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Fuzzy Pre*-γ-Open and Fuzzy Pre*-γ-Continuity Mappings in Fuzzy Topological Spaces

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Abstract

In topology, an open mapping is a function between two topological spaces that the image of an open set is an open. On the other hand, a continuous map is a continuous function between two topological spaces that the preimage of an open set is an open. Sivashanmugaraja introduced the concepts of fuzzy pre*-γ-continuous mappings in fuzzy topological spaces. In this paper, we formulate a definition of fuzzy pre*-γ-open mapping and fuzzy super pre-γ-open mapping in fuzzy topological spaces via pre-γ-open fuzzy sets. Also we study composite on these mappings. Moreover, we investigate relationships among these mappings and also prove some properties and theorems.

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Key words: Fuzzy topology, Fuzzy pre-γ-open, Fuzzy pre*-γ-open, Fuzzy super pre-γ-open, Fuzzy pre*-γ-continuous.

1 Introduction

The notion of fuzzy sets was introduced by Zadeh [11] in 1965. Fuzzy sets play a crucial role in all fields of mathematics. In 1968, Chang [1] introduced the idea of fuzzy topological space. Kasahara [4] defined an operation γ in topological spaces. A fuzzy operation γ on fuzzy topological space was introduced by Kalitha and Das [3]. Hariwan Z. Ibrahim [2] introduced the notion of pre-γ-open sets in topological spaces. As a generalization of these notions in fuzzy topology, Sivashanmugaraja and Vadivel [10] introduced notion of pre-γ-open fuzzy sets in fuzzy topological spaces. In this paper, we introduce the notion of pre*-γ-open and super fuzzy pre-γ-open mappings in fuzzy setting. Also we study the relationships between these mappings.

2 Preliminaries

In this paper, $(X,\ \tau_X)$, $(Y,\ \tau_Y)$ and $(Z,\ \tau_Z)$ (or simply X, Y and Z) are always mean fuzzy topological spaces (fts, in short). By $\underline{0}$ and $\underline{1}$, we mean the constant fuzzy sets taking on the values 0 and 1 on X, respectively. Now, we recall some definitions and results used in this paper.

Definition 2.1. A fuzzy set λ of a fts X is called fuzzy pre-γ-open [10] if $\lambda \leq \tau_\gamma\text{-}int(cl(\lambda))$. A fuzzy set λ of a fts X is called fuzzy pre-γ-closed [6] iff its complement is fuzzy pre-γ-open. The family of all pre-γ-open and pre-γ-closed fuzzy sets are denoted by $FP_\gamma O(X)$ and $FP_\gamma C(X)$ respectively.

Definition 2.2. [6] Let λ be a fuzzy set in a fts X. Then the pre-γ-interior of λ is defined as $pint_\gamma(\lambda) = \vee\{\mu : \mu \leq \lambda, \mu \in FP_\gamma O(X)\}$ and the pre-γ-closure of λ is defined as $pcl_\gamma(\lambda) = \wedge\{\mu : \mu \geq \lambda, \mu \in FP_\gamma C(X)\}$.

Definition 2.3. A fuzzy subset μ of a fts $(X,\ \tau)$ is called

(i) neighborhood [1] of a fuzzy point x_β iff $\exists$ a $\eta \in \tau$ such that $x_\beta \in \eta \leq \mu$;

(ii) pre-γ-neighborhood [9] of a fuzzy point $x_\beta \in X$, if $\exists$ a pre-γ-open fuzzy set η such that $x_\beta \in \eta \leq \lambda$.

Definition 2.4. A mapping $f : (X,\ \tau_X) \to (Y,\ \tau_Y)$ is called

(i) fuzzy pre-γ-open [8], if the image of each open fuzzy set of $(X,\ \tau_X)$ is pre-γ-open fuzzy set of $(Y,\ \tau_Y)$;

(ii) fuzzy pre-γ-continuous [9], if $f^{-1}(\lambda)$ is pre-γ-open fuzzy set in $(X,\ \tau_X)$, $\forall$ open fuzzy set λ in $(Y,\ \tau_Y)$.

