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A New Algorithm To Solve Fuzzy Transportation Problem Using Ranking Function

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Abstract:
In the literature, several methods are proposed for solving transportation problems in fuzzy environment . There are several papers in the literature in which generalized fuzzy numbers are used for solving real life problems . In this paper, a new method is proposed for solving fuzzy transportation problems by assuming the transportation cost, supply and demand of the product are fuzzy triangular numbers . The proposed method is a direct extension of classical method so the proposed method is very easy to understand and to apply on real life transportation problems for the decision makers. The solution procedure is illustrated through numerical example. The paper has significant implication in other variety study of linear programming problem.

Keywords: *Fuzzy cost matrix ,Fuzzy triangular number, Fuzzy transportation problem, Ranking function.*

I. INTRODUCTION

The transportation problem is an important linear programming problem which involves sources with homogeneous product and destinations which require these products, Dantzig [7], Dantzig et. al. [8]. It finds application in various real life models. The objective is to "determine a transportation schedule that meets all demands from the source with a minimum shipping cost". The introduction of fuzzy sets and its applications by H. J. Zimmermann [10] has motivated researchers to utilize its theory and methodology in different models of transportation problems.

In a fuzzy transportation problem the supply capacity (availability of products at the sources), the requirements of the destinations and the cost of unit transport are assumed to be fuzzy. All these values are taken to be intervals on the real line **R**. The objective of fuzzy transportation problem to "develop a shipping schedule that minimizes the transportation cost satisfying the fuzzy supply and fuzzy demand". Different methods derived by number of authors to solve the Fuzzy Transportation Problem for minimum shipping cost. Nagoor Gani et. al.[6] have already proposed a method to solve intuitionistic fuzzy transportation problem where they have considered the supply and demand to be fuzzy and also solved intuitionistic fuzzy transportation problem using zero suffix algorithm. H. Basirzadeh [9] and Narayanmoorthy et. al. [17], Chanas et. al. [18] have proposed alternative methods for similar problem which are available in the literature. Malini et.al.[16] have also proposed "a method to solve fuzzy transportation problem by using octagonal fuzzy numbers". Michael [12] has proposed "an algorithm for transportation problems with fuzzy constraints and investigated the relationship between the algebraic structure of the optimum solution of the deterministic problem and its fuzzy equivalent". Charnes and Cooper [1] have given a definition for "the optimal solution of a transportation problem and also proposed an algorithm to find optimal solution". M. Shanmugasundari and K. Ganesan[11] have derived "a novel approach for the fuzzy optimal solution of fuzzy transportation problem". Abdul Quddoos et.al. [2], P.Pandian et.al.[13], R.N.Mondal et.al. [14] and S.Ezhil Vannan et. al. [15] have derived "Various methods are developed to find the optimal solution of the fuzzy transportation problem" .

In this chapter we have discussed two different methods to solve fuzzy transportation problem .

II. PRELIMINARIES

***Definition .1 Fuzzy triangular number*:**

A fuzzy number $\tilde{a} = (a, b, c)$ is said to be a fuzzy triangular number if its membership function is given by

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$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ \frac{x-c}{b-c}, & b \le x \le c \\ 0 \quad , & \text{otherwise} \end{cases}$$

Definition . 2 Ranking function:

The function which maps from a set of fuzzy number into the set of real numbers i.e. R: F(**R**) → **R**, (where a natural order exists) is said to be ranking function, where F(**R**) is a set of fuzzy numbers. Let $\tilde{a} =$(a, b, c) be a triangular fuzzy number then a standard ranking function is R($\tilde{a}$)=(a + 2b + c)/4, which was used by Kumar Amit and Kaur Jagdeep [8]. The same function is used by us in our results.

