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A Fuzzy Inventory Model of Eco-Responsible Waste Disposal

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Abstract:

Our goal in this paper is to dispose of waste produced during manufacturing processes in order to maintain a clean and green environment.The trapezoidal fuzzy numbers have been introduced to us. Our aim is to find the best EOQ and lowest total cost for these models by defuzzing the fuzzy total production and inventory costs with the Beta distribution method and solving the problem with the Extension of Lagrangean Method. An example is used to verify the proposed model.

Introduction:

In general, it is the stock that is kept on hand for the next year's sale. When constructing an EOQ model to determine the lowest overall cost, the cost of waste disposal is often overlooked. Municipal waste is dealt with using methods such as landfill, decomposition, incineration, and a few others. In comparison to all of these processes, incineration is both cost-effective and environmentally friendly.

A analysis of Renewable Energy from Bio-Waste Municipal Corporation was established by Sawakhande S.M. and R.T. Ajdhav. Another EOQ model proposed by W. Ritha and Nivetha Martin describes the advantages of incineration as a waste disposal process. Another EOQ was developed by W. Ritha and I. Antonitte Vinolin [7], in which waste disposal is done with environmental consideration. Dobos and Richter [2] developed a production or recycling model, from which Jaber et al. [4, 5] suggested an EOQ of repair and waste disposal with entropy cost for further growth. The idea was developed by A. Roy and K. Maity, who proposed an EOQ with remanufacturing of faulty products. We will suggest an inventory model in this paper to dispose of wastages that occur during manufacturing processes in order to maintain a safe and green climate. The solution is explored in both a clear and a hazy light. To address the same in a fuzzy way, we use trapezoidal fuzzy numbers. The proposed model is numerically checked through an example, and the model's derivation concludes with a conclusion and recommendations for further study.

Definitions and Methodologies

Fuzzy Set:

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$.Here $\mu_{\tilde{A}}:X\rightarrow[0,1]$ is a mapping called the membership value of $x\in X$ in a fuzzy set $\tilde{A}$.

Trapezoidal Fuzzy Number:

A Trapezoidal Fuzzy Number $\tilde{\delta}=(z,y,x,v)$ is represented with membership function

$$\mu_{\tilde{\delta}}(g)=\begin{cases} L(g)=\dfrac{g-z}{y-z}, z\le g\le y \\ 1, y\le g\le x \\ R(g)=\dfrac{v-g}{v-x}, x\le g\le v \\ 0, Otherwise \end{cases}$$

Beta Distribution Method:

The Beta distribution function is a function from the set of all trapezoidal numbers to the real line R.

If $\tilde{A}=(a_1,a_2,a_3,a_4)$ is a trapezoidal fuzzy number, then the corresponding real number obtained by Beta distribution function is given by

$$\beta(\tilde{A})=\frac{2a_1+7a_2+7a_3+2a_4}{18}$$

Extension of the Lagrangean Method

Taha demonstrated how to use the Lagrangean Method to solve the optimal solution of a nonlinear programming problem with equality constraints, as well as how the Lagrangean Method can be expanded to solve inequality constraints The general principle behind extending the Lagrangean protocol is that if the problem's unbounded endpoint does not fulfil all requirements, the problem's constrained optimum must occur at a solution space boundary point. Suppose that the problem is given by Minimize y = f(x) Sub to gi(x) ≥0, i = 1,2, ….m. The non negativity constraints x ≥0 if any are included in them constraints. Then the procedure of the Extension of the Lagrangean method involves the following steps.

Step 1: Solve the unconstrained problem Min y = f(x) If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise , set k = 1 and go to step 2.

Step 2: Activate any k constraints (i. e., convert them into equality) and optimize f(x) subject to the k active constraints by the Lagrangean method. If the resulting solution is feasible with respect to the remaining constraints and repeat the step. If all sets of active constraints taken k at a time are considered without encountering a feasible solution, go to step 3.

Step 3: If K = m, stop; no feasible solution exists. Otherwise, set k = k + 1 and go to step 2.

Arithmetic Operations under Function Principle

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two hexagonal fuzzy numbers, then the arithmetic operations are defined as

1. $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} * \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4,)$
3. $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1} \right)$

$$\alpha \tilde{A} = \begin{cases} \alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha \geq 0 \\ \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_{1,} \alpha < 0 \end{cases}$$

Notations:

D_C -Ordering cost

E_C -Setup cost

L_C -Holding cost

T_C -Total cost

μ -Proportion of waste proposal

W -Waste disposal cost

α -Demand rate

β -Production rate

Q -Economic order quantity

$D_{\tilde{C}}$ -Fuzzy Ordering cost

$E_{\tilde{C}}$ - Fuzzy Setup cost

$L_{\tilde{C}}$ - Fuzzy Holding cost

$\tilde{W}$ - Fuzzy waste disposal cost

$\tilde{\mu}$ - Fuzzy Proportion of waste proposal

$\tilde{Q}$ - Fuzzy Economic order quantity

Assumptions

❖The demand rate is known and constant.
❖Shortages are not allowed.
❖The time limit is infinite.

