International Journal of Mathematics Trends and Technology Volume 67 Issue 4, 47-50, April, 2021
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V67I4P507 

# Perfect Prism Problem- A Better Way of Representing Perfect Cuboid Problem

Jayaram A S[#1]

[#] *Associate professor, Mechanical Deportment,*
*Dr Ambedkar Institute of Technology, Bangalore, India.*

***Abstract*** *- This paper illustrates that the perfect cuboid problem, which is also known as perfect Euler brick problem, can be easily and conveniently represented by a prism instead of a cuboid. This will make the concept simpler and easier to understand. It eliminates the use of solid diagonal of the cube, which used to be a hidden line. It makes the same line to be represented on the surface of the prism as face diagonal. All the seven lines representing the integer numbers are on the surfaces and on the edges. This eliminates repeated extra lines and makes the simpler visual meaning of the problem. The aim of this paper is not to prove or disprove the problem but to clearly illustrate that the key factor is common numbers in Pythagorean triplets, finally converting the whole problem into 2D with a set of triangles.*

**Keywords :** *Perfect Euler brick problem, Perfect cuboid problem, Prism representation, Pythagoras theorem, Seven integers.*
.

## I. INTRODUCTION

The perfect cuboid problem [1] or perfect Euler's brick problem[11] is finding Seven integers of a cuboid for the sides and diagonals as shown in the figure -1.

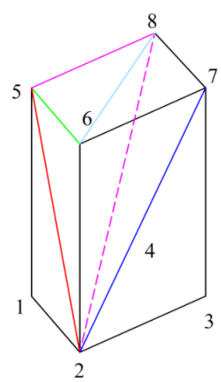


**Fig. 1 Perfect cuboid problem illustrated by the cuboid.**

The perfect cuboid problem is to find seven integers for each of the edges of the cube, face diagonals and the solid diagonal. In the figure-1, lengths of the lines between the corner points 1 to 2, 2 to 3 and 3 to 7 forming the edges of the cube, constructing three integers. The lengths of the lines between the corner points 5 to 2, 6 to 8 and 2 to 7 forming the face diagonals of the cube, constructing another three integers. Finally, the body diagonal is the line joining the corner points 2 to 8 forms the seventh integer. When one finds these seven integers, then the cuboid formed by these seven integers is called a *perfect cuboid*. This paper simplifies the above problem, as explained in the next section.

## II. PERFECT PRISM PROBLEM.

In perfect cuboid problem, there are many repetitive edges. Also, the body diagonals like the line joining points 2 to 8 are hidden lines. So, by cutting the cuboid along the line joining 6 and 8, we will get a triangular prism.

Figure-2 shows such a prism with the lines re-arranged in such a way as to make the illustration simple. It also makes one to move along the seven lines continuously, as explained later in this paper.

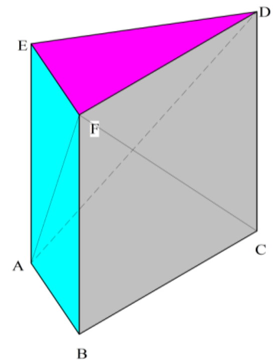


**Fig. 2. Perfect prism obtained by cutting the cuboid.**

To avoid confusion regarding the points that are cut away, the corners are re-named by letters, instead of numbers.

In the figure-2, lengths of the lines between the corner points E to F, F to D, D to E and A to E form the edges of the cube, constructing four integers. lengths of the lines between the corner points A to F, A to D form the face diagonals of the cube, constructing two integers. Finally, the original body diagonal of the cube now becomes another face diagonal becoming the line joining F to C, forms the seventh integer.

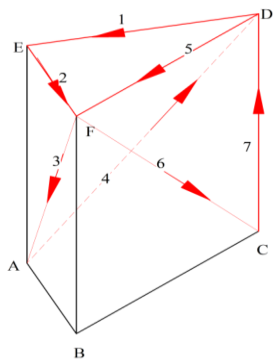


**Fig. 3. The seven lines representing legnths of seven integers.**

All the lines are on the surface and continuously traversable, starting from the corner D to finally ending at D. Since the volume of the prism or the area of the prism are not important and only the lengths are important, we can draw the developed surface of the prism, assuming it to be hollow, which means that we can consider only lateral surface of the prism.

## III. DEVELOPING THE SURFACE.

Figure-4 shows the developed lateral surface of the prism, cut along the edge FB.

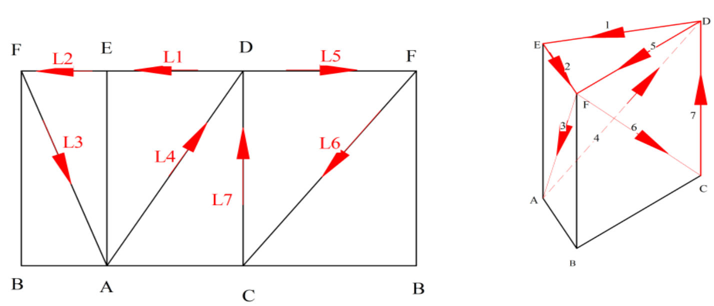


**Fig. 4. Developed surface along with prism.**

Since the lengths are to be represented, the number notations 1,2.. are replaced by L1, L2… to L7. Still the reversibility is maintained, even though it is not required to solve the problem. So, the path is from L1, L2 up to L7.

In this diagram, all the $90^0$ corners are still the same, except at the point E and A. So, that is the only thing that we have to remember, as the Pythagoras relation as

$$\boldsymbol{L1^2 + L2^2 = L5^2} \quad \textbf{(1)}$$

For the purpose of easy understanding, the repeated length are named again as shown in figure-5.

