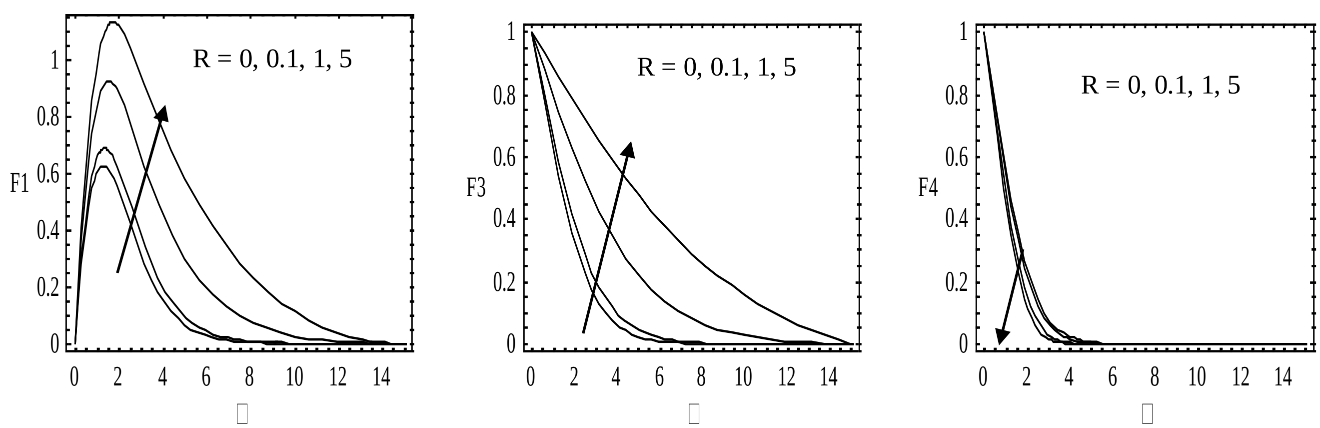
Figures 6(a-c) depict the influence of Gc. When Gc is raised, the velocity of the liquid dimishes and both boundary-layers are raised.

The outcomes of different parameters for certain fixed slopes of surface are portrayed in Figures (7-8). The liquid flows with higher speed on the vertical surface. When the surface is leaning at 30^{o}, the temperature&concentration along the boundary-layer are higher. Rising Sc reduces the temperature&concentration for a fixed α. At a fixed slope, the temperature rises and solutal boundary-layer thickness reduces for ascending R values.



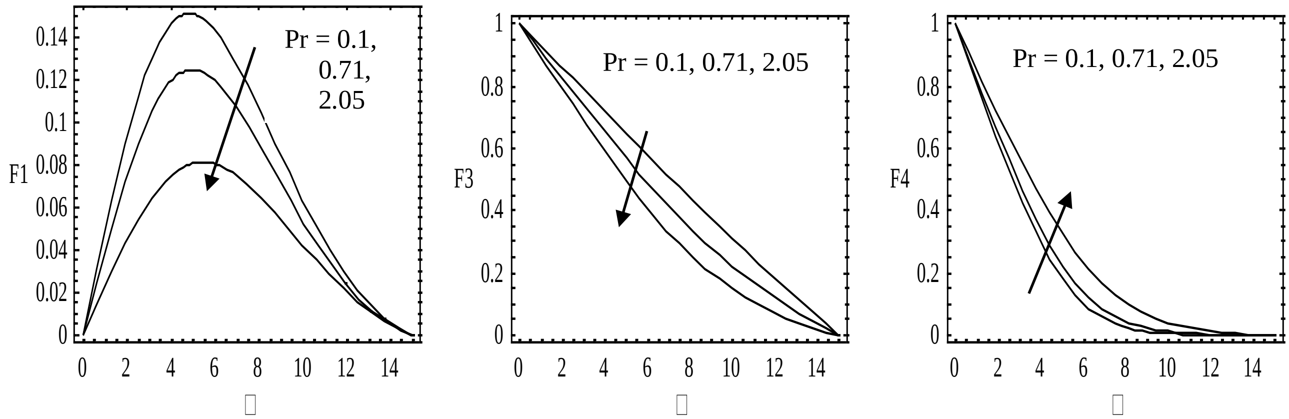
(a) (b) (c)

Fig. 1. Impacts of R when Gr = 0.1[Pr = 0.71,Gc = 0.1&Sc = 1]