Definition 2.5. A fts X is called

(i) fuzzy pre-γ-T_1 [7] if $\forall$ pair of fuzzy singletons p_0 and p_1 with different supports x_0 and x_1, $\exists$ a pre-γ-open fuzzy sets λ and μ such that $p_0 \leq \lambda \leq p_1^c$ and $p_1 \leq \mu \leq p_0^c$.

(ii) fuzzy pre-γ-Hausdorff or fuzzy pre-γ-T_2 [7] iff $\forall$ pair of fuzzy singletons p_0 and p_1 with different supports, $\exists$ a pre-γ-open fuzzy sets λ and μ such that $p_0 \leq \lambda \leq p_1^c$, $p_1 \leq \mu \leq p_0^c$ and $\lambda \leq \mu^c$;

(iii) fuzzy pre-γ-compact [5] if each pre-γ-open covering $\mathcal{A}$ of X contains a finite sub collection that also covers X.

3 Fuzzy Pre*-γ-Open and Fuzzy Pre*-γ-Closed Mappings

Definition 3.1. Let $(X,\ \tau_X)$ and $(Y,\ \tau_Y)$ be two fuzzy topological spaces and γ be a fuzzy operation on τ_X and τ_Y. A mapping $f:(X,\ \tau_X)\rightarrow(Y,\ \tau_Y)$ is called

(i) fuzzy pre*-γ-open, if the image of each pre-γ-open fuzzy set of $(X,\ \tau_X)$ is a pre-γ-open fuzzy set of $(Y,\ \tau_Y)$;

(ii) fuzzy pre*-γ-closed, if the image of each pre-γ-closed fuzzy set of $(X,\ \tau_X)$ is a pre-γ-closed fuzzy set of $(Y,\ \tau_Y)$.

Example 3.1 Let $X=Y=\{a,b,c\}$, λ and μ be the fuzzy subsets of X which are defined as $\lambda=\underline{0.4}$ and $\mu=\underline{0.5}$ Let $\tau_X=\{\underline{1},\ \underline{0},\ \lambda\}$ and $\tau_Y=\{\underline{1},\ \underline{0},\ \mu\}$. Then $(X,\ \tau_X)$ and $(Y,\ \tau_Y)$ are fts. Define an operation γ on τ_X by $\gamma(\underline{1})=\underline{1}$, $\gamma(\underline{0})=\underline{0}$, $\gamma(\lambda)=\lambda$ and also define a γ on τ_Y by $\gamma(\underline{1})=\underline{1}$, $\gamma(\underline{0})=\underline{0}$, $\gamma(\mu)=cl(\mu)$. A mapping $f:(X,\ \tau_X)\rightarrow(Y,\ \tau_Y)$ be an identity mapping. Then clearly f is a fuzzy pre*-γ-open mapping.

Theorem 3.1. Let $f:(X,\ \tau_X)\rightarrow(Y,\ \tau_Y)$ be a mapping. Then the following statements are equivalent:

(i) The mapping f is fuzzy pre*-γ-open,

(ii) For every fuzzy singleton $x_\alpha\in X$ and each fuzzy pre-γ-neighborhood P of a fuzzy singleton x_α, $\exists$ a pre-γ-open fuzzy set Q in Y such that $f(x_\alpha)\leq Q\leq f(P)$,

(iii) For every fuzzy set λ of X, $f(pint_\gamma(\lambda))\leq\ pint_\gamma(f(\lambda))$;

(iv) For every fuzzy set μ of Y, $pint_\gamma(f^{-1}(\mu))\leq f^{-1}(\ pint_\gamma(\mu))$;

(v) For every fuzzy set μ of Y, $f^{-1}(pcl_\gamma(\mu))\leq pcl_\gamma(f^{-1}(\mu))$.

