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Integer Graph Labeling

S.Kavitha[1], Dr.V.L.Stella Arputha Mary[2]

[1]Research Scholar (Full Time),Register Number 19212212092007
Department of Mathematics,
St.Mary's College (Autonomous), Thoothukudi,
Affiliated to Manonmaniam Sundaranar University,
Abishekapatti, Tirunelveli-627012,Tamilnadu, India.

[2]Assistant Professor, Department of Mathematics,
St.Mary's College (Autonomous), Thoothukudi

Abstract

In this paper, we investigate the results on Integer Labeling for union of Triangular graph, double Triangular graph, Quadrilateral graph, Double Quadrilateral graph and few more graphs with a cyclic graph.

Keywords: $C_m \cup T_n, C_m \cup Q_n, C_m \cup D(T_n), C_m \cup Q_n, C_m \cup (T_n \odot K_1), C_m \cup (T_n \odot K_1)$

AMS subject classification (2010): 05C78

1 Introduction

The graph we have considered here is a finite undirected, simple graph with the vertex set $V(G)$ and edge set $E(G)$. Sources of all details in this paper are "The dynamic survey of graph Labeling by "by Gallian and Harray's "Graph Theory"and V.L. Stella Arputha Mary and N. Nandhini's "Integer root Labeling". In this paper, We investigate the results on Integer Labeling for union of Triangular graph, Double Triangular graph, Quadilateral graph, Double Quadrilateral graph and few more graph with a cycle graph.

Definition 1.1

A walk in which $v_1, v_2, \ldots, v_n$ are distinct is called a Path. A path on n vertices is denoted by P_n.

Definition 1.2

A closed path is called a cycle.

Definition 1.3 A Triangular Snake T_n is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$. That is every edge of a path is replaced by a Triangle C_3.

Definition 1.4 A Quatrilateral Snake Q_n is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i and w_i respectively and It is found joining v_i and w_i. That is every edge of a path is replaced by a cycle C_4.

Definition 1.5 A Double Triangular Snake $D(T_n)$ consists of Two Triangular Snakes that have a common path.

Definition 1.6 A Double Quadrilateral Snake $D(Q_n)$ consists of two Quadrilateral snakes that have common path.

Definition 1.7 A graph that is not connected is disconnected. A graph G is said to be disconnected if there exist two nodes in G such that no path in G has those nodes as endpoints. A graph with just one vertex is connected. An edgeless graph with two (or) more vertices is disconnected.

2 Main results

Definition 2.1

A graph $G = (V, E)$ with p vertices and q edges is called an Integer graph if there exists a function $g : V \to \{1, 2, \ldots, q+1\}$ which is possible to label all vertices $v \in V$ with distinct elements from the range such that it induces an edge Labeling

$$g^* : E \to \{1, 2, \ldots, q\}$$

defined as

$$g^*(uv) = \left\lceil \sqrt{\frac{(f(u))^2 + (f(v))^2 + f(u)f(v)}{3}} \right\rceil$$

Or

$$\left\lceil \sqrt{\frac{(f(u))^2 + (f(v))^2 + f(u)f(v)}{3}} \right\rceil$$

is distinct for all $uv \in E$.

Theorem 2.2

$P_m \cup P_n$ is an Integer graph.

Proof. Let P_m and P_n be two paths $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ respectively.

Let g be a function such that $g : V(P_m \cup P_n) \rightarrow \{1, 2, 3, \ldots, q+1\}$ defined by

$$g(u_i) = i; 1 \leq \ i \leq \ m$$

$$g(v_i) = m + i; 1 \leq \ i \leq \ n$$

We find the edges to be labeled with

$$g^*(u_i u_{i+1}) = i; 1 \leq \ i \leq \ m$$

$$g^*(v_i v_{i+1}) = m + i; 1 \leq \ i \leq \ n$$

and are all distinct. Thus $P_m \cup P_n$ is an Integer graph as it admits Integer Labeling □