Arithmetic operations:

Let $\tilde{a} = (a, b, c)$ and $\tilde{b} = (e, f, g)$ be two triangular fuzzy numbers defined on the set of real numbers R. Then

(i) $\tilde{a} \oplus \tilde{b} = (a+e, b+f, c+g)$

(ii) $-\tilde{a} = (-c, -b, -a)$

(iii) $\tilde{a} - \tilde{b} = (a-g, b-f, c-e)$

(iv) $\tilde{a} \otimes \tilde{b} = \begin{Bmatrix} (ae, bf, cg)\ , a \ge 0 \\ (ag, bf, cg)\, , a < 0, c \ge 0 \\ (ag, bf, ce), c < 0 \end{Bmatrix}$

(v) $\tilde{a} \le \tilde{b}$ iff $R(\tilde{a}) \le R(\tilde{b})$

III. MATHEMATICAL MODEL OF FUZZY TRANSPORTATION PROBLEM

Let us consider a Fuzzy Transportation Problem with m-sources and n-destinations. Let $\tilde{c}_{ij}$ be the cost of transporting one unit of product from i-th source to j-th destination for i = 1, 2 ,…….,m. and j = 1, 2,……, n.

Let $\tilde{a}_i$ be the quantity of commodity available at source i for i = 1, 2……, m and $\tilde{b}_j$ be the quantity of commodity needed of the destinations j for j = 1, 2, …………, n.

Assume that $\tilde{x}_{ij}$, i = 1, 2,……., m and j = 1, 2,……, n is quantity of commodity transported from i-th source to j-th destination.

Mathematical model of the Fuzzy Transportation Problem is:

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{x}_{ij}\,\tilde{c}_{ij}$$

Subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} \le \tilde{a}_i \ , i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \ge \tilde{b}_j \ , j = 1, 2, ..., n$$

$\tilde{x}_{ij} \ge 0$ for all i and j.

If the problem is balanced then the model is

Minimize $z = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{x}_{ij}\,\tilde{c}_{ij}$

Subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \;,\; i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \;,\; j = 1, 2, ..., n$$

$\tilde{x}_{ij} \geq 0$ for all i and j

IV. EXISTENCE OF FUZZY FEASIBLE SOLUTION

The necessary and sufficient condition for the existence of fuzzy feasible solution to the Fuzzy Transportation Problem is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ i.e. total supply = total demand .

Proof: ***Necessary condition*:**

Let there exist a fuzzy feasible solution to the fuzzy TP.

maximize $\tilde{z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i \;,\; i = 1, 2, 3, ..., m \qquad (2.2.1)$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}j \;,\; j = 1, 2, 3, ..., n \qquad (2.2.2)$$

and $\quad x_{ij} \geq \tilde{0}$ for all i and j

From the equation (2.2.1) $\quad \sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i \;,\; i = 1, 2, 3, ..., m$

$$\Rightarrow \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{x}_{ij} \approx \sum_{i=1}^{m} \tilde{a}_i \qquad (2.2.3)$$

Also from the equation (2.2.2), $\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}j \;,\; j = 1, 2, 3, ..., n$

$$\Rightarrow \sum_{j=1}^{n}\sum_{i=1}^{m} \tilde{x}_{ij} \approx \sum_{j=1}^{n} \tilde{b}j \qquad (2.2.4)$$

From equation (2. 2. 3) and (2. 2. 4) we get $\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}j$

Sufficient condition:

Since all $\tilde{a}_i$ and $\tilde{b}_j$ are positive therefore $\tilde{x}_{ij}$ must be positive .Hence Fuzzy Transportation Problem posses feasible solution as described by Edward Samuel et. al. [6]

V. PROPOSED ALGORITHM

Here we discuss the second method to solve fuzzy transportation problem**.**

Step.I: Solve the given transportation problem by North –West corner method.

Step.II: Determine $\tilde{C} = \max\{\tilde{c}_{ij} \mid \tilde{x}_{ij} > \tilde{0}\}$ and set up the Pseudo cost matrix $\tilde{C}$ with

$$\tilde{C}_{ij} = \begin{Bmatrix} M & \text{if} & \tilde{C}_{ij} > \tilde{Z} \\ \tilde{1} & \text{if} & \tilde{C}_{ij} = \tilde{Z} \\ \tilde{0} & \text{if} & \tilde{C}_{ij} < \tilde{Z} \end{Bmatrix}$$

Step.III: Solving the new fuzzy transportation problem and determine its Pseudo cost $\tilde{Z}_k = \sum_i \sum_j \tilde{C}_{ij} \tilde{x}_{ij}$ for $k = 1, 2, 3, ...$

Step.IV: Is $\tilde{Z}_k = \tilde{0}$?