Proof. (i) $\rightarrow$ (ii) Let P be fuzzy pre-γ-neighborhood of x_α in X. Then $\exists$ an pre-γ-open fuzzy set Q in X such that $x_\alpha\in Q\leq P$ and hence $f(x_\alpha)\leq f(Q)\leq f(P)$. Since f is fuzzy pre*-γ-open, then $f(Q)$ is pre-γ-open fuzzy set in Y. Put $f(Q)=N$, then $f(x_\alpha)\leq N\leq f(P)$.

(ii) $\rightarrow$ (i) Let P be a pre-γ-open fuzzy set of X containing x_α. Then P is fuzzy pre-γ-neighborhood of each $x_\alpha\in P$. By hypothesis, $\exists$ a pre-γ-open fuzzy set N of Y such that $f(x_\alpha)\in N\leq f(P)$. Hence, $f(P)$ is fuzzy pre-γ-neighborhood of each $f(x_\alpha)\in f(P)$. So $f(P)$ is pre-γ-open fuzzy set in Y. Therefore, f is fuzzy pre*-γ-open mapping.

(i) $\rightarrow$ (iii) Since $pint_\gamma(\lambda)\leq\lambda\leq X$, is fuzzy pre-$\gamma$-open and f is a fuzzy pre*-γ-open, then $f(pint_\gamma(\lambda))\leq f(\lambda)$ is pre-γ-open fuzzy set in Y. Hence, $f(pint_\gamma(\lambda))\leq pint_\gamma(f(\lambda))$.

(iii) $\rightarrow$ (iv) By replacing $f^{-1}(\mu)$ instead of λ in (iii), we have $f(pint_\gamma(f^{-1}(\mu))) \leq pint_\gamma(f(f^{-1}(\mu))) \leq pint_\gamma(\mu)$ and then $pint(f^{-1}(\mu)) \leq f^{-1}(pint_\gamma(\mu))$.

(iv) $\rightarrow$ (i) Let λ be a pre-γ-open fuzzy set in X. Then $f(\lambda) \in Y$ and by hypothesis, $pint_\gamma(f^{-1}(f(\lambda))) \leq f^{-1}(pint_\gamma(f(\lambda)))$. This implies that, $pint_\gamma(\lambda) \leq f^{-1}(pint_\gamma(f(\lambda)))$. Thus $f(pint_\gamma(\lambda)) \leq pint_\gamma(f(\lambda))$. Therefore by (iii), f is fuzzy pre*-γ-open.

(iv) $\rightarrow$ (v) and (v) $\rightarrow$ (iv) Obvious.

(i) $\rightarrow$ (v) Let μ be a fuzzy set of Y and $x_\alpha \in f^{-1}(pcl_\gamma(\mu))$. Then $f(x_\alpha) \in pcl_\gamma(\mu)$. Assume that P is a pre-γ-open fuzzy set containing x_α. Since f is fuzzy pre*-γ-open, we obtain $f(P)$ is a pre-γ-open fuzzy set in Y. Hence, $\mu \wedge f(P) \neq \phi$. Thus $f^{-1}(\mu) \wedge P \neq \phi$. Therefore, $x_\alpha \in pcl_\gamma(f^{-1}(\mu))$. So, $f^{-1}(pcl_\gamma(\mu)) \leq pcl_\gamma(f^{-1}(\mu))$.

(v) $\rightarrow$ (i) Let μ be a fuzzy set in Y. Then $Y \setminus \mu \leq Y$. According to the assumption $f^{-1}(pcl_\gamma(Y \setminus \mu)) \leq pcl_\gamma(f^{-1}(Y \setminus \mu))$ and hence $[X \setminus f^{-1}(pint_\gamma(\mu))] \leq [X \setminus int(f^{-1}(\mu))]$ that implies $pint_\gamma(f^{-1}(\mu)) \leq f^{-1}(pint_\gamma(\mu))$. Then by (iv), f is fuzzy pre*-γ-open. ■

Theorem 3.2. If $f_1 : (X, \tau_X) \rightarrow (Y, \tau_Y)$ is fuzzy pre-γ-open mapping and $f_2 : (Y, \tau_Y) \rightarrow (Z, \tau_Z)$ is fuzzy pre*-γ-open mapping, then $(f_2 \circ f_1)$ is fuzzy pre-γ-open.

