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# *OVERVIEW OF STAR-LAPLACE TRANSFORM $S$-STEP AND ITS INVERSE*

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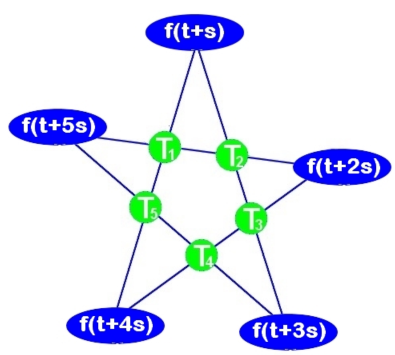
Abstract. This paper presents abrief overview of Star-Laplace Transform $s$-step. The definition of Star-Laplace Transform $s$-step and most of its important properties have been mentioned with detailed proofs. It also includes a brief overview of Inverse Star-Laplace Transform $s$-step. The Star-Laplace Transform $s$-step can be interpreted as a transformation from the time domain where inputs and outputs are functions of time to the frequency domain.

## 1. INTRODUCTION

This paper deals with a brief overview of what Star-Laplace Transform $s$-step is and its application. The Star-Laplace Transform $s$-step is a specific type of integral transform. Considering a function $f(t)$, its corresponding Star-Laplace Transform $s$-step will be denoted as $\mathcal{L}_s^\star[f(t)]$, where $\mathcal{L}_s^\star$ is the operator operated on the time domain function $f(t)$. When we solve a linear Star-System with $\alpha$ Coefficient in five unknowns $\star[f(t+s), f(t+2s), f(t+3s), f(t+4s), f(t+5s); \alpha] = \alpha$ (See [4], [5]),




(Fig 1)

In addition to having the sum $\alpha_s^\star$ in each line.

*Key words and phrases.* Star Laplace Transform s-step, Laplace Transform, Star-Coefficient, Star-System, Equations Matrix,Heaviside's.

We will place the 5 Star-Vectors that are the solution (linearly independent) in the columns of a matrix, So what we've done is create the Star-Coefficient $\alpha_s^\star$. Directly from the coefficient $\alpha_s^\star$, the Star-Laplace transform $s$-step or Generalized Laplace operator of fairly regular function $f$ are defined : $\mathbb{R}_+ \to \mathbb{C}$ by

$$\mathcal{L}_s^\star f(\omega) = \int_0^{+\infty} (\alpha_s^\star f)(t) e^{-\omega t} dt. \tag{1.1}$$

The Star-Laplace transform $s$-step of a function results in a new function of complex frequency $\omega$. It is also predominantly used in the analysis of transient events in the electrical circuits where frequency domain analysis is used.
The contents of the paper is as follows :
In the second section, we present some preliminary results and notations that will be useful in the sequel.
The section 3 we defined the Star Laplace Transform $s$-step (SLT) and certain of its Applications.
The section 4 we investigate further the properties and Theorems of the Star-Laplace Transform $s$-step.
The section 5 we introduce the inverse of the Star Laplace Transform $s$-step and a few special functions like the Heaviside step function were also discussed in detail.

## 2. Some basic definitions and notations

### 2.1. **Laplace Transform.** Some of the very important properties of Laplace transforms on are described as follows: [2]

**Definition 1.** *Let $f(t)$; $t \geq 0$ be a given function. We call*

$$F(x) = \int_0^{+\infty} f(t) e^{-tx} dt \tag{2.1}$$

*the Laplace transform of $f(t)$ and write*

$$F(t) = \mathcal{L}(f)(t), \quad f(t) = \mathcal{L}^{-1}(F)(t).$$

One can prove that the Laplace transform $\mathcal{L}$ is injective, that is the reason why $\mathcal{L}^{-1}$ is well defined (for a precise formula of $\mathcal{L}^{-1}$, see page 10 in [1]).
To compute Laplace transforms, we need:

$$d(fg) = fdg + gdf, \quad \int_a^b df = f(b) - f(a), \tag{2.2}$$

where $df := f'(t)dt$.

Laplace transform is linear: By linearity, we mean for all real numbers $a$, $b$,

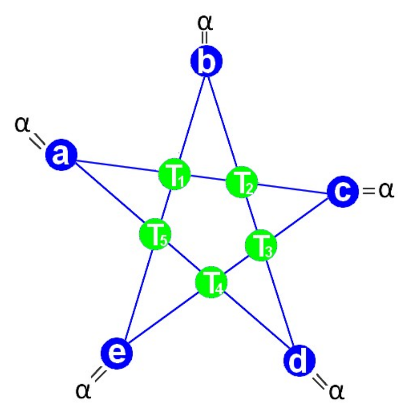
$$\mathcal{L}(af + bg) = a\mathcal{L}(f) + b\mathcal{L}(g)$$

Following pages, we introduce some notations and star-system with $\alpha$ coefficient defined in [4], [5] and [3].

### 2.2. A star-system with $\alpha$ coefficient:

**Definition 2.** *Let $a$, $b$, $c$, $d$, $e$ and $\alpha$ be real numbers, and let $T_1$, $T_2$, $T_3$, $T_4$, $T_5$ be unknowns (also called variables or indeterminates). Then a system of the form*

$$\begin{cases} T_1 + T_2 = \alpha - a - c \\ T_2 + T_3 = \alpha - b - d \\ T_3 + T_4 = \alpha - c - e \\ T_4 + T_5 = \alpha - a - d \\ T_1 + T_5 = \alpha - b - e \end{cases}$$




(Fig 2)

is called a star-system with coefficient $\alpha$ in five unknowns.

We have also noted $\star[a, b, c, d, e; \alpha] = \alpha$.

The scalars $a$, $b$, $c$, $d$, $e$ are called the coefficients of the unknowns, and $\alpha$ is called the constant "$Chaff$" of the star-system in five unknowns. A vector $(T_1, T_2, T_3, T_4, T_5)$ in $R^5$ is called a star-solution vector of this star-system if and only if $\star[a, b, c, d, e; \alpha] = \alpha$.
The solution of a Star-system is the set of values for $T_1$ , $T_2$ , $T_3$ , $T_4$ and $T_5$ that satisfies five equations simultaneously.