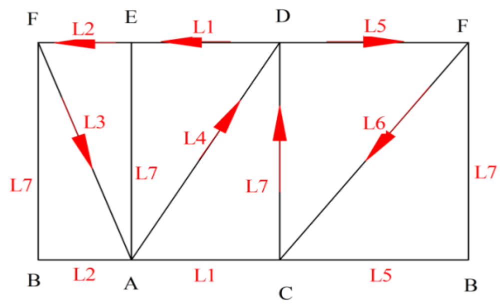


**Fig. 5. Developed surface with repeated lengths marked.**

Now the basic equations to be satisfied, in addition to equation (1) are:

$$\boldsymbol{L7^2 + L2^2 = L3^2} \quad \textbf{(2)}$$

$$\boldsymbol{L7^2 + L1^2 = L4^2} \quad \textbf{(3)}$$

$$\boldsymbol{L7^2 + L5^2 = L6^2} \quad \textbf{(4)}$$

So, it is very clear from these equations that L7 is common to three equations! That means we should find a number common to three Pythagorean triplets [6]. By calculations, we can find that the following triplets with the number 15 as common.

$$\boldsymbol{15^2 + 20^2 = 25^2} \quad \textbf{(5)}$$

$$\boldsymbol{15^2 + 36^2 = 39^2} \quad \textbf{(6)}$$

$$\boldsymbol{15^2 + 112^2 = 113^2} \quad \textbf{(7)}$$

But by comparing equations (2),(3) and (4) with (5),(6) and (7), we can conclude that to satisfy equation (1), we should have **$25^2 + 39^2$ must be equal to $112^2$**, but it is not equal. Thus we should try the next common triplet and so on. In the figure-6, the triangles are re- arranged to get more clarity.

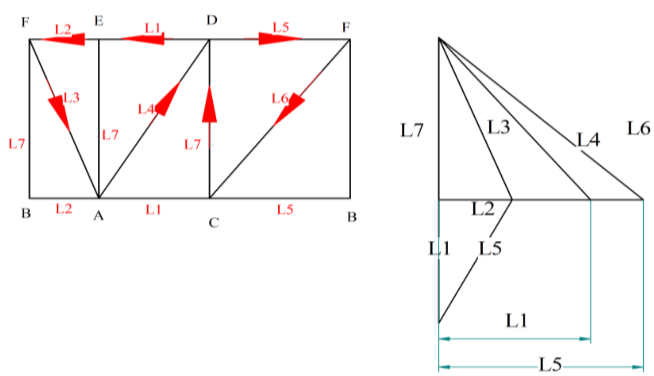


**Fig. 6. Rearranging the triangles.**

This will clearly gives an idea that exactly these common numbers are making it difficult. From advanced computers, has been found out that such a perfect prism does not exist within the existing range of the computers of today. It is easy to get stress in an element[13] or a plate with holes by finite element method using computers [14] but since the numbers are infinite, a logical proof is requires in case of numbers to disprove it. But even if one set of such numbers are found, it becomes a proof! Also, many have claimed to have it proved that such a set of integers does not exist [1-10].

## IV. CONCLUSIONS

This paper establishes a clear understanding of perfect cuboid problem by converting it in to perfect prism problem. Finally, it converts the problem into 2D visualization by developing the surface and rearranging it into simple rightangled triangles..

## REFERENCES


[1] Maiti, S., The Non-existence of Perfect Cuboid., arXiv preprint arXiv:2005.07514 (2020).

[2] Sharipov, R. A., Symmetry-Based Approach to the Problem of a Perfect Cuboid., Journal of Mathematical Sciences 252.2 (2021) 266-282.

[3] Masharov, A. A., and R. A. Sharipov., A strategy of numeric search for perfect cuboids in the case of the second cuboid conjecture., arXiv preprint arXiv:1504.07161 (2015).

[4] Gallyamov, R. R., I. R. Kadyrov, D. D. Kashelevskiy, N. G. Kutlugallyamov, and R. A. Sharipov., A fast modulo primes algorithm for searching perfect cuboids and its implementation., arXiv preprint arXiv:1601.00636 (2016).

[5] Reiter, C., and J. Tirrell., Pursuing the perfect parallelepiped., JP Jour. Algebra Number Theory & Appl 6(2006) 279-274.

[6] Nguhi, Alex, and Cleophas Kweyu., On the Pythagorean Triples' Equations and the Perfect Cuboid Problem., OSF Preprints. April 4 (2021).

[7] Dawit, Bambore., The Proof for Non-existence of Perfect Cuboid., (2014).

[8] DRAGAN, VALERIU.,Notes on a Particular Class of Perfect Cuboids., WSEAS TRANSACTIONS ON MATHEMATICS 13(2014) 811-819.

[9] Druzhinin, V., It is impossible for a perfect cuboid to exist., Norwegian Journal of Development of the International Science 18-1 (2018).

[10] Sharipov, Ruslan., On a pair of cubic equations associated with perfect cuboids., arXiv preprint arXiv:1208.0308 (2012).

[11] Ortan, Alexandra, and Vincent Quenneville-Bélair., Euler's brick., Delta Epsilon, McGill Undergraduate Mathematics Journal 1 (2006): 30-33.

[12] Sawyer, Jorge, and Clifford Reiter., Perfect parallelepipeds exist., Mathematics of computation 80(274)(2011) 1037-1040.

[13] Jayaram A S, Shifting of Maximum Stress from a critical section., International Journal of Engineering Applied Sciences and Technology, 9 (2019) 164-167.

[14] Jayaram A S., Effect of Stress Concentration Factor on Maximum Stress for A Rectangular Plate With Cutout, Subjected to Tensile Load" International Journal of Innovative Science and Research Technology, 9(2017) 294-297.