Proof. Assume that λ be an open fuzzy set in X. Since f_1 is fuzzy pre-γ-open, $f_1(\lambda)$ is a pre-γ-open fuzzy set in Y. Also since f_2 is fuzzy pre*-γ-open mapping, hence $f_2(f_1(\lambda)) = (f_2 \circ f_1)(\lambda)$ is a pre-γ-open fuzzy set in Z. Therefore $(f_2 \circ f_1)$ is fuzzy pre-γ-open mapping. ■

Theorem 3.3. Let $f_1 : (X, \tau_X) \rightarrow (Y, \tau_Y)$ and $f_2 : (Y, \tau_Y) \rightarrow (Z, \tau_Z)$ be two mappings. If f_1 is an onto fuzzy pre-γ-continuous and $(f_2 \circ f_1)$ is fuzzy pre*-γ-closed, then f_2 is fuzzy pre-γ-closed.

Proof. Let λ be a closed fuzzy set in Y and f_1 be a fuzzy pre-γ-continuous mapping. Therefore $f_1^{-1}(\lambda)$ is a pre-γ-closed fuzzy set in X. But $(f_2 \circ f_1)$ is a fuzzy pre*-γ-closed, then $(f_2 \circ f_1)(f_1^{-1}(\lambda))$ is pre-γ-closed in Z. Also, by onto of f_1, $f_2(\lambda)$ is pre-γ-closed fuzzy set in Z. Hence, f_2 is fuzzy pre-γ-closed. ■

Theorem 3.4. If f_1 and f_2 be two fuzzy pre*-γ-open mappings, then the composite map $(f_2 \circ f_1)$ is also a fuzzy pre*-γ-open mapping.

Proof. Suppose that λ be a pre-γ-open fuzzy set in X. According to the assumption, f_1 be a fuzzy pre*-γ-open mapping, we obtain $f_1(\lambda)$ is a pre-γ-open fuzzy set in Y. Also by hypothesis f_2 is fuzzy pre*-γ-open, then $f_2(f_1(\lambda))$ is pre-γ-open fuzzy set in Z. But $f_2(f_1(\lambda)) = (f_2 \circ f_1)(\lambda)$. Hence, the composite map $f_2 \circ f_1$ is fuzzy pre*-γ-open. ■

4 Fuzzy Super Pre-γ-Open and Fuzzy Super Pre-γ-Closed Mappings

Definition 4.1. Let $(X,\ \tau_X)$ and $(Y,\ \tau_Y)$ be two fuzzy topological spaces and γ be a fuzzy operation on τ_X. A mapping $f:(X,\ \tau_X)\to(Y,\ \tau_Y)$ is called

(i) fuzzy super pre-γ-open if the image of every pre-γ-open fuzzy set of X is an open fuzzy set of Y.

(i) fuzzy super pre-γ-closed if the image of every pre-γ-closed fuzzy set of X is an closed fuzzy set of Y.

Example 4.1 Let $X=Y=\{a,b,c\}$ and $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4\in I^X$ defined as $\lambda_1(a)=0.3,\ \lambda_1(b)=0.6,\ \lambda_1(c)=0.3;\ \lambda_2(a)=0.6,\ \lambda_2(b)=0.2,\ \lambda_2(c)=0.7;$ $\lambda_3(a)=0.3,\ \lambda_3(b)=0.2,\ \lambda_3(c)=0.3;\ \lambda_4(a)=0.6,\ \lambda_4(b)=0.6,\ \lambda_4(c)=0.7.$ Let $\tau_X=\{\underline{1},\ \underline{0},\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4\}$ and τ_Y be the fuzzy discrete topology. Then (X,τ_X) and (Y,τ_Y) are fts. Define an operation $\gamma:\tau_X\to I^X$ by $\gamma(\underline{1})=\underline{1},\ \gamma(\underline{0})=\underline{0},\ \gamma(\lambda_1)=\lambda_1,\ \gamma(\lambda_2)=\underline{0.8},\ \gamma(\lambda_3)=\lambda_3,\ \gamma(\lambda_4)=cl(\lambda_4)$ and also define $\gamma:\tau_Y\to I^Y$ by $\gamma(\underline{1})=\underline{1},\ \gamma(\underline{0})=\underline{0}$. A mapping $f:(X,\ \tau_X)\to(Y,\ \tau_Y)$ defined as $f(a)=a,\ f(b)=c,\ f(c)=b$. Then f is a fuzzy super pre-γ-open mapping.