2.3. **Star-Coefficient:** The star-Coefficient [4] is also noted by $\alpha_\star$ and is a solution of equation

$$\alpha = T_1(\alpha) + T_2(\alpha) + T_3(\alpha) + T_4(\alpha) + T_5(\alpha),$$

where $(T_1\ ,\ T_2\ ,\ T_3\ ,\ T_4\ ,\ T_5)$ is solution of a star-system:

$$\star[a, b, c, d, e; \alpha] = \alpha$$

On the other hand, the star-system with coefficient $\alpha$ [4] can be written in matrix form:

$$T^\star = M^\star F^\star \text{ Where } M^\star = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} \end{pmatrix},$$

vector $T^\star = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{pmatrix}$ and $F^\star = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix}$.

$M^\star$ called the star-Matrix of the star-system with $\alpha$ coefficient $\star[a, b, c, d, e; \alpha] = \alpha$

## 3. Definition of Star-Laplace Transform s-step

Consider the following star-system (Fig 1):

$$\star[f(t+s), f(t+2s), f(t+3s), f(t+4s), f(t+5s); \alpha] = \alpha,$$

of five equations in five unknowns:

$$\begin{cases} T_1 + T_2 = \alpha - f(t+5s) - f(t+2s) \\ T_2 + T_3 = \alpha - f(t+s) - f(t+3s) \\ T_3 + T_4 = \alpha - f(t+2s) - f(t+4s) \\ T_4 + T_5 = \alpha - f(t+3s) - f(t+5s) \\ T_5 + T_1 = \alpha - f(t+4s) - f(t+s)) \end{cases}$$

In a particular case if $\alpha = T_2(\alpha) + T_2(\alpha) + T_3(\alpha) + T_4(\alpha) + T_5(\alpha)$, we obtain

$$T^\star = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{4}{3} & \frac{-2}{3} & \frac{1}{3} \end{pmatrix} . \begin{pmatrix} f(t+s) \\ f(t+2s) \\ f(t+3s) \\ f(t+4s) \\ f(t+5s) \end{pmatrix}$$

In this case

$$\alpha_s^\star f(t) = \frac{2}{3}\left(f(t+s) + f(t+2s) + f(t+3s) + f(t+4s) + f(t+5s)\right)$$

Behold, the Star-Laplace Transform s-step is born!
To productively use the Star-Laplace Transform, we need to be able to transform functions from the time domain to the Star-Laplace domain.

**Definition 3.** *Let $f(t)$, $t > 0$ be a given function. The Star-Laplace transform s-step (SLT) of f(t) and write*

$$\forall\ s \geq 0, \mathcal{L}_s^\star\left[f(t)\right](\omega) = \int_0^{+\infty} (\alpha_s^\star f)(t) e^{-\omega t} dt. \tag{3.1}$$

## 4. Properties and Theorems of Star-Laplace Transform $s$-step

4.1. **Linearity Property.** If $a$ and $b$ are constants, then $\forall\ s \geq 0$,

$$\mathcal{L}_s^\star\left[af(t) + bg(t)\right](\omega) = a\mathcal{L}_s^\star\left[f(t)\right](\omega) + b\mathcal{L}_s^\star\left[g(t)\right](\omega). \tag{4.1}$$

Proof.
Use linearity property of integral

4.2. **Change of Scale Property.** A linear multiplication or division of $a$ constant with the variable is known as scaling.
By change of scale property: For all $s \geq 0$ and $a > 0$, we have

$$\mathcal{L}_s^\star\left[f(at)\right](\omega) = \frac{1}{a}\mathcal{L}_{as}^\star\left[f(t)\right](\frac{\omega}{a}). \tag{4.2}$$

Proof.
To find the Star-Laplace Transform $s$-step of $f(at)$, we apply the definition: For all $s \geq 0$ and $a > 0$, we have

$$
\begin{aligned}
\mathcal{L}_s^\star\left[f(at)\right](\omega) &= \int_0^{+\infty} (\alpha_s^\star f)(at)e^{-\omega t}dt \\
&= \frac{2}{3}\int_0^{+\infty}\left(f(a(t+s))\right)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty}\left(f(a(t+2s))\right)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty}\left(f(a(t+3s))\right)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty}\left(f(a(t+4s))\right)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty}\left(f(a(t+5s))\right)e^{-\omega t}dt \\
&= \frac{2}{3}\int_s^{+\infty} f(at)e^{-\omega(t-s)}dt + \frac{2}{3}\int_{2s}^{+\infty} f(at)e^{-\omega(t-2s)}dt \\
&+ \frac{2}{3}\int_{3s}^{+\infty} f(at)e^{-\omega(t-3s)}dt + \frac{2}{3}\int_{4s}^{+\infty} f(at)e^{-\omega(t-4s)}dt \\
&+ \frac{2}{3}\int_{5s}^{+\infty} f(at)e^{-\omega(t-5s)}dt \\
&= \frac{2}{3a}\int_{as}^{+\infty} f(t)e^{-\frac{\omega}{a}(t-as)}dt + \frac{2}{3a}\int_{2as}^{+\infty} f(t)e^{-\frac{\omega}{a}(t-2as)}dt \\
&+ \frac{2}{3a}\int_{3as}^{+\infty} f(t)e^{-\frac{\omega}{a}(t-3as)}dt + \frac{2}{3a}\int_{4as}^{+\infty} f(t)e^{-\frac{\omega}{a}(t-4as)}dt \\
&+ \frac{2}{3a}\int_{5as}^{+\infty} f(t)e^{-\frac{\omega}{a}(t-5as)}dt \\
&= \frac{1}{a}\mathcal{L}_{as}^\star\left[f(t)\right](\frac{\omega}{a}).
\end{aligned}
$$

4.3. **Shifting Theorem.** The Shifting Theorem of Star-Laplace Transform $s$-step states that if $s \geq 0$ and $a \in \mathbb{R}$, then

$$\mathcal{L}_s^\star\left[e^{-at}f(t)\right](\omega) = \mathcal{L}_s^\star\left[f(t)\right](\omega + a). \tag{4.3}$$