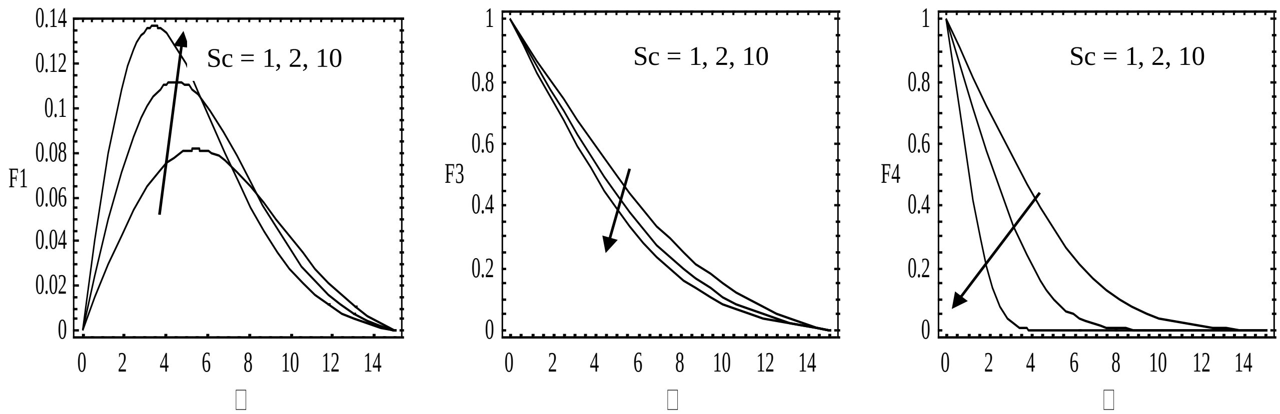
(a) (b) (c)

Fig. 2. Impacts of R when Gr = 2.5 [Pr = 0.71,Gc = 0.1&Sc = 1]



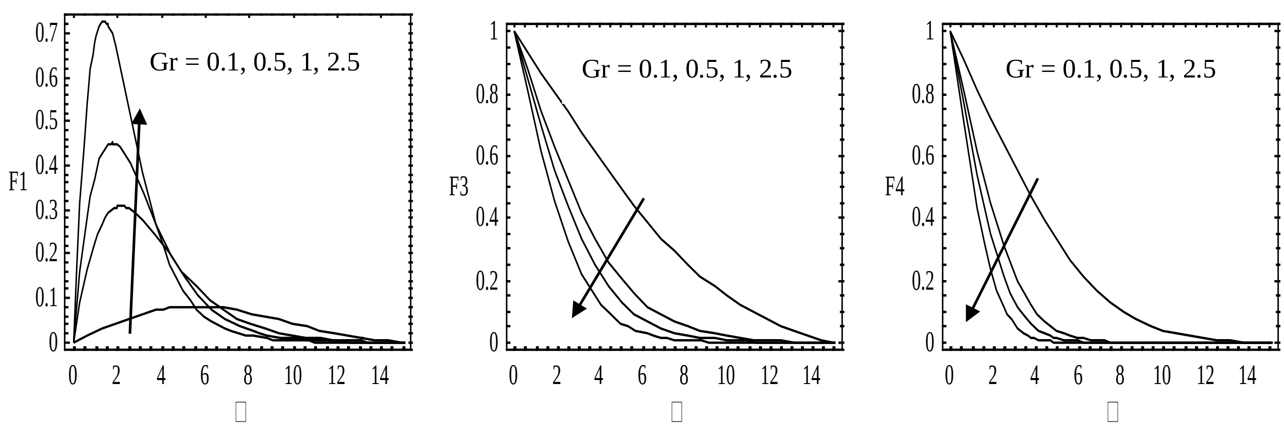
(a) (b) (c)

Fig. 3. Impacts of Pr [R=1,SGr = 2.5,Gc = 0.1&Sc = 1]



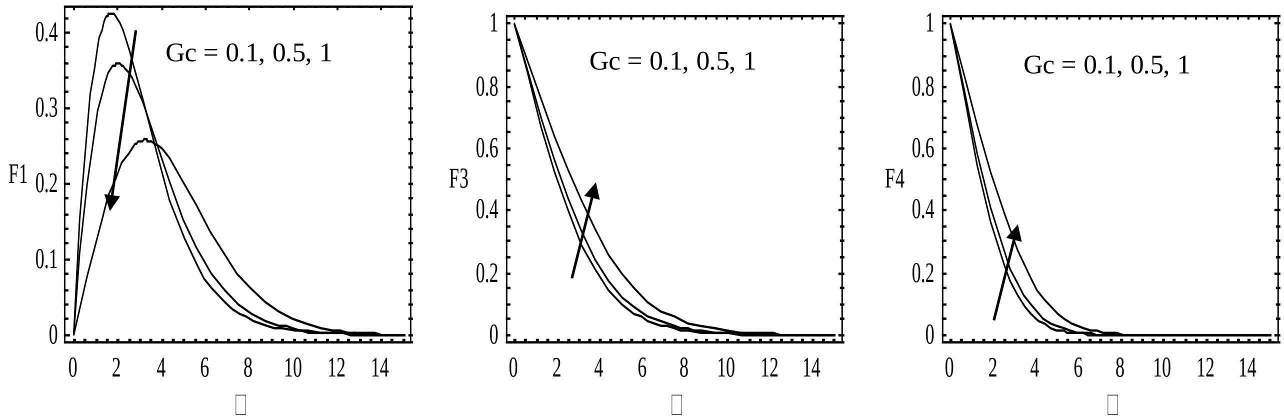
(a) (b) (c)