Mathematical Model in Crisp Sense

The total cost of this mathematical inventory model is the sum of the Ordering cost, Holding cost, Setup Cost and the Waste disposal cost.

$$T_C = \frac{Q}{2}\left[L_C\alpha + \left(1-\frac{\beta}{\alpha}\right)D_C\right] + \frac{1}{Q}\left[D_C\alpha + E_C + W\mu\right]$$

Differentiate partially with respect to ‘q’ and equate it to zero

$$q^* = \sqrt{\frac{2\left(D_C\alpha + E_C + W\mu\right)}{L_C\alpha + \left(1-\frac{\beta}{\alpha}\right)D_C}}$$

Mathematical Model in fuzzy Sense

us consider the Ordering cost, Setup cost, Holding cost, Waste disposal cost and the proportion of waste produced as trapezoidal fuzzy numbers.

$$T_{\tilde{C}} = \frac{\tilde{Q}}{2}\left[L_{\tilde{C}}\alpha \oplus \left(1\Theta\frac{\beta}{\alpha}\right)D_{\tilde{C}}\right] \oplus \frac{1}{\tilde{Q}}\left[D_{\tilde{C}} + E_{\tilde{C}} + \tilde{W}\tilde{\mu}\right]$$

$D_{\tilde{C}}, E_{\tilde{C}}, L_{\tilde{C}}, \tilde{W}, \tilde{\mu}, \tilde{Q}$ be the trapezoidal fuzzy numbers.

$$T_{\tilde{C}} = \frac{Q_1}{2}\left[L_{C_1}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_1}\right] + \frac{1}{Q_1}\left[D_{C_1}\alpha + E_{C_1} + W_1\mu_1\right] + \frac{Q_2}{2}\left[L_{C_2}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_2}\right] + \frac{1}{Q_2}\left[D_{C_2}\alpha + E_{C_2} + W_2\mu_2\right] +$$
$$\frac{Q_3}{2}\left[L_{C_3}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_3}\right] + \frac{1}{Q_3}\left[D_{C_3}\alpha + E_{C_3} + W_3\mu_3\right] + \frac{Q_4}{2}\left[L_{C_4}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_4}\right] + \frac{1}{Q_4}\left[D_{C_4}\alpha + E_{C_4} + W_4\mu_4\right]$$

We use Beta distribution method for defuzzification

$$T_{\tilde{C}} = \frac{1}{18}\left\{\begin{array}{l} 2\left(\frac{Q_1}{2}\left[L_{C_1}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_1}\right] + \frac{1}{Q_4}\left[D_{C_1}\alpha + E_{C_1} + W_1\mu_1\right]\right) + \\ 7\left(\frac{Q_2}{2}\left[L_{C_2}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_2}\right] + \frac{1}{Q_3}\left[D_{C_2}\alpha + E_{C_2} + W_2\mu_2\right]\right) + \\ 7\left(\frac{Q_3}{2}\left[L_{C_3}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_3}\right] + \frac{1}{Q_2}\left[D_{C_3}\alpha + E_{C_3} + W_3\mu_3\right]\right) + \\ 2\left(\frac{Q_4}{2}\left[L_{C_4}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_4}\right] + \frac{1}{Q_1}\left[D_{C_4}\alpha + E_{C_4} + W_4\mu_4\right]\right) \end{array}\right\}$$

In these following steps,we use extension of lagrangean method to find the solution of Q_1, Q_2, Q_3, Q_4 to minimize $T_C(Q)$