Example 2.3

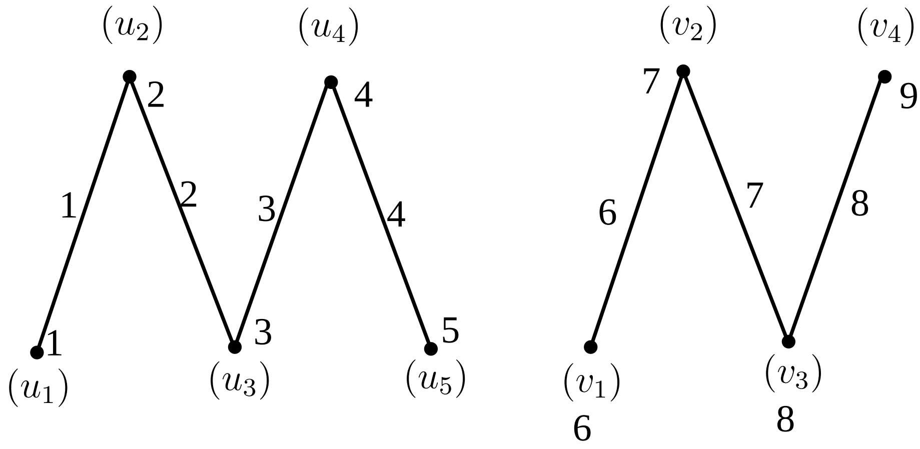

Figure 1: $P_5 \cup P_4$

Theorem 2.4 $C_m \cup P_n$ is an Integer graph for $m \geq 3$ and $n > 1$.

Proof. Let C_m be the cycle $u_1, u_2, \ldots, u_m, u_1$ and P_n be a path $v_1, v_2, \ldots, v_n$ where $m \geq 3$ and $n > 1$.

Let g be a function such that $g : V(C_m \cup P_n) \to \{1, 2, 3, \ldots, q+1\}$ be defined by

$$g(u_i) = i; 1 \leq \ i \leq \ m$$

$$g(v_i) = m + i; 1 \leq \ i \leq \ n$$

Then we find the edges to be Labeled with

$$g^*(u_i u_{i+1}) = i \ for \ i = 1, 2$$

$$g^*(u_i u_{i+1}) = i + 1 \text{ for } 1 \leq \ i \leq \ m; i = 3, 4, \ldots, m-1$$

$$g^*(u_m u_1) = 3$$

Edges of P_n are labeled by $\{m+1, m+2, \ldots, 4m+n-1\}$.

Hence $C_m \cup P_n$ is an Integer graph if $m \geq 3$ and $n > 1$ □

Example 2.5

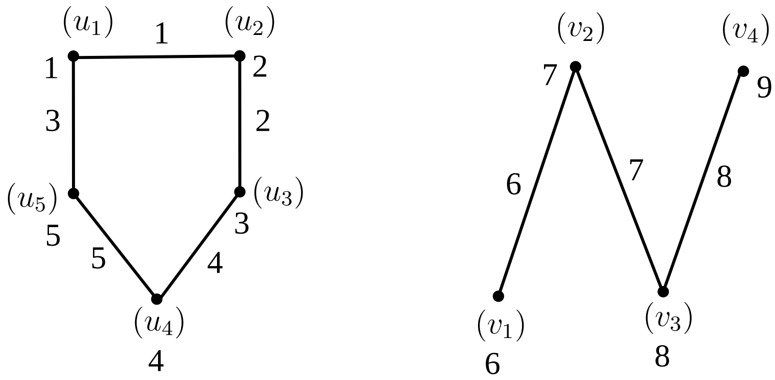

Figure 2: $C_5 \cup P_4$

Theorem 2.6 $C_m \cup (P_n \odot K_1)$ is an Integer graph for $m \geq 3$ and $n > 1$

Proof. Let C_m be the cycle $u_1, u_2, \ldots, u_m, u_1$.

Let $P_n \odot K_1$ be a graph obtained from a path $v_1, v_2, \ldots, v_n$ by joining the vertices v_i to pendent vertices w_i respectively

Define a function $g : V(C_m \cup (P_n \odot K_1)) \rightarrow \{1, 2, 3, \ldots, q+1\}$ by

$$g(u_i) = i; 1 \leq \ i \leq \ m$$

$$g(v_j) = m + 2j - 1; 1 \leq \ j \leq \ n$$

$$g(w_j) = m + 2j; 1 \leq \ j \leq \ n$$

We get distinct edge labels for C_m.

Edges of $P_n \odot K_1$ are Labeled by $\{m+1, m+2, \ldots, m+2n-1\}$.

Hence $C_m \cup (P_n \odot K_1)$ is an Integer graph as it satisfies Integer Labeling. □