Theorem 4.1. Let $f:(X,\ \tau_X)\to(Y,\ \tau_Y)$ be a mapping. Then the following statements are equivalent:

(i) The mapping f is fuzzy super pre-γ-open,

(ii) For every fuzzy singleton $x_\alpha\in X$ and each fuzzy pre-γ-neighborhood P of a fuzzy singleton x_α, $\exists$ a fuzzy neighborhood N of $f(x_\alpha)$ in Y such that $f(x_\alpha)\leq N\leq f(P)$,

(iii) For every fuzzy set λ of X, $f(pint_\gamma(\lambda))\leq\ int(f(\lambda))$;

(iv) For every fuzzy set μ of Y, $pint_\gamma(f^{-1}(\mu))\leq f^{-1}(\ int(\mu))$;

(v) For every fuzzy set μ of Y, $f^{-1}(cl(\mu))\leq pcl_\gamma(f^{-1}(\mu))$.

(vi) If f is an onto, then for every fuzzy subset μ of Y and for any pre-γ-closed fuzzy set λ in X containing $f^{-1}(\mu)$, there exists a closed fuzzy subset η of Y containing μ such that $f^{-1}(\eta)\leq\lambda$.

Proof. (i) $\to$ (ii) Let P be fuzzy pre-γ-neighborhood of x_α in X. So $\exists$ an pre-γ-open fuzzy set Q in X such that $x_\alpha\in Q\leq P$ and therefore $f(x_\alpha)\leq f(Q)\leq f(P)$. Since f is fuzzy super pre-γ-open, $f(Q)$ is open fuzzy set in Y. Put $f(Q)=R$, then $f(x_\alpha)\leq R\leq f(P)$.

(ii) $\to$ (i) Let P be a pre-γ-open fuzzy set of X containing x_α. Therefore $f(x_\alpha) \in f(P)$. By hypothesis, $\exists$ an open fuzzy set R of Y such that $f(x_\alpha) \in R \leq f(P)$. Thus, $f(P)$ is fuzzy neighborhood for $f(x_\alpha) \in f(P)$. So $f(P)$ is an open fuzzy set in Y. Therefore, f is super pre-γ-open mapping.

(i) $\to$ (iii) Since $pint_\gamma(\lambda) \leq \lambda \leq X$, is pre-$\gamma$-open fuzzy set and f is fuzzy super pre-γ-open, we obtain $f(pint_\gamma(\lambda)) \leq f(\lambda)$ is an open fuzzy set in Y. Thus, $f(pint_\gamma(\lambda)) \leq int(f(\lambda))$.

(iii) $\to$ (iv) By replacing $f^{-1}(\mu)$ instead of λ in (iii), we have $f(pint_\gamma(f^{-1}(\mu)) \leq int(f(f^{-1}(\mu))) \leq int(\mu)$ and therefore $pint(f^{-1}(\mu)) \leq f^{-1}(int(\mu))$.

(iv) $\to$ (i) Let λ be a pre-γ-open fuzzy set of X. Then $f(\lambda)$ is in Y and by hypothesis, $pint_\gamma(f^{-1}(f(\lambda))) \leq f^{-1}(int(f(\lambda)))$. Therefore, $pint_\gamma(\lambda) \leq f^{-1}(int(f(\lambda)))$. Hence $f(pint_\gamma(\lambda)) \leq int(f(A))$. Thus, f is fuzzy super pre-γ-open.