If Yes go to step. II

If No stop and the current solution is optimal

VI. NUMERICAL EXAMPLE

Example.1: We take the transportation problem having supplies $\widetilde{20}, \widetilde{30}$ *and* $\widetilde{50}$ respectively and their destinations with demand $\widetilde{10}$, $\widetilde{20}$ *and* $\widetilde{40}$ respectively. The cost matrix is

$$\begin{bmatrix} \tilde{5} & \tilde{7} & \tilde{9} \\ \tilde{3} & \tilde{8} & \tilde{3} \\ \tilde{4} & \tilde{6} & \tilde{6} \end{bmatrix}$$

where, $\widetilde{20} = (19,20,22)$, $\widetilde{30} = (29,30,32)$, $\widetilde{50} = (45,50,50)$, $\widetilde{10} = (9,10,12)$, $\widetilde{40} = (39,40,42)$, $\tilde{5} = (8,5,6)$, $\tilde{7} = (6,7,8)$ and $\tilde{9} = (7,9,11)$, $\tilde{3} = (2,7,5)$, $\tilde{8} = (7,8,9)$, $\tilde{4} = (3,4,6)$ *and* $\tilde{6} = (5,6,7)$ are fuzzy triangular number .

Using North-West corner method we get the solution is $\begin{bmatrix} (3,5,6)^{(9,10,12)} & (6,7,8)^{(9,10,10)} & (9,10,12) & (0,0,0) \\ (2,3,5) & (7,8,9)^{(9,10,12)} & (2,3,5)^{(18,20,20)} & (0,0,0) \\ (3,4,6) & (5,6,7) & (5,6,7)^{(20,20,22)} & (0,0,0)^{(28,30,32)} \end{bmatrix}$

So, $\tilde{C} = \max\{(3,5,6),(6,7,8),(7,8,9),(2,3,5),(5,6,7),(0,0,0)\}$

=(7,8,9) (using ranking function)

Now the fuzzy Transportation problem by algorithm is:

$$\begin{bmatrix} \tilde{0} & \tilde{0} & M & \tilde{0} \\ \tilde{0} & \tilde{1} & \tilde{0} & \tilde{0} \\ \tilde{0} & \tilde{0} & \tilde{0} & \tilde{0} \end{bmatrix}$$

Using least cost method we get

$$\begin{bmatrix} \tilde{0}^{(9,10,12)} & \tilde{0}^{(10,10,10)} & M & \tilde{0} \\ \tilde{0} & \tilde{1} & \tilde{0}^{(29,30,32)} & \tilde{0} \\ \tilde{0} & \tilde{0}^{(9,10,12)} & \tilde{0}^{(10,10,10)} & \tilde{0}^{(29,30,32)} \end{bmatrix}$$

$$\Rightarrow \tilde{Z}_1 = 0$$

So, Z= min{(3,5,6,),(6,7,8),(2,3,5),(5,6,7),(0,0,0)} =(6,7,8)

Using algorithm we get new Transportation problem

$$\begin{pmatrix} \tilde{0} & \tilde{1} & M & \tilde{0} & (19,20,22) \\ \tilde{0} & M & \tilde{0} & \tilde{0} & (29,30,32) \\ \tilde{0} & \tilde{0} & \tilde{0} & \tilde{0} & (48,50,54) \\ (9,10,12) & (19,20,22) & (39,40,42) & (29,30,32) & \end{pmatrix}$$

Using least cost method we get

$$\begin{bmatrix} \tilde{0}^{(9,10,12)} & \tilde{1} & M & \tilde{0}^{(10,10,10)} \\ \tilde{0} & \tilde{1} & \tilde{0}^{(29,30,32)} & \tilde{0} \\ \tilde{0} & \tilde{0}^{(19,20,22)} & \tilde{0}^{(10,10,10)} & \tilde{0}^{(20,20,22)} \end{bmatrix}$$

$$\Rightarrow \tilde{Z}_2 = 0$$

So,

$$\tilde{Z} = \max\{(3,5,6),(0,0,0,),(2,3,5),(5,6,7),(5,6,7),(0,0,0)\} = (5,6,7)$$