Example 2.7

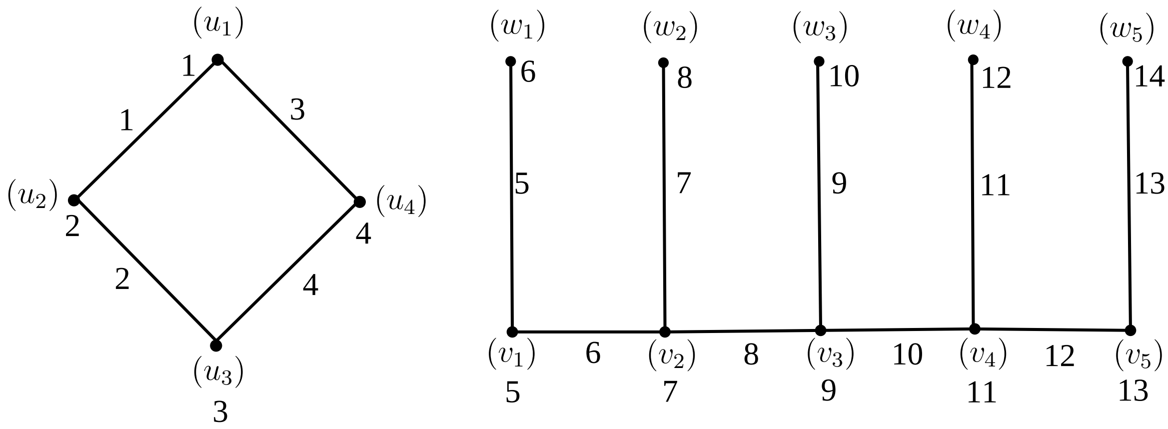

Figure 3: $C_4 \cup (P_5 \odot K_1)$

Theorem 2.8 $C_m \cup L_n$ is an Integer graph for $m \geq 3$ and $n > 1$

Proof. Let C_m be the cycle $u_1, u_2, \ldots, u_m, u_1$.

Let L_n be a ladder formed by connecting two paths $v_1, v_2, \ldots, v_n$ and $w_1, w_2, \ldots, w_n$

Define a function $g : V(C_m \cup L_n) \rightarrow \{1, 2, 3, \ldots, q+1\}$ by

$$g(u_i) = i; 1 \leq \ i \leq \ m$$

$$g(v_j) = m + (3j-2); 1 \leq \ j \leq \ n$$

$$g(w_j) = m + (3j-1); 1 \leq \ i \leq \ n$$

We get distinct edge labels for C_n and edges of L_n are labeled by $\{m+1, m+2, \ldots, m+3, n-2\}$.

Hence $C_m \cup L_n$ admits Integer labeling and it is an Integer graph if $m \geq 3$ and $n > 1$

$$g^*(w_i w_{i+1}) = m + 3i - 1; 1 \leq \ i \leq \ n-1$$

$$g^*(v_i w_{i+1}) = m + 3i; 1 \leq \ i \leq \ n-1$$

$$g^*(w_i v_i) = m + 3i - 2; 1 \leq \ i \leq \ n-1$$

We find all edge label to be distinct. Hence $C_m \cup L_n$ is an Integer graph. □

Example 2.9

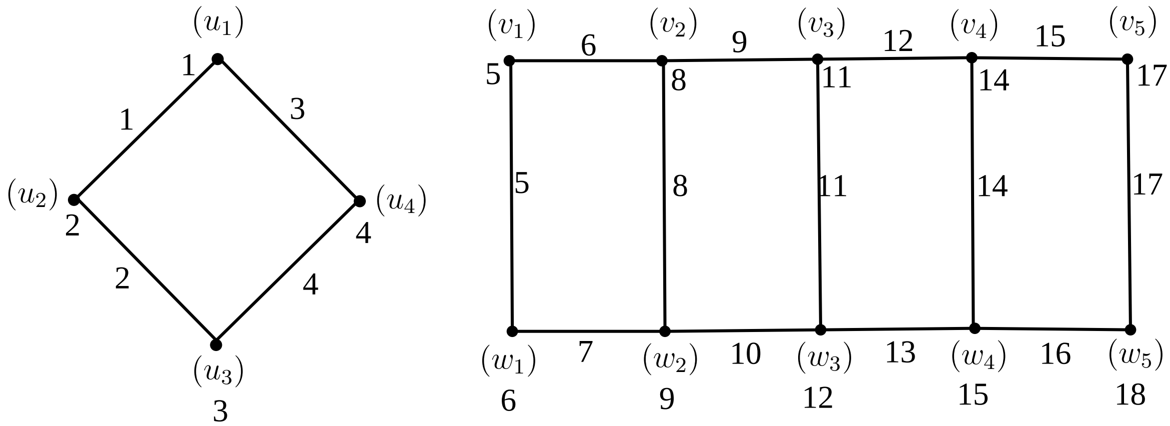

Figure 4: $C_4 \cup L_5$

Theorem 2.10

$C_m \cup T_n$ is an Integer graph

Proof.

Let $G = C_m \cup T_n$ be a graph.

Let $C_m = u_1, u_2, \ldots, u_m, u_1$ be a cycle .