Proof.
By definition, for all $s \geq 0$ and $a \in \mathbb{R}$, we have

$$\begin{aligned}
\mathcal{L}_s^\star\left[e^{at}f(t)\right](\omega) &= \frac{2}{3}\int_0^{+\infty}\left(e^{-at}f(t+s)\right)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty}\left(e^{-at}f(t+2s)\right)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty}\left(e^{-at}f(t+3s)\right)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty}\left(e^{-at}f(t+4s)\right)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty}\left(e^{-at}f(t+5s)\right)e^{-\omega t}dt \\
&= \frac{2}{3}\int_0^{+\infty} f(t+s))e^{-(\omega+a)t}dt + \frac{2}{3}\int_0^{+\infty} f(t+2s)e^{-(\omega+a)t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} f(t+3s)e^{-(\omega+a)t}dt + \frac{2}{3}\int_0^{+\infty} f(t+4s))e^{-(\omega+a)t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} f(t+5s)e^{-(\omega+a)t}dt \\
&= \mathcal{L}_s^\star\left[f(t)\right](\omega+a).
\end{aligned}$$

4.4. **Multiplication of powers of the variable.** The variable that has been used so far is "t". Thus, if we multiply powers of t with the original function f (t), the Star-Laplace Transform $s$-step can be expressed as, $\forall\ s \geq 0$

$$\mathcal{L}_s^\star\left[t^n f(t)\right](\omega) = (-1)^n \frac{d^n}{d\omega^n}\mathcal{L}_s^\star\left[f(t)\right](\omega). \tag{4.4}$$

Proof.
This result can be proved by the use of Mathematical Induction.
Step 1 To prove that the result is true when $n = 1$. Let

$$\begin{aligned}
\mathcal{L}_s^\star\left[f(t)\right](\omega) &= \int_0^{+\infty}(\alpha_s^\star f)(t)e^{-\omega t}dt \\
&= \frac{2}{3}\int_0^{+\infty} f(t+s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} f(t+2s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} f(t+3s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} f(t+4s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} f(t+5s)e^{-\omega t}dt
\end{aligned}$$

Differentiating with respect to $\omega$, $s \geq 0$ and applying the rule of differentiation under the integral sign,

$$\begin{aligned}
\frac{d}{d\omega}\mathcal{L}_s^\star\left[f(t)\right](\omega) &= \frac{2}{3}\int_0^{+\infty} f(t+s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt + \frac{2}{3}\int_0^{+\infty} f(t+2s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt \\
&+ \frac{2}{3}\int_0^{+\infty} f(t+3s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt + \frac{2}{3}\int_0^{+\infty} f(t+4s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt \\
&+ \frac{2}{3}\int_0^{+\infty} f(t+5s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt \\
&= -\mathcal{L}_s^\star\left[tf(t)\right](\omega)
\end{aligned}$$

We obtain

$$\forall\ s \geq 0, \mathcal{L}_s^\star\left[tf(t)\right](\omega) = (-1)\frac{d}{d\omega}\mathcal{L}_s^\star\left[f(t)\right](\omega) \tag{4.5}$$

Which proves the result for $n = 1$.

Step 2 Since the result holds true for n=1, it can be assumed that the result is true when n is any natural number "p". We have

$$\forall\ s \geq 0, \mathcal{L}_s^\star\left[t^p f(t)\right](\omega) = (-1)^p\frac{d^p}{d\omega^p}\mathcal{L}_s^\star\left[f(t)\right](\omega) \tag{4.6}$$

Step 3 To prove that the result holds true when n=p+1. From Step 2

$$(-1)^p\frac{d^p}{d\omega^p}\mathcal{L}_s^\star\left[f(t)\right](\omega) = \mathcal{L}_s^\star\left[t^p f(t)\right](\omega) \tag{4.7}$$

Differentiating with respect to $\omega$, $s \geq 0$ and applying the rule of differentiation under the integral sign,

$$\begin{aligned}
(-1)^p\frac{d^{p+1}}{d\omega^{p+1}}\mathcal{L}_s^\star\left[f(t)\right](\omega) &= \frac{2}{3}\int_0^{+\infty} t^p f(t+s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt + \frac{2}{3}\int_0^{+\infty} t^p f(t+2s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt \\
&+ \frac{2}{3}\int_0^{+\infty} t^p f(t+3s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt + \frac{2}{3}\int_0^{+\infty} t^p f(t+4s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt \\
&+ \frac{2}{3}\int_0^{+\infty} t^p f(t+5s)\frac{d}{d\omega}\left(e^{-\omega t}\right)dt \\
&= -\mathcal{L}_s^\star\left[t^{p+1}f(t)\right](\omega)
\end{aligned}$$

We obtain

$$\forall\ s \geq 0, \mathcal{L}_s^\star\left[t^{p+1}f(t)\right](\omega) = (-1)^{p+1}\frac{d^p}{d\omega^p}\mathcal{L}_s^\star\left[f(t)\right](\omega). \tag{4.8}$$

Which proves the result for $n = p + 1$.

Thus, by the rule of Mathematical Induction, it can be said that the result is true for any value of $n$.