Step 1:

$$T_{\tilde{C}} = \frac{1}{18}\left\{\begin{array}{l} 2\left(\frac{Q_1}{2}\left[L_{C_1}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_1}\right] + \frac{1}{Q_4}\left[D_{C_1}\alpha + E_{C_1} + W_1\mu_1\right]\right) + \\ 7\left(\frac{Q_2}{2}\left[L_{C_2}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_2}\right] + \frac{1}{Q_3}\left[D_{C_2}\alpha + E_{C_2} + W_2\mu_2\right]\right) + \\ 7\left(\frac{Q_3}{2}\left[L_{C_3}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_3}\right] + \frac{1}{Q_2}\left[D_{C_3}\alpha + E_{C_3} + W_3\mu_3\right]\right) + \\ 2\left(\frac{Q_4}{2}\left[L_{C_4}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_4}\right] + \frac{1}{Q_1}\left[D_{C_4}\alpha + E_{C_4} + W_4\mu_4\right]\right) \end{array}\right\}$$

$$0 < Q_1 \le Q_2 \le Q_3 \le Q_4,$$

With $Q_2 - Q_1 \ge 0, Q_3 - Q_2 \ge 0, Q_4 - Q_3 \ge 0$

$$Where Q_1 > 0.$$

$$\frac{\partial T_{\tilde{C}}}{\partial Q_1}=0 \Rightarrow Q_1=\sqrt{\frac{2\left(2D_{C_4}\alpha+2E_{C_4}+2W_4\mu_4\right)}{2L_{C_1}\alpha+2\left(1-\frac{\beta}{\alpha}\right)D_{C_1}}}$$

$$\frac{\partial T_{\tilde{C}}}{\partial Q_2}=0 \Rightarrow Q_2=\sqrt{\frac{2\left(7D_{C_3}\alpha+7E_{C_3}+7W_3\mu_3\right)}{7L_{C_2}\alpha+7\left(1-\frac{\beta}{\alpha}\right)D_{C_2}}}$$

$$\frac{\partial T_{\tilde{C}}}{\partial Q_3}=0 \Rightarrow Q_3=\sqrt{\frac{2\left(7D_{C_2}\alpha+7E_{C_2}+7W_2\mu_2\right)}{7L_{C_3}\alpha+7\left(1-\frac{\beta}{\alpha}\right)D_{C_3}}}$$

$$\frac{\partial T_{\tilde{C}}}{\partial Q_4}=0 \Rightarrow Q_4=\sqrt{\frac{2\left(2D_{C_1}\alpha+2E_{C_1}+2W_1\mu_1\right)}{2L_{C_4}\alpha+2\left(1-\frac{\beta}{\alpha}\right)D_{C_4}}}$$

As the above results,show that $Q_1 \geq Q_2, Q_3 \geq Q_4$ it doesno satisfy the local optimum

Step 2:

Convert the inequality constraint $Q_2-Q_1 \geq 0$, into equality constraint $Q_2-Q_1=0$ and optimize $T_{\tilde{C}}(Q)$ Subject to $Q_2-Q_1=0$ by the lagrangean method,

Fix the constraint as 1,

(i.e) $\lambda(Q_2-Q_1)$

$$T_{\tilde{C}}=\frac{1}{18}\left\{\begin{array}{l} 2\left(\frac{Q_1}{2}\left[L_{C_1}\alpha+\left(1-\frac{\beta}{\alpha}\right)D_{C_1}\right]+\frac{1}{Q_4}\left[D_{C_1}\alpha+E_{C_1}+W_1\mu_1\right]\right)+ \\ 7\left(\frac{Q_2}{2}\left[L_{C_2}\alpha+\left(1-\frac{\beta}{\alpha}\right)D_{C_2}\right]+\frac{1}{Q_3}\left[D_{C_2}\alpha+E_{C_2}+W_2\mu_2\right]\right)+ \\ 7\left(\frac{Q_3}{2}\left[L_{C_3}\alpha+\left(1-\frac{\beta}{\alpha}\right)D_{C_3}\right]+\frac{1}{Q_2}\left[D_{C_3}\alpha+E_{C_3}+W_3\mu_3\right]\right)+ \\ 2\left(\frac{Q_4}{2}\left[L_{C_4}\alpha+\left(1-\frac{\beta}{\alpha}\right)D_{C_4}\right]+\frac{1}{Q_1}\left[D_{C_4}\alpha+E_{C_4}+W_4\mu_4\right]\right) \end{array}\right\}+\lambda(Q_2-Q_1)$$

Differentiate partially with respect to Q_1, Q_2, Q_3, Q_4 and λ to find the minimization of $T_{\tilde{C}}(Q)$,we get five kind of results.