Let $w_1, w_2, \ldots, w_m$ be the path P_n

Let T_n be the triangular snake obtained from the path P_n by joining w_i and $w_(i+1)$ to a new vertex $v_i; 1 \leq\ i \leq\ n-1$

A function $g : V(G) \rightarrow \{1, 2, 3, \ldots, q+1\}$ is defined by

$$g(u_i) = i; 1 \leq\ i \leq\ m$$

$$g(w_(i)) = m + 3i - 2; 1 \leq\ i \leq\ n$$

$$g(v_i) = m + 3i - 1; 1 \leq\ i \leq\ n-1$$

Edges are labeled using Integer labeling as

$$g^*(w_i w_{i+1}) = m + 3i - 1; 1 \leq\ i \leq\ n-1$$

$$g^*(v_i w_{i+1}) = m + 3i; 1 \leq\ i \leq\ n-1$$

$$g^*(w_i v_i) = m + 3i - 2; 1 \leq\ i \leq\ n-1$$

We find all edges labels to be distinct.

Hence $C_m \cup T_n$ is an Integer graph

□

Example 2.11

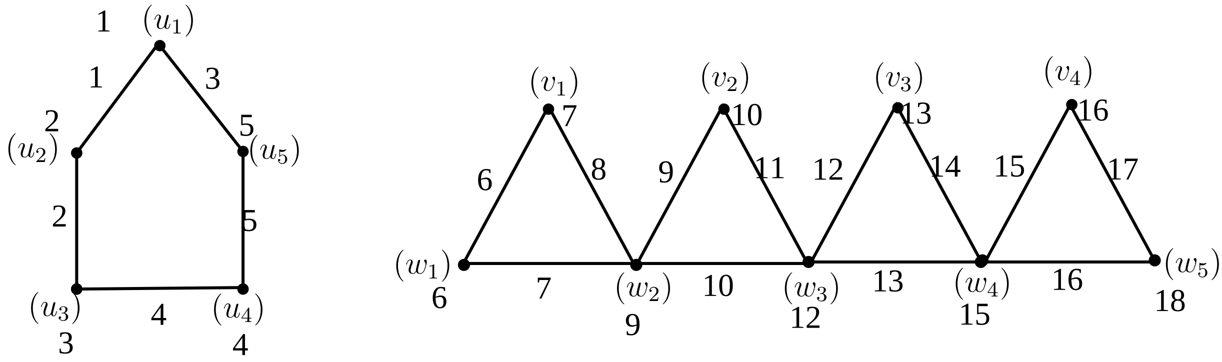

Figure 5: $C_5 \cup P_5$

Theorem 2.12 $C_m \cup Q_n$ is an Integer graph.

Proof. Let $C_m \cup Q_n$ be a graph G.

$C_m = u_1, u_2, \ldots, u_m, u_1$ be a cycle .

Let $P_n = v_1, v_2, \ldots, v_n$ be a path

A Quadrilateral snack Q_n is obtained by joining v_i and v_{i+1} to new vertices S_i and $t_i; 1 \leq\ i \leq\ n-1$ respectively and also by joining S_i andt_i.

A function $g : V(G) \to \{1, 2, 3, \ldots, q+1\}$ is defined by

$$g(u_i) = i; 1 \leq\ i \leq\ m$$

$$g(v_(i)) = m + 4i - 3; 1 \leq\ i \leq\ n$$

$$g(s_i) = m + 4i - 2; 1 \leq\ i \leq\ n-1$$

$$g(t_i) = m + 4i - 1; 1 \leq\ i \leq\ n-1$$

Using the Integer Labeling the edges are Labeled as

$$g^*(u_i u_{i+1}) = i; 1 \leq\ i \leq\ m-1$$

$$g^*(u_m u_1) = 3$$

$$g^*(v_(i)v_{i+1}) = m + 4i - 1; 1 \leq\ i \leq\ n - 1$$

$$g^*(t_i v_{i+1}) = m + 4i; 1 \leq\ i \leq\ n - 1$$

$$g^*(v_i s_i) = m + 4i - 3; 1 \leq\ i \leq\ n - 1$$

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$$g^*(s_i t_i) = m + 4i - 2; 1 \leq\ i \leq\ n - 1$$

Edge labels are found to be distinct.