## 5. Star-Laplace Transform $s$-step of Derivatives

Let $f(t)$ be the time domain function. The Star-Laplace Transform $s$-step of its derivative can be expressed as
(5.1)
$\mathcal{L}_s^\star\left[f'(t)\right](\omega) = \omega\mathcal{L}_s^\star\left[f(t)\right](\omega) - (f(s) + f(2s) + f(3s) + f(4s) + f(5s))$

Proof.
By definition, $\mathcal{L}_s^\star\left[f'(t)\right](\omega) = \int_0^{+\infty}(\alpha_s^\star f')(t)e^{-\omega t}dt$
Integrating by parts,

$$\begin{aligned}
\mathcal{L}_s^\star\left[f'(t)\right](\omega) &= \int_0^{+\infty}(\alpha_s^\star f')(t)e^{-\omega t}dt \\
&= \frac{2}{3}\int_0^{+\infty} f'(t+s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} f'(t+2s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} f'(t+3s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} f'(t+4s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} f'(t+5s)e^{-\omega t}dt \\
&= \frac{2\omega}{3}\int_0^{+\infty} f(t+s)e^{-\omega t}dt - f(s) + \frac{2\omega}{3}\int_0^{+\infty} f(t+2s)e^{-\omega t}dt - f(2s) \\
&+ \frac{2\omega}{3}\int_0^{+\infty} f(t+3s)e^{-\omega t}dt - f(3s) + \frac{2\omega}{3}\int_0^{+\infty} f(t+4s)e^{-\omega t}dt - f(s) \\
&+ \frac{2\omega}{3}\int_0^{+\infty} f(t+5s)e^{-\omega t}dt - f(s) \\
&= \omega\mathcal{L}_s^\star\left[f(t)\right](\omega) - (f(s) + f(2s) + f(3s) + f(4s) + f(5s))
\end{aligned}$$

Differentiating equation (5.1) again with respect to variable "t",

$$\begin{aligned}
\mathcal{L}_s^\star\left[f"(t)\right](\omega) &= \omega\mathcal{L}_s^\star\left[f'(t)\right](\omega) - (f'(s) + f'(2s) + f'(3s) + f'(4s) + f'(5s)) \\
&= \omega^2\mathcal{L}_s^\star\left[f(t)\right](\omega) - (\omega f(s) + f'(s)) - (\omega f(2s) + f'(2s)) - (\omega f(3s) + f'(3s)) \\
&- (\omega f(4s) + f'(4s)) - (\omega f(5s) + f'(5s))
\end{aligned}$$

Thus, in general, the $n^{th}$ derivative can be expressed as,

$$\begin{aligned}
\mathcal{L}_s^\star\left[f^n(t)\right](\omega) &= \omega^n\mathcal{L}_s^\star\left[f(t)\right](\omega) - \omega^{n-1}f(s) - \omega^{n-2}f'(s) - \ldots - \omega^2 f^{n-3}(s) - \omega f^{n-2}(s) \\
&- f^{n-1}(s) - \omega^{n-1}f(2s) - \omega^{n-2}f'(2s) - \ldots - \omega^2 f^{n-3}(2s) - \omega f^{n-2}(2s) \\
&- f^{n-1}(2s) - \omega^{n-1}f(3s) - \omega^{n-2}f'(3s) - \ldots - \omega^2 f^{n-3}(3s) - \omega f^{n-2}(3s) \\
&- f^{n-1}(3s) - \omega^{n-1}f(4s) - \omega^{n-2}f'(4s) - \ldots - \omega^2 f^{n-3}(4s) - \omega f^{n-2}(4s) \\
&- f^{n-1}(4s) - \omega^{n-1}f(5s) - \omega^{n-2}f'(5s) - \ldots - \omega^2 f^{n-3}(5s) - \omega f^{n-2}(5s) \\
&- f^{n-1}(5s).
\end{aligned}$$

The above mentioned results are put to incredible use in solving Differential Equations.

## 6. Star-Laplace Transform $s$-step of Integrals

When the time domain function is integrated, its Star-Laplace Transform $s$-step can be expressed as,

$$\mathcal{L}_s^\star\left[\int_0^t f(\xi)d\xi\right](\omega) = \frac{1}{\omega}\mathcal{L}_s^\star\left[f(t)\right](\omega). \tag{6.1}$$

Proof.
By definition,

$$\mathcal{L}_s^\star\left[\int_0^t f(\xi)d\xi\right](\omega) = \int_0^{+\infty} \alpha_s^\star\left(\int_0^t f(\xi)d\xi\right)e^{-\omega t}dt.$$

Integrating by parts,

$$\begin{aligned}
\mathcal{L}_s^\star\left[\int_0^t f(\xi)d\xi\right](\omega) &= \int_0^{+\infty} \alpha_s^\star\left(\int_0^t f(\xi)d\xi\right)e^{-\omega t}dt \\
&= \frac{2}{3}\int_0^{+\infty}\left(\int_0^t f(\xi+s)d\xi\right)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty}\left(\int_0^t f(\xi+2s)d\xi\right)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty}\left(\int_0^t f(\xi+3s)d\xi\right)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty}\left(\int_0^t f(\xi+4s)d\xi\right)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty}\left(\int_0^t f(\xi+5s)d\xi\right)e^{-\omega t}dt \\
&= \frac{2}{3}\left[\left(-\frac{e^{-\omega t}}{\omega}\right)\int_0^t f(\xi+s)d\xi\right]_0^{+\infty} + \frac{2}{3\omega}\int_0^{+\infty} f(t+s)e^{-\omega t}dt \\
&+ \frac{2}{3}\left[\left(-\frac{e^{-\omega t}}{\omega}\right)\int_0^t f(\xi+2s)d\xi\right]_0^{+\infty} + \frac{2}{3\omega}\int_0^{+\infty} f(t+2s)e^{-\omega t}dt \\
&+ \frac{2}{3}\left[\left(-\frac{e^{-\omega t}}{\omega}\right)\int_0^t f(\xi+3s)d\xi\right]_0^{+\infty} + \frac{2}{3\omega}\int_0^{+\infty} f(t+3s)e^{-\omega t}dt \\
&+ \frac{2}{3}\left[\left(-\frac{e^{-\omega t}}{\omega}\right)\int_0^t f(\xi+4s)d\xi\right]_0^{+\infty} + \frac{2}{3\omega}\int_0^{+\infty} f(t+4s)e^{-\omega t}dt \\
&+ \frac{2}{3}\left[\left(-\frac{e^{-\omega t}}{\omega}\right)\int_0^t f(\xi+5s)d\xi\right]_0^{+\infty} + \frac{2}{3\omega}\int_0^{+\infty} f(t+5s)e^{-\omega t}dt \\
&= \frac{1}{\omega}\mathcal{L}_s^\star\left[f(t)\right](\omega)
\end{aligned}$$