Fig. 4. Impacts of Sc [Pr = 2.05, R = 1,Gr = 0.1,Gc = 0.1&Sc = 1]



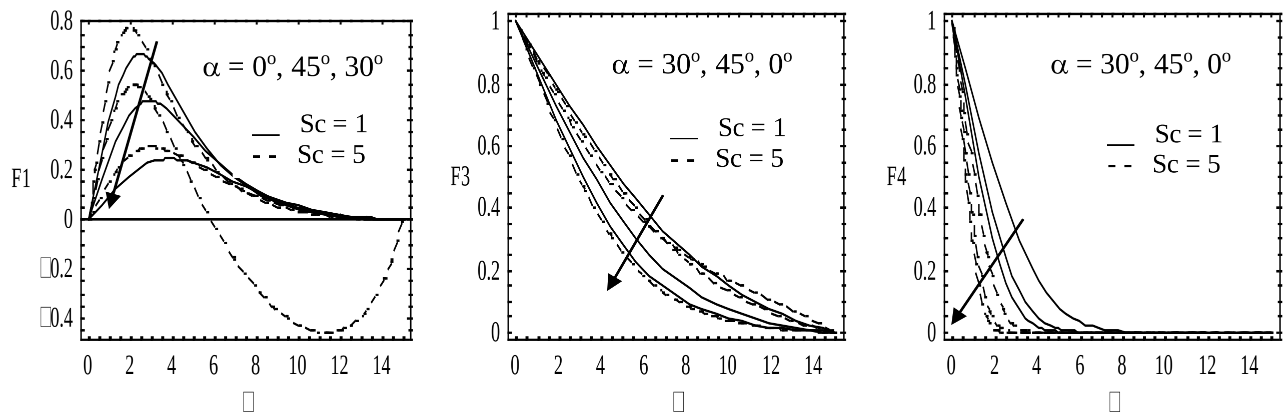
(a) (b) (c)

Fig. 5. Impacts of Gr [Pr = 2.05, R = 1,Gc = 0.1&Sc = 1]



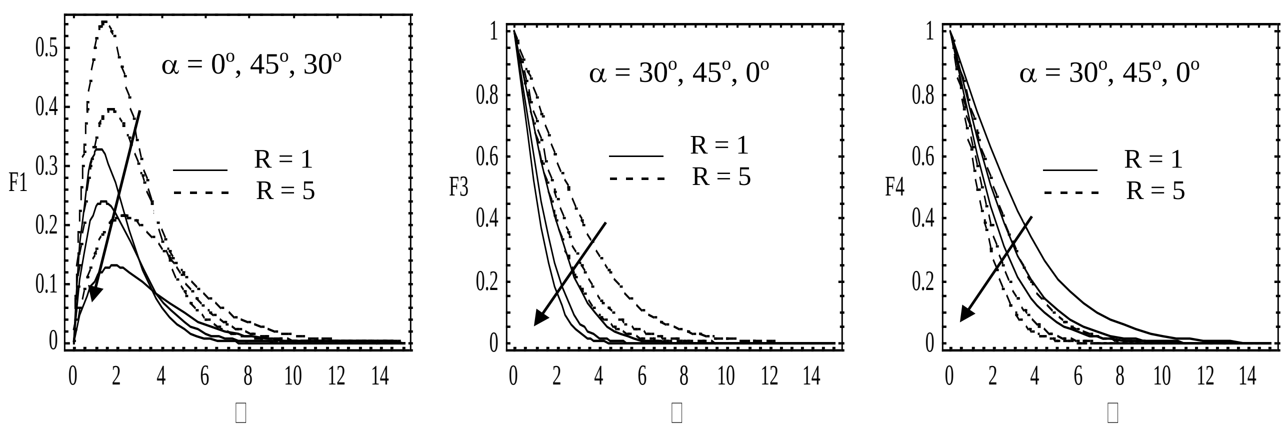
(a) (b) (c)

Fig. 6. Impacts of Gc [Pr = 0.71, R =0.1, Gr = 1&Sc = 1]



(a) (b) (c)

Fig. 7. Impacts of Sc for different slopes [Pr = 0.71,R = 1,Gr = 1&Gc = 1]



(a) (b) (c)

Fig. 8. Impacts of R for different slopes [Pr=13.67,Gc =0.1,Gr = 1&Sc = 1]

VII. CONCLUSIONS

PDEs that model the liquid motion are reduced to ODEs by symmetries. Approximate solution got by applying IV order R-K algorithm with trajectory shoot technique. Following conclusions are evolved.

- Thinning of thermal and concentration boundary-layer thicknesses resulted as Gr & Sc rise; but when Gc rises, these thicknesses increase.
- Velocity and temperature of the liquid rise with R and concentration shows the opposite as R intensifies.
- As Pr rises, temperature rises but not the concentration.
- Velocity and thickness of momentum boundary-layer rise with increasing Gr & Sc; but the opposite trend noticed with increasing Pr & Gc.

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