Thus $C_m \cup Q_n$ is an Integer graph □

Example 2.13

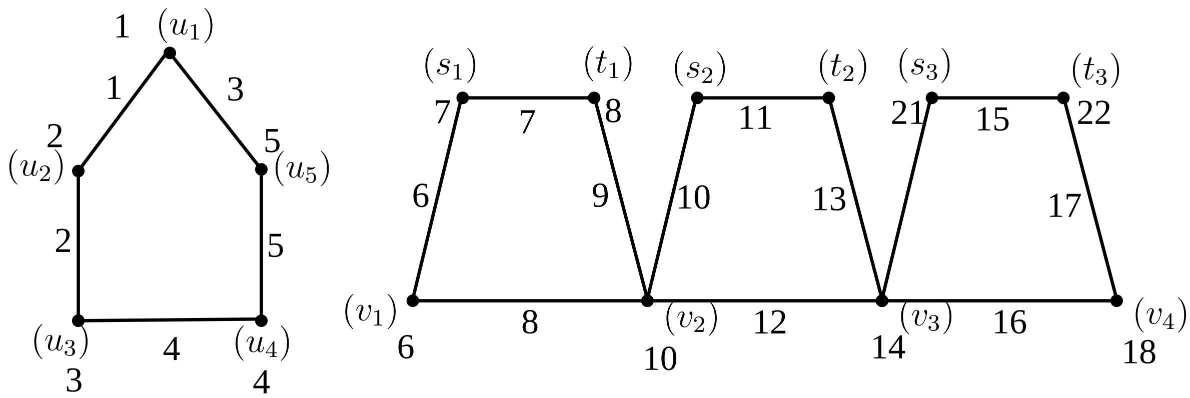

Figure 6: $C_5 \cup Q_4$

Theorem 2.14 $C_m \odot D(T_n)$ is an Integer graph.

Proof.

Let $G = C_m \odot D(T_n)$ be a graph with the cycle $u_1, u_2, \ldots, u_m, u_1$ and Double Triangular snack $D(T_n)$ is obtained from the path

$P_n = v_1, v_2, \ldots, v_n$ by joing v_i and v_{i+1} to new vertexs S_i and $t_i; 1 \leq\ i \leq\ n$

Let the function $g : V(G) \rightarrow \{1, 2, 3, \ldots, q+1\}$ be defined by

$$g(u_i) = i; 1 \leq\ i \leq\ m$$

$$g(v_i) = m + 5i - 4; 1 \leq\ i \leq\ n$$

$$g(s_i) = m + 5i - 3; 1 \leq\ i \leq\ n$$

$$g(t_i) = m + 5i - 1; 1 \leq\ i \leq\ n$$

Edges are Labeled using Integer labeling as,

$$g^*(u_i u_{i+1}) = i; 1 \leq\ i \leq\ m - 1$$

$$g^*(v_i s_i) = m + 5i - 4; 1 \leq\ i \leq\ n$$

$$g^*(u_m u_1) = 3$$

$$g^*(s_i v_{i+1}) = m + 5i - 1; 1 \leq\ i \leq\ n - 1$$

$$g^*(v_i v_{i+1}) = m + 5i - 2; 1 \leq\ i \leq\ n - 1$$

$$g^*(v_i s_i) = m + 5i - 4; 1 \leq\ i \leq\ n - 1$$

$$g^*(v_i t_i) = m + 5i - 3; 1 \leq\ i \leq\ n - 1$$

$$g^*(v_{i+1} t_i) = m + 5i; 1 \leq\ i \leq\ n - 1$$

Edge label are all distinct.