The above mentioned result can be generalized as,

$$\mathcal{L}_s^{\star}\left[\int_0^t\int_0^t...\int_0^t f(\xi)(d\xi)^n\right](\omega)=\frac{1}{\omega^n}\mathcal{L}_s^{\star}\left[f(t)\right](\omega). \tag{6.2}$$

## 7. Inverse Star-Laplace Transform $s$-step

**Definition 4.** *If $\mathcal{L}_s^{\star}[f(t)](\omega)=\int_0^{+\infty}\alpha_s^{\star}(f(t))\,e^{-\omega t}dt$, then $\alpha_s^{\star}(f)(t)$ is called the Inverse Star-Laplace Transform s-step of $\mathcal{L}_s^{\star}[f(t)]$. It can be denoted as,*

$$(\mathcal{L}_s^{\star})^{-1}\left[\int_0^{+\infty}\alpha_s^{\star}(f(t))\,e^{-\omega t}dt\right]=\alpha_s^{\star}(f)(t) \tag{7.1}$$

*If $s=0$ then $f(t)=\frac{3}{10}\alpha_0^{\star}(f)(t)$*

Thus, the frequency domain function $\omega\rightarrow\int_0^{+\infty}\alpha_s^{\star}(f)(t)e^{-\omega t}dt$ can be converted to its corresponding time domain equivalent $\alpha_s^{\star}(f)(t)$ using the Star-Laplace Transform $s$-step Inverse operator $(\mathcal{L}_s^{\star})^{-1}$.

7.1. **Using Standard Results.** A few standard results which can be used to find the inverse Star-Laplace Transform $s$-step have been tabulated below. These results can be easily proven using the standard definitions and (7.1).

**Example 1.** *The Heaviside step function is defined as*

$$\Omega(t)=\begin{cases}0\; if\; t<0\\ 1\; if\; t\geq 0\end{cases}$$

*To find the Star-Laplace Transform s-step, we apply the definition.*

$$\begin{aligned}\mathcal{L}_s^{\star}\Omega(\omega) &= \int_0^{+\infty}(\alpha_s^{\star}\Omega)(t)e^{-\omega t}dt\\ &= \frac{2}{3}\int_0^{+\infty}\left(f(t+s)+f(t+2s)+f(t+3s)+f(t+4s)+f(t+5s)\right)e^{-\omega t}dt\\ &= \frac{2}{3}\int_s^{+\infty}C(x)e^{-\omega(x-s)}dx+\frac{2}{3}\int_{2s}^{+\infty}C(x)e^{-\omega(x-2s)}dx+\frac{2}{3}\int_{3s}^{+\infty}C(x)e^{-\omega(x-3s)}dx\\ &+ \frac{2}{3}\int_{4s}^{+\infty}C(x)e^{-\omega(x-4s)}dx+\frac{2}{3}\int_{5s}^{+\infty}C(x)e^{-\omega(x-5s)}dx\end{aligned}$$

*In the first case if $s\leq 0$ then:*

$$\mathcal{L}_s^{\star}\Omega(\omega)=\frac{2}{3\omega}\left(e^{\omega s}+e^{2\omega s}+e^{3\omega s}+e^{4\omega s}+e^{5\omega s}\right)$$

*In the second case if $s > 0$ then:*

$$\mathcal{L}_s^\star\Omega(\omega) = \frac{10}{3\omega}$$

*Reciprocally, Let the frequency Domain Function*

$$\phi_s(t) = \begin{cases} \frac{2}{3t}\left(e^{ts} + e^{2ts} + e^{3ts} + e^{4ts} + e^{5ts}\right) & if\ s < 0 \\ \frac{10}{3t} & if\ s \geq 0 \end{cases}$$

*We obtain the inverse Star-Laplace Transform s-step or the Star-Coefficient* $\alpha_s^\star(f)(t)$

$$\alpha_s^\star\,(f)\,(t) = \frac{10}{3}$$

*and we deduce the origin function of the star-system:*

$$\star[f(t+s), f(t+2s), f(t+3s), f(t+4s), f(t+5s); \alpha] = \alpha$$

*Therefore* $f(t) = \frac{3}{10}\alpha_0^\star\,(f)\,(t) = 1$

**Example 2.** *The Cosine function is defined as*

$$f_\xi(t) = cos(\xi t)\Omega(t) = \begin{cases} 0 & if\ t < 0 \\ cos(\xi t) & if\ t \geq 0 \end{cases}$$

*As before, start with the definition of the Star-Laplace Transform s-step*

$$\begin{aligned}
\mathcal{L}_s^\star f_\xi(\omega) &= \int_0^{+\infty}(\alpha_s^\star f_\xi)(t)e^{-\omega t}dt \\
&= \frac{2}{3}\int_0^{+\infty} cos(\xi(t+s))\Omega(t+s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} cos(\xi(t+2s))\Omega(t+2s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} cos(\xi(t+3s))\Omega(t+3s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} cos(\xi(t+4s))\Omega(t+4s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} cos(\xi(t+5s))\Omega(t+5s)e^{-\omega t}dt \\
&= \frac{2}{3}\int_s^{+\infty} cos(\xi x)\Omega(x)e^{-\omega(x-s)}dx + \frac{2}{3}\int_{2s}^{+\infty} cos(\xi x)\Omega(x)e^{-\omega(x-2s)}dx \\
&+ \frac{2}{3}\int_{3s}^{+\infty} cos(\xi x)\Omega(x)e^{-\omega(x-3s)}dx + \frac{2}{3}\int_{4s}^{+\infty} cos(\xi x)\Omega(x)e^{-\omega(x-4s)}dx \\
&+ \frac{2}{3}\int_{5s}^{+\infty} cos(\xi x)\Omega(x)e^{-\omega(x-5s)}dx
\end{aligned}$$