Let all the above partial derivatives equal to zero and solve Q_1, Q_2, Q_3, Q_4 we get,

$$Q_1=Q_2=\sqrt{\frac{2\left(2D_{C_4}\alpha+2E_{C_4}+2W_4\mu_4\right)+2(7D_{C_3}\alpha+7E_{C_3}+7W_3\mu_3)}{2L_{C_1}\alpha+2\left(1-\frac{\beta}{\alpha}\right)D_{C_1}+7L_{C_2}\alpha+7\left(1-\frac{\beta}{\alpha}\right)D_{C_2}}}$$

$$Q_3=\sqrt{\frac{2(7D_{C_2}\alpha+7E_{C_2}+7W_2\mu_2)}{7L_{C_3}\alpha+7\left(1-\frac{\beta}{\alpha}\right)D_{C_3}}}$$

$$Q_4=\sqrt{\frac{2(2D_{C_1}\alpha+2E_{C_1}+2W_1\mu_1)}{2L_{C_4}\alpha+2\left(1-\frac{\beta}{\alpha}\right)D_{C_4}}}$$

Because the above result shows that $Q_3 \geq Q_4$ it doesnot satisfy the constraint.

$0 < Q_1 \leq Q_2 \leq Q_3 \leq Q_4$,It is not a local optimum.

Step 3:

Fix the constraints as 2,

$\lambda_1(Q_2 - Q_1) + \lambda_2(Q_3 - Q_2)$

$$T_{\tilde{C}} = \frac{1}{18}\left\{\begin{array}{l} 2\left(\frac{Q_1}{2}\left[L_{C_1}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_1}\right] + \frac{1}{Q_4}\left[D_{C_1}\alpha + E_{C_1} + W_1\mu_1\right]\right) + \\ 7\left(\frac{Q_2}{2}\left[L_{C_2}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_2}\right] + \frac{1}{Q_3}\left[D_{C_2}\alpha + E_{C_2} + W_2\mu_2\right]\right) + \\ 7\left(\frac{Q_3}{2}\left[L_{C_3}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_3}\right] + \frac{1}{Q_2}\left[D_{C_3}\alpha + E_{C_3} + W_3\mu_3\right]\right) + \\ 2\left(\frac{Q_4}{2}\left[L_{C_4}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_4}\right] + \frac{1}{Q_1}\left[D_{C_4}\alpha + E_{C_4} + W_4\mu_4\right]\right) \end{array}\right\} + \lambda_1(Q_2 - Q_1) + \lambda_2(Q_3 - Q_2)$$

Differentiate partially with respect to $Q_1, Q_2, Q_3, Q_4, \lambda$ to find the minimization of $T_{\tilde{C}}(Q)$ we ge five kind of results. Let all the above partial derivatives equal to zero, solve Q_1, Q_2, Q_3, Q_4

We get

$$Q_1 = Q_2 = Q_3 = Q_1 = Q_2 = \sqrt{\frac{2\left(2D_{C_4}\alpha + 2E_{C_4} + 2W_4\mu_4\right) + 2(7D_{C_3}\alpha + 7E_{C_3} + 7W_3\mu_3) + 2(7D_{C_2}\alpha + 7E_{C_2} + 7W_2\mu_2)}{2L_{C_1}\alpha + 2\left(1-\frac{\beta}{\alpha}\right)D_{C_1} + 7L_{C_2}\alpha + 7\left(1-\frac{\beta}{\alpha}\right)D_{C_2} + 7L_{C_3}\alpha + 7\left(1-\frac{\beta}{\alpha}\right)D_{C_3}}}$$

$$Q_4 = \sqrt{\frac{2(2D_{C_1}\alpha + 2E_{C_1} + 2W_1\mu_1)}{2L_{C_4}\alpha + 2\left(1-\frac{\beta}{\alpha}\right)D_{C_4}}}$$

Step 4:

Fix the constraints as 3,(i.e)
$\lambda_1(Q_2 - Q_1) + \lambda_2(Q_3 - Q_2) + \lambda_3(Q_4 - Q_3)$

$$T_{\tilde{C}} = \frac{1}{18}\left\{\begin{array}{l} 2\left(\frac{Q_1}{2}\left[L_{C_1}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_1}\right] + \frac{1}{Q_4}\left[D_{C_1}\alpha + E_{C_1} + W_1\mu_1\right]\right) + \\ 7\left(\frac{Q_2}{2}\left[L_{C_2}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_2}\right] + \frac{1}{Q_3}\left[D_{C_2}\alpha + E_{C_2} + W_2\mu_2\right]\right) + \\ 7\left(\frac{Q_3}{2}\left[L_{C_3}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_3}\right] + \frac{1}{Q_2}\left[D_{C_3}\alpha + E_{C_3} + W_3\mu_3\right]\right) + \\ 2\left(\frac{Q_4}{2}\left[L_{C_4}\alpha + \left(1-\frac{\beta}{\alpha}\right)D_{C_4}\right] + \frac{1}{Q_1}\left[D_{C_4}\alpha + E_{C_4} + W_4\mu_4\right]\right) \end{array}\right\} + \lambda_1(Q_2 - Q_1) + \lambda_2(Q_3 - Q_2) + \lambda_3(Q_4 - Q_3)$$