Hence $C_m \odot D(T_n)$ is an Integer graph. □

Example 2.15

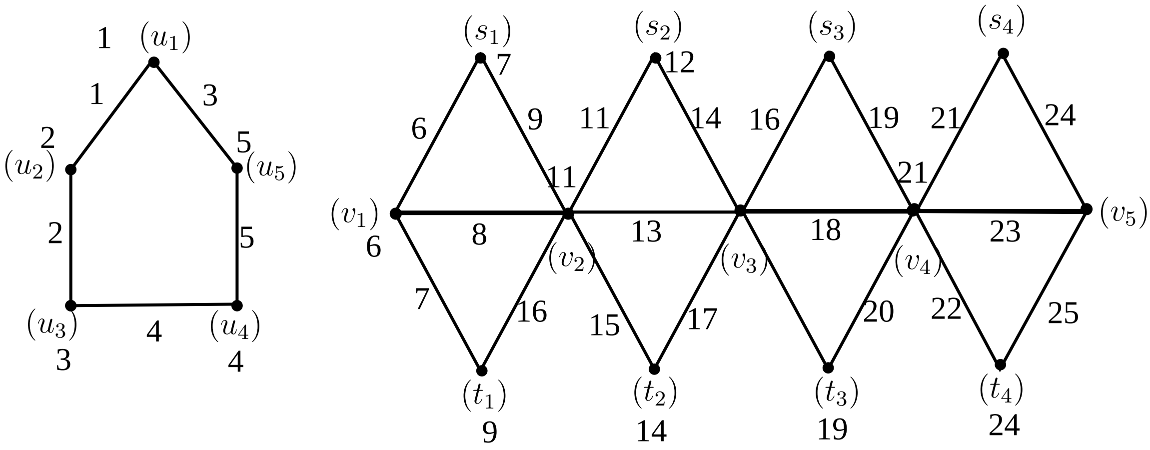

Figure 7: $C_5 \odot D(T_4)$

Theorem 2.16 $C_m \odot D(Q_n)$ is an Integer graph

Proof. Let $G = C_m \odot D(Q_n)$ be a graph where C_m is the cycle $u_1, u_2, \ldots, u_m, u_1$ and Let v_i, w_i, r_i, s_i, t_i be the vertices of $D(Q_n)$ Let $g : V(G) \to \{1, 2, 3, \ldots, q+1\}$ be defined by

$$g(u_i) = i; 1 \leq \ i \leq \ m$$

$$g(v_i) = m + 7i - 6; 1 \leq \ i \leq \ n$$

$$g(w_i) = m + 7i - 5; 1 \leq \ i \leq \ n-1$$

$$g(r_i) = m + 7i - 2; 1 \leq \ i \leq \ n-1$$

$$g(s_i) = m + 7i - 4; 1 \leq \ i \leq \ n-1$$

$$g(t_i) = m + 7i; 1 \leq \ i \leq \ n-1$$

Now the edge labels are given by

$$g^*(u_i u_{i+1}) = i; 1 \leq \ i \leq \ m-1$$

$$g^*(u_m u_1) = 3$$

$$g^*(s_i v_{i+1}) = m + 7i - 2; 1 \leq \ i \leq \ n - 1$$

$$g^*(v_{i+1} v_i) = m + 7i - 3; 1 \leq \ i \leq \ n - 1$$

$$g^*(w_i v_i) = m + 7i - 6; 1 \leq \ i \leq \ n - 1$$

$$g^*(v_i r_i) = m + 7i; 1 \leq \ i \leq \ n - 1$$

$$g^*(v_{i+1} t_i) = m + 7i; 1 \leq \ i \leq \ n - 1$$

$$g^*(w_i s_i) = m + 7i - 5; 1 \leq \ i \leq \ n - 1$$

$$g^*(t_i x_i) = m + 7i - 1; 1 \leq \ i \leq \ n - 1$$

Edge labels are all distinct.

Hence $C_m \odot D(Q_n)$ is an Integer graph □

Example 2.17

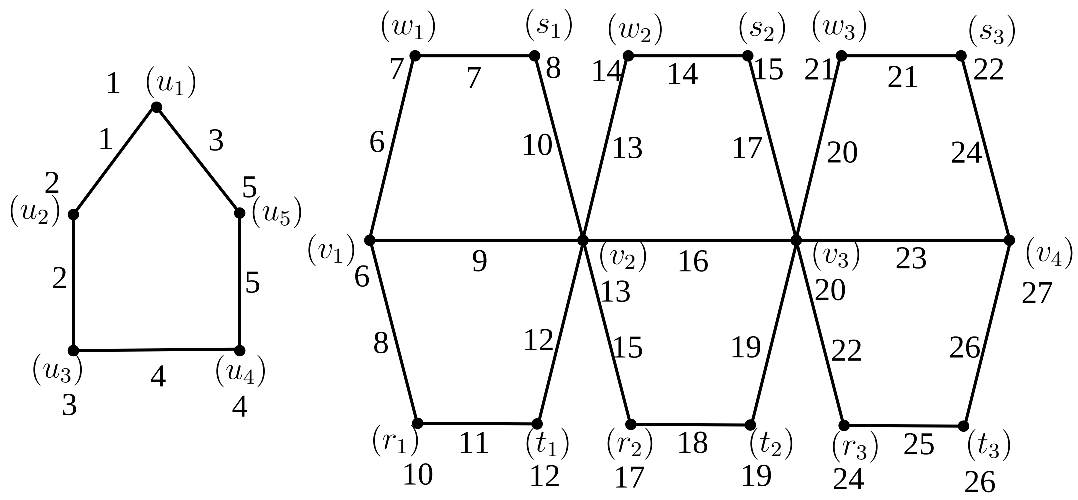

Figure 8: $C_5 \odot D(Q_4)$

Theorem 2.18 $C_m \cup (T_n \odot K_1)$ is an Integer graph

Proof. Let $C_m \cup (T_n \odot K_1)$ be a graph G, where C_m is the Cycle $u_1, u_2, \ldots, u_m, u_1$.