*If $s \leq 0$*

$$\begin{aligned}
\mathcal{L}^{\star}_{s} f_{\xi}(\omega) &= \frac{2}{3}\int_{0}^{+\infty} cos(\xi x)e^{-\omega(x-s)}dx + \frac{2}{3}\int_{0}^{+\infty} cos(\xi x)e^{-\omega(x-2s)}dx \\
&+ \frac{2}{3}\int_{0}^{+\infty} cos(\xi x)e^{-\omega(x-3s)}dx + \frac{2}{3}\int_{0}^{+\infty} cos(\xi x)e^{-\omega(x-4s)}dx \\
&+ \frac{2}{3}\int_{0}^{+\infty} cos(\xi x)e^{-\omega(x-5s)}dx \\
&= \frac{2\omega e^{\omega s}}{3(\omega^2+\xi^2)} + \frac{2\omega e^{2\omega s}}{3(\omega^2+\xi^2)} + \frac{2\omega e^{3\omega s}}{3(\omega^2+\xi^2)} + \frac{2\omega e^{4\omega s}}{3(\omega^2+\xi^2)} + \frac{2\omega e^{5\omega s}}{3(\omega^2+\xi^2)}
\end{aligned}$$

*If $s \geq 0$*

$$\begin{aligned}
\mathcal{L}^{\star}_{s} f_{\xi}(\omega) &= \frac{2}{3}\int_{s}^{+\infty} cos(\xi x)e^{-\omega(x-s)}dx + \frac{2}{3}\int_{2s}^{+\infty} cos(\xi x)e^{-\omega(x-2s)}dx \\
&+ \frac{2}{3}\int_{3s}^{+\infty} cos(\xi x)e^{-\omega(x-3s)}dx + \frac{2}{3}\int_{4s}^{+\infty} cos(\xi x)e^{-\omega(x-4s)}dx \\
&+ \frac{2}{3}\int_{5s}^{+\infty} cos(\xi x)e^{-\omega(x-5s)}dx \\
&= \left[\frac{2e^{\omega(s-x)}(\xi sin(\xi x) - \omega cos(\xi x))}{3(\omega^2+\xi^2)}\right]_{s}^{+\infty} + \left[\frac{2e^{\omega(2s-x)}(\xi sin(\xi x) - \omega cos(\xi x))}{3(\omega^2+\xi^2)}\right]_{2s}^{+\infty} \\
&+ \left[\frac{2e^{\omega(3s-x)}(\xi sin(\xi x) - \omega cos(\xi x))}{3(\omega^2+\xi^2)}\right]_{3s}^{+\infty} + \left[\frac{2e^{\omega(4s-x)}(\xi sin(\xi x) - \omega cos(\xi x))}{3(\omega^2+\xi^2)}\right]_{4s}^{+\infty} \\
&+ \left[\frac{2e^{\omega(5s-x)}(\xi sin(\xi x) - \omega cos(\xi x))}{3(\omega^2+\xi^2)}\right]_{5s}^{+\infty} \\
&= \frac{2(\omega cos(\xi s) - \xi sin(\xi s))}{3(\omega^2+\xi^2)} + \frac{2(\omega cos(\xi 2s)) - \xi sin(\xi 2s)}{3(\omega^2+\xi^2)} \\
&+ \frac{2(\omega cos(\xi 3s) - \xi sin(\xi 3s))}{3(\omega^2+\xi^2)} + \frac{2(\omega cos(\xi 4s) - \xi sin(\xi 4s))}{3(\omega^2+\xi^2)} \\
&+ \frac{2(\omega cos(\xi 5s) - \xi sin(\xi 5s))}{3(\omega^2+\xi^2)}
\end{aligned}$$

*Reciprocally, if $s \leq 0$*
*Let the frequency Domain Function,*

$$\begin{aligned}
\phi_s(t) &= \frac{2te^{ts}}{3(t^2+\xi^2)} + \frac{2te^{2ts}}{3(t^2+\xi^2)} + \frac{2te^{3ts}}{3(t^2+\xi^2)} \\
&+ \frac{2te^{4ts}}{3(t^2+\xi^2)} + \frac{2te^{5ts}}{3(t^2+\xi^2)}
\end{aligned}$$

*We obtain the inverse Star-Laplace Transform s-step or the Star-Coefficient* $\alpha_s^\star(f)(t)$*:*

$$\begin{aligned}\alpha_s^\star(f)(t) &= \frac{2}{3}cos(\xi(t+s)) + \frac{2}{3}cos(\xi(t+2s)) + \frac{2}{3}cos(\xi(t+3s)) \\ &+ \frac{2}{3}cos(\xi(t+4s)) + \frac{2}{3}cos(\xi(t+5s))\end{aligned}$$

*and we deduce the origin function of the star-system:*

$$\star[f(t+s), f(t+2s), f(t+3s), f(t+4s), f(t+5s); \alpha] = \alpha$$

*Therefore* $f(t) = \frac{3}{10}\alpha_0^\star(f)(t) = cos(\xi t)$
<u>*Reciprocally, if* $s \geq 0$</u>
*Let the frequency Domain Function,*

$$\begin{aligned}\phi_s(t) &= \frac{2(tcos(s\xi) - \xi sin(s\xi))}{3(t^2+\xi^2)} + \frac{2(tcos(2s\xi) - \xi sin(2s\xi))}{3(t^2+\xi^2)} \\ &+ \frac{2(tcos(3s\xi) - \xi sin(3s\xi))}{3(t^2+\xi^2)} + \frac{2(tcos(4s\xi) - \xi sin(4s\xi))}{3(t^2+\xi^2)} \\ &+ \frac{2(tcos(5s\xi) - \xi sin(5s\xi))}{3(t^2+\xi^2)}\end{aligned}$$

*We obtain the inverse Star-Laplace Transform s-step or the Star-Coefficient* $\alpha_s^\star(f)(t)$*:*

$$\begin{aligned}\alpha_s^\star(f)(t) &= \frac{2}{3}cos(\xi(t+s)) + \frac{2}{3}cos(\xi(t+2s)) + \frac{2}{3}cos(\xi(t+3s)) \\ &+ \frac{2}{3}cos(\xi(t+4s)) + \frac{2}{3}cos(\xi(t+5s))\end{aligned}$$