(iv) $\to$ (v) Let μ be any fuzzy set in Y. Therefore $Y \setminus \mu \leq Y$, thus by hypothesis, we obtain $pint_\gamma(f^{-1}(Y \setminus \mu)) \leq f^{-1}(int(Y \setminus \mu))$. Thus $X \setminus pcl_\gamma(f^{-1}(\mu)) \leq X \setminus f^{-1}(cl(\mu))$. Hence, $f^{-1}(cl(\mu)) \leq pcl_\gamma(f^{-1}(\mu))$.

(v) $\to$ (iv) Let μ be any fuzzy set in Y. Then $Y \setminus \mu \leq Y$. Therefore by hypothesis, we have $f^{-1}(cl(Y \setminus \mu)) \leq pcl_\gamma(f^{-1}(Y \setminus \mu))$ and thus $X \setminus f^{-1}(int(\mu)) \leq X \setminus pint_\gamma(f^{-1}(\mu))$. Hence, $pint_\gamma(f^{-1}(\mu)) \leq f^{-1}(int(\mu))$.

(iv) $\to$ (i) Let μ be a pre-γ-open fuzzy set in X. Then $f(\mu)$ is in Y and by hypothesis, $pint(f^{-1}(f(\mu))) \leq f^{-1}(int(f(\mu)))$. Therefore, $pint_\gamma(\mu) \leq f^{-1}(int(f(\mu)))$. Thus $f(pint_\gamma(\mu)) \leq int(f(\mu))$. Thus by (iii), f is fuzzy super pre-γ-open.

(i) $\to$ (vi) Let $\eta = Y \setminus f(X \setminus \lambda)$ and λ be a pre-γ-closed fuzzy set of X containing $f^{-1}(\mu)$. Then $X \setminus \lambda$ is a pre-γ-open fuzzy set. Since f is a fuzzy super-pre-γ-open mapping, $f(X \setminus \lambda)$ is an open fuzzy set in Y. Therefore, η is a closed fuzzy set of Y and $f^{-1}(\eta) = X \setminus f^{-1} f(X \setminus \lambda) \leq X \setminus (X \setminus \lambda) = \lambda$.

(vi) $\to$ (i) Let P be a pre-γ-open fuzzy set in X and put $\mu = Y \setminus f(P)$. Then $X \setminus P$ is a pre-γ-closed fuzzy set in X with $f^{-1}(\mu) \leq (X \setminus P)$. By hypothesis, there exists a closed fuzzy set η of Y such that $\mu \leq \eta$ and $f^{-1}(\eta) \leq (X \setminus P)$. Hence, $f(P) \leq (Y \setminus \eta)$ and since $\mu \leq \eta$, then $(Y \setminus \eta) \leq (Y \setminus \mu) = f(P)$. This implies $f(P) = Y \setminus \eta$ which is open. Therefore, f is fuzzy super pre-γ-open. ■

Remark 4.1. The composition of two fuzzy super pre-γ-open mappings need not be fuzzy super pre-γ-open.

Proof. Obvious.

Theorem 4.2. Let $f : (X, \tau_X) \to (Y, \tau_Y)$ be a bijective fuzzy super pre-γ-open mapping. Then the following statements are hold:

(i) If X is a fuzzy pre-γ-T_i-space, then Y is fuzzy T_i, where $i = 1,\ 2$.

(ii) If Y is a fuzzy compact space, then X is fuzzy pre-γ-compact.