Let $v_1, v_2, \ldots, v_n$ be a path. Let $r_i, 1 \leq i \leq n-1$ be the new vertex joining v_i and v_{i+1}.

Let S_i be the vertex joined to $v_i; 1 \leq i \leq n$

Let t_i be the vertex joined to $v_i; 1 \leq i \leq n-1$

Define a function $g : V(G) \rightarrow \{1, 2, 3, \ldots, q+1\}$ by

$$g(v_i) = i; 1 \leq i \leq m$$

$$g(v_i) = m + 5i - 3; 1 \leq i \leq n$$

$$g(r_i) = m + 5i - 2; 1 \leq i \leq n-1$$

$$g(s_i) = m + 5i - 4; 1 \leq i \leq n$$

$$f(t_i) = m + 5i - 1; 1 \leq i \leq n-1$$

Now the edge labels are

$$g^*(u_i u_{i+1}) = i; 1 \leq i \leq m-1$$

$$g^*(u_m u_1) = 3$$

$$g^*(v_i v_{i+1}) = m + 5i - 1; 1 \leq i \leq n-1$$

$$g^*(v_i v_{i+1}) = m + 5i - 1; 1 \leq i \leq n-1$$

$$g^*(v_i s_i) = m + 5i - 4; 1 \leq \; i \leq \; n$$

$$g^*(v_i r_i) = m + 5i - 3; 1 \leq \; i \leq \; n - 1$$

$$g^*(r_i t_i) = m + 5i - 2; 1 \leq \; i \leq \; n - 1$$

Each edge label distinct. Hence $C_m \cup (T_n \odot K_1)$ is an Integer graph . □

Example 2.19

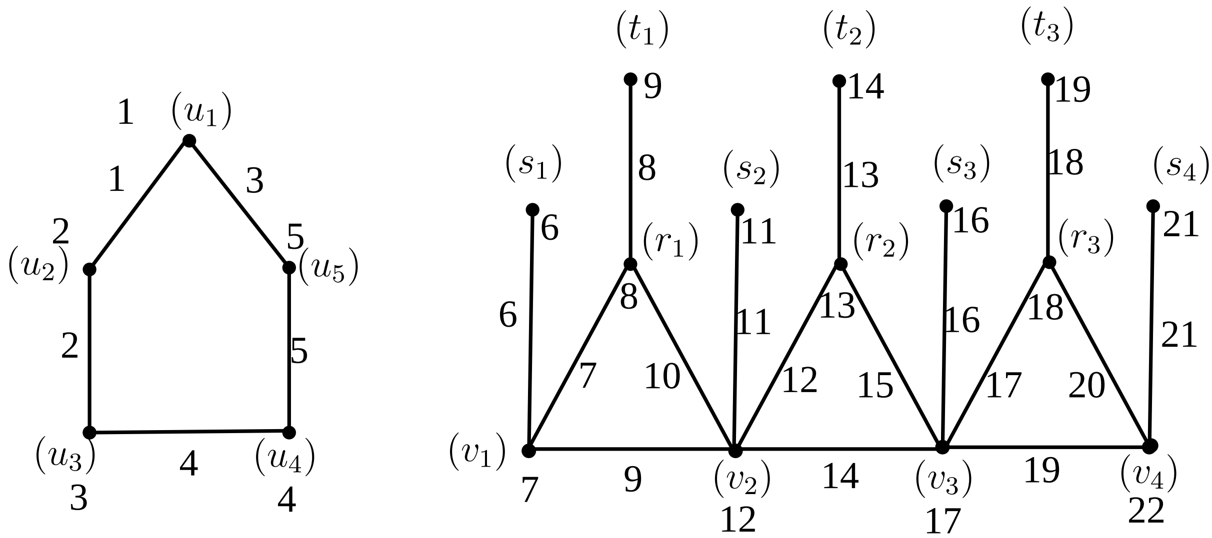

Figure 9: $C_5 \cup (T_4 \odot K_1)$

Theorem 2.20 $C_m \cup (Q_n \odot K_1)$ is an Integer graph .

Proof. Let $G = C_m \cup (Q_n \odot K_1)$

Let C_m be the cycle $u_1, u_2, \ldots, u_m, u_1$

Let $v_1, v_2, \ldots, v_n$ be a path .