*and we deduce the origin function of the star-system:*

$$\star[f(t+s), f(t+2s), f(t+3s), f(t+4s), f(t+5s); \alpha] = \alpha$$

*Therefore* $f(t) = \frac{3}{10}\alpha_0^\star(f)(t) = cos(\xi t)$

**Example 3.** *The Sine function is defined as*

$$f_\xi(t) = sin(\xi t)\Omega(t) = \begin{cases} 0 & if\ t < 0 \\ sin(\xi t) & if\ t \geq 0 \end{cases}$$

*As before, start with the definition of the Star-Laplace Transform s-step*

$$\begin{aligned}
\mathcal{L}_s^{\star} f_\xi(\omega) &= \int_0^{+\infty} (\alpha_s^{\star} f_\xi)(t) e^{-\omega t} dt \\
&= \frac{2}{3}\int_0^{+\infty} sin(\xi(t+s))\Omega(t+s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} sin(\xi(t+2s))\Omega(t+2s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} sin(\xi(t+3s))\Omega(t+3s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} sin(\xi(t+4s))\Omega(t+4s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} sin(\xi(t+5s))\Omega(t+5s)e^{-\omega t}dt \\
&= \frac{2}{3}\int_s^{+\infty} sin(\xi x)\Omega(x)e^{-\omega(x-s)}dx + \frac{2}{3}\int_{2s}^{+\infty} sin(\xi x)\Omega(x)e^{-\omega(x-2s)}dx \\
&+ \frac{2}{3}\int_{3s}^{+\infty} sin(\xi x)\Omega(x)e^{-\omega(x-3s)}dx + \frac{2}{3}\int_{4s}^{+\infty} sin(\xi x)\Omega(x)e^{-\omega(x-4s)}dx \\
&+ \frac{2}{3}\int_{5s}^{+\infty} sin(\xi x)\Omega(x)e^{-\omega(x-5s)}dx
\end{aligned}$$

*If* $s \leq 0$

$$\mathcal{L}_s^{\star} f_\xi(\omega) = \frac{2\xi e^{\omega s}}{3(\omega^2+\xi^2)} + \frac{2\xi e^{2\omega s}}{3(\omega^2+\xi^2)} + \frac{2\xi e^{3\omega s}}{3(\omega^2+\xi^2)} + \frac{2\xi e^{4\omega s}}{3(\omega^2+\xi^2)} + \frac{2\xi e^{5\omega s}}{3(\omega^2+\xi^2)}$$

*If* $s \geq 0$

$$\begin{aligned}
\mathcal{L}_s^{\star} f_\xi(\omega) &= \frac{2(\omega sin(s\xi)+\xi cos(s\xi))}{3(\omega^2+\xi^2)} + \frac{2(\omega sin(2s\xi)+\xi cos(2s\xi))}{3(\omega^2+\xi^2)} \\
&+ \frac{2(\omega sin(3s\xi)+\xi cos(3s\xi))}{3(\omega^2+\xi^2)} + \frac{2(\omega sin(4s\xi)+\xi cos(4s\xi))}{3(\omega^2+\xi^2)} \\
&+ \frac{2(\omega sin(5s\xi)+\xi cos(5s\xi))}{3(\omega^2+\xi^2)}
\end{aligned}$$

*Reciprocally, if* $s \leq 0$

*Let the frequency Domain Function,*

$$\phi_s(t) = \frac{2\xi e^{ts}}{3(t^2+\xi^2)} + \frac{2\xi e^{2ts}}{3(t^2+\xi^2)} + \frac{2\xi e^{3ts}}{3(t^2+\xi^2)} + \frac{2\xi e^{4ts}}{3(t^2+\xi^2)} + \frac{2\xi e^{5ts}}{3(t^2+\xi^2)}$$

*We obtain the inverse Star-Laplace Transform s-step or the Star-Coefficient* $\alpha_s^{\star}(f)(t)$*:*

$$\begin{aligned}
\alpha_s^{\star}(f)(t) &= \frac{2}{3}sin(\xi(t+s)) + \frac{2}{3}sin(\xi(t+2s)) + \frac{2}{3}sin(\xi(t+3s)) \\
&+ \frac{2}{3}sin(\xi(t+4s)) + \frac{2}{3}sin(\xi(t+5s))
\end{aligned}$$

*and we deduce the origin function of the star-system:*

$$\star[f(t+s), f(t+2s), f(t+3s), f(t+4s), f(t+5s); \alpha] = \alpha$$

*Therefore* $f(t) = \frac{3}{10}\alpha_0^\star(f)(t) = sin(\xi t)$
*Reciprocally, if* $s \geq 0$
*Let the frequency Domain Function,*

$$\begin{aligned} \phi_s(t) &= \frac{2(tsin(s\xi)+\xi cos(s\xi))}{3(t^2+\xi^2)} + \frac{2(tsin(2s\xi)+\xi cos(2s\xi))}{3(t^2+\xi^2)} \\ &+ \frac{2(tsin(3s\xi)+\xi cos(3s\xi))}{3(t^2+\xi^2)} + \frac{2(tsin(4s\xi)+\xi cos(4s\xi))}{3(t^2+\xi^2)} \\ &+ \frac{2(tsin(5s\xi)+\xi cos(5s\xi))}{3(t^2+\xi^2)} \end{aligned}$$

*We obtain the inverse Star-Laplace Transform s-step or the Star-Coefficient* $\alpha_s^\star(f)(t)$

$$\begin{aligned} \alpha_s^\star(f)(t) &= \frac{2}{3}sin(\xi(t+s)) + \frac{2}{3}sin(\xi(t+2s)) + \frac{2}{3}sin(\xi(t+3s)) \\ &+ \frac{2}{3}sin(\xi(t+4s)) + \frac{2}{3}sin(\xi(t+5s)) \end{aligned}$$

*and we deduce the origin function of the star-system:*

$$\star[f(t+s), f(t+2s), f(t+3s), f(t+4s), f(t+5s); \alpha] = \alpha$$

*, Therefore* $f(t) = \frac{3}{10}\alpha_0^\star(f)(t) = sin(\xi t)$

**Example 4.** *The Exponential function is defined as*

$$f_\xi(t) = e^{\xi t}\Omega(t) = \begin{cases} 0 \;\; if \; t < 0 \\ e^{\xi t} \; if \; t \geq 0 \end{cases}$$