In the following steps we use extension of the lagrngean method to find the solution of Q_1, Q_2, Q_3, Q_4 to minimize $T_{\tilde{C}}(Q)$

$$Q^* = \sqrt{\frac{2\left[\left(2D_{C_1}+7D_{C_2}+7D_{C_3}+2D_{C_4}\right)\alpha+\left(2E_{C_1}+7E_{C_2}+7E_{C_3}+2E_{C_4}\right)+\left(2W_1\mu_1+7W_2\mu_2+7W_3\mu_3+2W_4\mu_4\right)\right]}{\left[\left(2L_{C_1}+7L_{C_2}+7L_{C_3}+2L_{C_4}\right)\alpha+\left(2D_{C_1}+7D_{C_2}+7D_{C_3}+2D_{C_4}+\right)\left(1-\frac{\beta}{\alpha}\right)\right]}}$$

Numerical Example:

Crisp sense

Case I:

Given α -Rs.3000, β -Rs.1500, D_C - Rs.750, L_C - Rs.5, E_C - Rs.70000, W - Rs.75, μ - Rs.750 Find the order quantity and total cost.

$$q^* = \sqrt{\frac{2\left(D_C\alpha+E_C+W\mu\right)}{L_C\alpha+\left(1-\frac{\beta}{\alpha}\right)D_C}}$$

=17.58140

Case II:

$$T_{\tilde{C}} = \frac{\tilde{Q}}{2}\left[L_{\tilde{C}}\alpha \oplus \left(1\Theta\frac{\beta}{\alpha}\right)D_{\tilde{C}}\right] \oplus \frac{1}{\tilde{Q}}\left[D_{\tilde{C}}+E_{\tilde{C}}+\tilde{W}\tilde{\mu}\right]$$

=Rs.142766.25

Fuzzy sense:

α =3000, β =1500, $\tilde{D}_C$ =Rs.(650,700,800,850), $\tilde{L}_C$ =(1,3,7,9), $\tilde{E}_C$ =(50000,60000,80000,90000), $\tilde{W}$ =(55,65,85,95), $\tilde{\mu}$ =(710,730,770,790) 750 Find the fuzzy order quantity and fuzzy total cost.

Case I:

$$Q^* = \sqrt{\frac{2\left[\left(2D_{C_1}+7D_{C_2}+7D_{C_3}+2D_{C_4}\right)\alpha+\left(2E_{C_1}+7E_{C_2}+7E_{C_3}+2E_{C_4}\right)+\left(2W_1\mu_1+7W_2\mu_2+7W_3\mu_3+2W_4\mu_4\right)\right]}{\left[\left(2L_{C_1}+7L_{C_2}+7L_{C_3}+2L_{C_4}\right)\alpha+\left(2D_{C_1}+7D_{C_2}+7D_{C_3}+2D_{C_4}+\right)\left(1-\frac{\beta}{\alpha}\right)\right]}}$$

=21.47231

Case II:

$$T_{\tilde{C}} = \frac{Q_1}{2}\left[L_{C_1}\alpha+\left(1-\frac{\beta}{\alpha}\right)D_{C_1}\right]+\frac{1}{Q_1}\left[D_{C_1}\alpha+E_{C_1}+W_1\mu_1\right]+\frac{Q_2}{2}\left[L_{C_2}\alpha+\left(1-\frac{\beta}{\alpha}\right)D_{C_2}\right]+\frac{1}{Q_2}\left[D_{C_2}\alpha+E_{C_2}+W_2\mu_2\right]+$$
$$\frac{Q_3}{2}\left[L_{C_3}\alpha+\left(1-\frac{\beta}{\alpha}\right)D_{C_3}\right]+\frac{1}{Q_3}\left[D_{C_3}\alpha+E_{C_3}+W_3\mu_3\right]+\frac{Q_4}{2}\left[L_{C_4}\alpha+\left(1-\frac{\beta}{\alpha}\right)D_{C_4}\right]+\frac{1}{Q_4}\left[D_{C_4}\alpha+E_{C_4}+W_4\mu_4\right]$$

=132326.31

Conclusion:

We suggest a fuzzy inventory model to dispose of wastages that arise during manufacturing processes in this paper. The problem is described in both a strict and a hazy way. Defuzzification is accomplished using the beta distribution process. In addition, trapezoidal fuzzy numbers are used to represent ordering costs, setup costs, holding costs, waste disposal costs, and the proportion of waste generated. Numerical examples are used to verify the derived solutions. This paper may be expanded upon in the future for additional research studies.

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