Proof. (i) We prove that for the case of a pre-γ-T_2-space. Suppose y_α and y_β be two fuzzy singleton with different supports in Y. Then there exist two fuzzy singletons x_α and x_β in X such that $f(x_\alpha) = y_\alpha$ and $f(x_\beta) = y_\beta$. According to the assumption, X is a fuzzy pre-γ-T_2-space, then there exist two disjoint pre-γ-open fuzzy sets λ and μ of X such that $x_\alpha \leq \lambda \leq x^c_\beta$ and $x_\beta \leq \mu \leq x^c_\alpha$ and $\lambda \leq \mu^c$. Since f is a bijective fuzzy super pre-γ-open map, we obtain $f(\lambda)$ and $f(\mu)$ are open fuzzy sets of Y with $y_\alpha \leq f(\lambda)$, $y_\beta \leq f(\mu)$, and $f(\lambda) \wedge f(\mu) = \phi$. Hence, Y is fuzzy T_2.

(ii) Let Y be a fuzzy compact space. Let $\{\mu_j : j \in J\}$ be a family of pre-γ-open fuzzy set cover of X and f be an onto super pre-γ-open mapping. Then $\{f(\mu_j) : j \in J\}$ is an open cover fuzzy set of Y. But, Y is a fuzzy compact space, hence there exists a finite subfamily I of J such that $Y = \vee\{f(\mu_j) : j \in I\}$. Then by one-one of f, we obtain $\{\mu_j : j \in I\}$ is a finite subfamily of X. Therefore, X is fuzzy pre-γ-compact.. ■

Theorem 4.3. If $f : (X,\ \tau_X) \to (Y,\ \tau_Y)$ is an onto fuzzy super pre-γ-open mapping and Y is a fuzzy connected space, then X is fuzzy pre-γ-connected.

Proof. Obvious. ■

5 Fuzzy Pre*-γ-Continuous or Fuzzy Pre-γ-Irresolute Mappings

Definition 5.1. [9] Let $(X,\ \tau_X)$ and $(Y,\ \tau_Y)$ be two fuzzy topological spaces and γ be a fuzzy operation on τ_X and τ_Y. A mapping $f : (X,\ \tau_X) \to (Y,\ \tau_Y)$ is said to be fuzzy pre*-γ-continuous (or pre-γ-irresolute), if $f^{-1}(\lambda)$ is a pre-γ-open fuzzy set of X, $\forall$ pre-γ-open fuzzy set λ of Y.

Example 5.1 Let $X = Y = \{a,\ b,\ c\}$. Let τ_X be the indiscrete fuzzy topology and τ_Y be the discrete fuzzy topology. Define $\gamma : \tau_X \to I^X$ by $\gamma(\underline{1}) = \underline{1}$, $\gamma(\underline{0}) = \underline{0}$ and also define $\gamma : \tau_Y \to I^Y$ by $\gamma(\lambda) = \lambda$, $\forall\ \lambda \in \tau_Y$. A mapping $f : (X,\ \tau_X) \to (Y,\ \tau_Y)$ defined as $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is a fuzzy pre*-γ-continuous mapping.

Theorem 5.1. Let $f : (X,\ \tau_X) \to (Y,\ \tau_Y)$ be a bijective mapping. Then the following statements are equivalent:

(i) f is fuzzy pre*-γ-open;

(i) f is fuzzy pre*-γ-closed;

(i) f^{-1} is fuzzy pre*-γ-continuous.

Proof. Obvious.

Theorem 5.2. A mapping $f : (X,\ \tau_X) \to (Y,\ \tau_Y)$ is fuzzy pre*-γ-continuous iff $f^{-1}(\lambda)$ is pre-γ-closed fuzzy set in X, $\forall$ pre-γ-closed fuzzy set λ in Y.

Proof. Obvious. ■

Theorem 5.3. Let $f : (X,\ \tau_X) \to (Y,\ \tau_Y)$ be a mapping. Then the following are equivalent:

(i) f is a fuzzy pre*-γ-continuous mapping;

(ii) $f(pcl_\gamma(\lambda)) \leq cl(f(\lambda))$, $\forall$ fuzzy set λ in X.