*As before, start with the definition of the Star-Laplace Transform s-step*

$$\begin{aligned}
\mathcal{L}_s^\star f_\xi(\omega) &= \int_0^{+\infty} (\alpha_s^\star f_\xi)(t) e^{-\omega t} dt \\
&= \frac{2}{3}\int_0^{+\infty} e^{\xi(t+s)}\Omega(t+s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} e^{\xi(t+2s)}\Omega(t+2s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} e^{\xi(t+3s)}\Omega(t+3s)e^{-\omega t}dt + \frac{2}{3}\int_0^{+\infty} e^{\xi(t+4s)}\Omega(t+4s)e^{-\omega t}dt \\
&+ \frac{2}{3}\int_0^{+\infty} e^{\xi(t+5s)}\Omega(t+5s)e^{-\omega t}dt \\
&= \frac{2e^{\omega s}}{3}\int_s^{+\infty} \Omega(x)e^{-x(\omega-\xi)}dx + \frac{2e^{\omega 2s}}{3}\int_{2s}^{+\infty} \Omega(x)e^{-x(\omega-\xi)}dx \\
&+ \frac{2e^{\omega 3s}}{3}\int_{3s}^{+\infty} \Omega(x)e^{-x(\omega-\xi)}dx + \frac{2e^{\omega 4s}}{3}\int_{4s}^{+\infty} \Omega(x)e^{-x(\omega-\xi)}dx \\
&+ \frac{2e^{\omega 5s}}{3}\int_{5s}^{+\infty} \Omega(x)e^{-x(\omega-\xi)}dx
\end{aligned}$$

*If $s \leq 0$*

$$\begin{aligned}
\mathcal{L}_s^\star f_\xi(\omega) &= \left(\frac{2e^{\omega s}}{3} + \frac{2e^{2\omega s}}{3} + \frac{2e^{3\omega s}}{3} + \frac{2e^{4\omega s}}{3} + \frac{2e^{4\omega s}}{3}\right)\int_0^{+\infty} e^{-x(\omega-\xi)}dx \\
&= \frac{2\left(e^{\omega s} + e^{2\omega s} + e^{3\omega s} + e^{4\omega s} + e^{5\omega s}\right)}{3(\omega-\xi)}
\end{aligned}$$

*If $s \geq 0$*

$$\begin{aligned}
\mathcal{L}_s^\star f_\xi(\omega) &= \frac{2e^{\omega s}}{3}\int_s^{+\infty} e^{-x(\omega-\xi)}dx + \frac{2e^{\omega 2s}}{3}\int_{2s}^{+\infty} e^{-x(\omega-\xi)}dx \\
&+ \frac{2e^{\omega 3s}}{3}\int_{3s}^{+\infty} e^{-x(\omega-\xi)}dx + \frac{2e^{\omega 4s}}{3}\int_{4s}^{+\infty} e^{-x(\omega-\xi)}dx \\
&+ \frac{2e^{\omega 5s}}{3}\int_{5s}^{+\infty} e^{-x(\omega-\xi)}dx \\
&= \frac{2\left(e^{s\xi} + e^{2s\xi} + e^{3s\xi} + e^{4s\xi} + e^{5s\xi}\right)}{3(\omega-\xi)}
\end{aligned}$$

*Reciprocally, if $s \leq 0$*
*Let the frequency Domain Function,*

$$\phi_s(t) = \frac{2\left(e^{ts} + e^{2ts} + e^{3ts} + e^{4ts} + e^{5ts}\right)}{3(t-\xi)}$$

*We obtain the inverse Star-Laplace Transform $s$-step or the Star-Coefficient $\alpha_s^\star(f)(t)$:*

$$\alpha_s^\star\left(f\right)\left(t\right)=\frac{2}{3}\left(e^{\xi(t+s)}+e^{\xi(t+2s)}+e^{\xi(t+3s)}+e^{\xi(t+4s)}+e^{\xi(t+5s)}\right)$$

*and we deduce the origin function of the star-system:*

$$\star[f(t+s),f(t+2s),f(t+3s),f(t+4s),f(t+5s);\alpha]=\alpha$$

*Therefore* $f(t)=\frac{3}{10}\alpha_0^\star\left(f\right)\left(t\right)=e^{\xi t}$

*Reciprocally, if* $s\geq 0$.

*Let the frequency Domain Function,*

$$\phi_s(t)=\frac{2\left(e^{s\xi}+e^{2s\xi}+e^{3s\xi}+e^{4s\xi}+e^{5s\xi}\right)}{3(t-\xi)}.$$

*We obtain the inverse Star-Laplace Transform $s$-step or the Star-Coefficient $\alpha_s^\star(f)(t)$:*

$$\alpha_s^\star\left(f\right)\left(t\right)=\frac{2}{3}\left(e^{\xi(t+s)}+e^{\xi(t+2s)}+e^{\xi(t+3s)}+e^{\xi(t+4s)}+e^{\xi(t+5s)}\right)$$

*and we deduce the origin function of the star-system:*

$$\star[f(t+s),f(t+2s),f(t+3s),f(t+4s),f(t+5s);\alpha]=\alpha.$$

*Therefore* $f(t)=\frac{3}{10}\alpha_0^\star\left(f\right)\left(t\right)=e^{\xi t}$.

## 8. Conclusion

This paper thus, consisted of a brief overview of what Star-Laplace Transform $s$-step is, and what is it used for. The major properties of Star-Laplace Transform $s$-step discussed and a few special functions like the Heaviside step function, Sine Function, Cosine Function and Exponential Functions were also discussed in detail. It also included a detailed explanation of Inverse Star-Laplace Transform $s$-step and the method that can be employed in finding the Inverse Star-Laplace Transform $s$-step. It goes without saying that Star-Laplace Transform $s$-step is put to tremendous use in many branches of Applied Sciences.

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