Let w_i and s_i be two vertices joined to v_i and v_{i+1} respectively

Let x_i be the vertex joined to $v_i; 1 \leq i \leq n$

Let y_i be the new vertex joined to $w_i; 1 \leq i \leq n-1$

Let t_i be the new vertex joined to $s_i; 1 \leq i \leq n-1$

Define a function $g^* : V(G) \rightarrow \{1, 2, 3, \ldots, q+1\}$ by

$$g(u_i) = i; 1 \leq i \leq m$$

$$g(v_i) = m + 7i - 6; 1 \leq i \leq n$$

$$g(x_i) = m + 7i - 5; 1 \leq i \leq n$$

$$g(y_i) = m + 7i - 3; 1 \leq i \leq n-1$$

$$g(v_i) = m + 7i - 6; 1 \leq i \leq n$$

$$g(w_i) = m + 7i - 4; 1 \leq i \leq n-1$$

$$g(t_i) = m + 7i - 1; 1 \leq i \leq n-1$$

$$g(s_i) = m + 7i; 1 \leq i \leq n-1$$

Now the edge labels are given by

$$g^*(u_i u_{i+1}) = i; 1 \leq i \leq n-1$$

$$g^*(u_m u_i) = 3$$

$$g^*(w_i v_i) = m + 7i - 5; 1 \leq i \leq n-1$$

$$g^*(v_i v_{i+1}) = m + 7i - 3; 1 \leq i \leq n-1$$

$$g^*(v_{i+1} s_i) = m + 7i; 1 \leq i \leq n-1$$

$$g^*(w_i s_i) = m + 7i - 2; 1 \leq i \leq n-1$$

$$g^*(v_i x_i) = m + 7i - 6; 1 \leq i \leq n-1$$

$$g^*(w_i y_i) = m + 7i - 4; 1 \leq i \leq n-1$$

$$g^*(s_i t_i) = m + 7i - 1; 1 \leq i \leq n-1$$

All edges are Labeled distinctly. Hence $C_m \cup (Q_n \odot K_1)$ is an Integer graph. □

Example 2.21

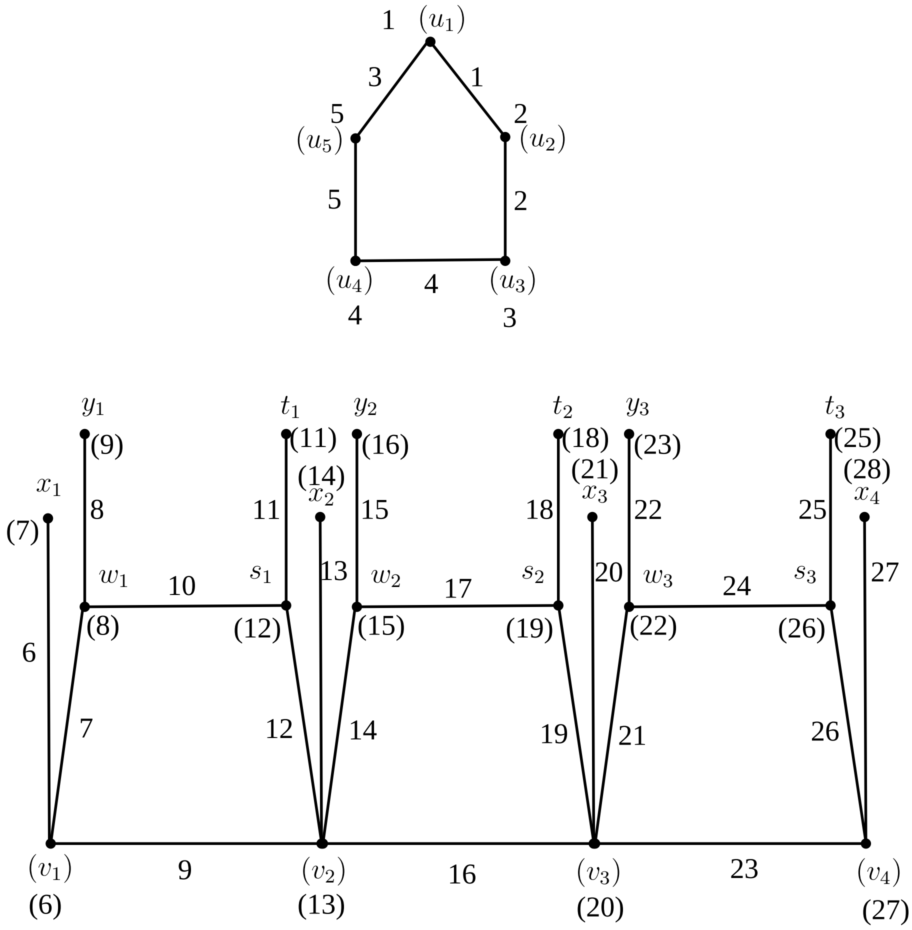

Figure 10: $C_5 \cup (Q_4 \odot K_1)$

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