Proof.
(i)$\Rightarrow$(ii) Let λ be a fuzzy set in X. Then $cl(f(\lambda))$ is closed fuzzy set in Y. Since f is fuzzy pre*-γ-continuous mapping, $f^{-1}(cl(f(\lambda)))$ is a pre-γ-closed fuzzy set in X and so $f^{-1}(cl(f(\lambda))) = pcl_\gamma(f^{-1}(cl(f(A))))$. Since $\lambda \leq f^{-1}(f(\lambda))$, we have $pcl_\gamma(\lambda) \leq pcl_\gamma(f^{-1}(f(\lambda))) \leq pcl_\gamma(f^{-1}(cl(f(\lambda)))) = f^{-1}(cl(f(\lambda)))$. Hence $f(pcl(\lambda)) \leq cl(f(\lambda))$.

(ii)$\Rightarrow$(i) Let μ be a closed fuzzy set in Y and let $\lambda = f^{-1}(\mu)$. According to the assumption, $f(pcl_\gamma(f^{-1}(\mu))) \leq cl(f(f^{-1}(\mu)))$, which implies $pcl_\gamma(f^{-1}(\mu)) \leq f^{-1}(cl(f(f^{-1}(\mu))) \leq f^{-1}(cl(\mu)) \leq f^{-1}(\mu)$. But $f^{-1}(\mu) \leq pcl_\gamma(f^{-1}(\mu))$, so $f^{-1}(\mu) = pcl_\gamma(f^{-1}(\mu))$. Hence $f^{-1}(\mu)$ is pre-γ-closed fuzzy set in X. Therefore f is fuzzy pre*-γ-continuous mapping. ■

Theorem 5.4. If $f_1 : (X,\ \tau_X) \to (Y,\ \tau_Y)$ and $f_2 : (Y,\ \tau_Y) \to (Z,\ \tau_Z)$ are fuzzy pre*-γ-continuous mappings, then the composite map $(f_2 \circ f_1) : (X,\ \tau_X) \to (Z,\ \tau_Z)$ is also fuzzy pre*-γ-continuous.

Proof. Let μ be a pre-γ-open fuzzy set in Z. Since f_2 be a fuzzy pre*-γ-continuous,, $f_2^{-1}(\mu)$ is pre-γ-open fuzzy set in Y. Also since f_1 is fuzzy pre*-γ-continuous, $f_1^{-1}(f_2^{-1}(\mu))$ is pre-γ-open fuzzy set in X. But $f_1^{-1}(f_2^{-1}(\mu)) = (f_2 \circ f_1)^{-1}(\mu)$. So $(f_2 \circ f_1)^{-1}(\mu)$ is a pre-γ-open fuzzy set in X. Therefore $(f_2 \circ f_1)$ is fuzzy pre*-γ-continuous. ■

Theorem 5.5. If $f_1 : (X,\ \tau_X) \to (Y,\ \tau_Y)$ is fuzzy pre*-γ-continuous and $f_2 : (Y,\ \tau_Y) \to (Z,\ \tau_Z)$ be a fuzzy pre-γ-continuous, then $(f_2 \circ f_1) : (X,\ \tau_X) \to (Z,\ \tau_Z)$ is fuzzy pre-γ-continuous.

Proof. Let μ be an open fuzzy set in Z. Since f_2 be a fuzzy pre-γ-continuous, $f_2^{-1}(\mu)$ is a pre-γ-open fuzzy set in Y. Also since f_1 is fuzzy pre*-γ-continuous, $f_1^{-1}(f_2^{-1}(\mu))$ is a pre-γ-open fuzzy set in X. Hence $(f_2 \circ f_1)^{-1}(\mu)$ is pre-γ-open fuzzy set in X. Therefore $(f_2 \circ f_1)$ is fuzzy pre-γ-continuous. ■

6 Conclusion

In this paper, the concepts of fuzzy pre*-γ-open and fuzzy super pre-γ-open mappings are introduced. Also we studied about fuzzy pre*-γ-continuous. It is reported that the notions of these three mappings are independent. Moreover we have discussed the relationships between these mappings. There is a scope to study and extend these newly defined mappings.

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