New Transportation problem is: $\begin{bmatrix} \tilde{0} & M & M & \tilde{0} \\ \tilde{0} & M & \tilde{0} & \tilde{0} \\ \tilde{0} & \tilde{1} & \tilde{1} & \tilde{0} \end{bmatrix}$

Using least cost method we get

$$\begin{bmatrix} \tilde{0}^{(9,10,12)} & M & M & \tilde{0}^{(10,10,10)} \\ \tilde{0} & M & \tilde{0}^{(29,30,32)} & \tilde{0} \\ \tilde{0} & \tilde{1}^{(19,20,22)} & \tilde{1}^{(10,10,10)} & \tilde{0}^{(20,20,22)} \end{bmatrix}$$

$$\Rightarrow \tilde{Z}_4 = \tilde{1}(19,20,22) + \tilde{1}(10,10,10) \neq \tilde{0}$$

So, $\tilde{Z}=(3,5,6),(3,10,12)+(2,3,5),(29,30,32)+(5,6,7),(19,20,22)+(5,6,7)(10,10,10)=(230,320,356)$ which is optimal solution i.e. a fuzzy solution.

Example-2

We take the transportation problem having supplies $\widetilde{15}, \widetilde{20}$ *and* $\widetilde{25}$ respectively and their destinations with demand $\widetilde{22}$, $\widetilde{13}$ *and* $\widetilde{15}, \widetilde{10}$ respectively. The cost matrix is

$$\begin{bmatrix} \tilde{5} & \tilde{3} & \tilde{8} & \tilde{2} \\ \tilde{4} & \tilde{6} & \tilde{7} & \tilde{1} \\ \tilde{6} & \tilde{3} & \tilde{2} & \tilde{9} \end{bmatrix}$$

where, $\widetilde{15} = (13,15,17)$, $\widetilde{20} = (19,20,23)$, $\widetilde{25} = (22,25,27), \widetilde{22} = (20,22,24)$, $\widetilde{13} = (12,13,14), \widetilde{15} = (14,15,17), \widetilde{10} = (8,10,12)$ are fuzzy triangular number .

Using North-West corner method we get the solution is $\begin{bmatrix} (3,5,6)^{(13,15,17)} & (1,3,4) & (1,2,3) & (7,9,10) \\ (2,4,5)^{(7,7,7)} & (5,6,8)^{(12,13,14)} & (5,7,8)^{(0,0,2)} & (0,1,3) \\ (4,6,8) & (7,8,9) & (1,2,4)^{(14,15,15)} & (7,8,9)^{(8,10,12)} \end{bmatrix}$

$$\tilde{Z} = \max\ \{(3,5,6),(2,4,5),(5,6,8),(5,7,8),(1,2,4),(7,8,9)\}$$

=(7,8,9) (using ranking function)

Now the fuzzy Transportation problem by algorithm is:

$$\begin{bmatrix} \tilde{0} & \tilde{0} & \tilde{0} & M \\ \tilde{0} & \tilde{0} & \tilde{0} & \tilde{0} \\ \tilde{0} & \tilde{1} & \tilde{0} & \tilde{1} \end{bmatrix}$$

Using least cost method we get

$$\begin{bmatrix} \tilde{0}^{(13,15,17)} & \tilde{0} & \tilde{0} & M \\ \tilde{0}^{(7,7,7)} & \tilde{0}^{(12,13,14)} & \tilde{0}^{(0,0,2)} & \tilde{0} \\ \tilde{0} & \tilde{1} & \tilde{0}^{(14,15,15)} & \tilde{1}^{(8,10,12)} \end{bmatrix}$$

$$\Rightarrow \tilde{Z}_4 = \tilde{1}(8,10,12) \neq \tilde{0}$$

So, optimal transportation cost is

$$\tilde{Z}=(3,5,6).(13,15,17)+(2,4,5).(7,7,7)+(5,6,8).(12,13,14)+(5,7,8)(0,0,2)+(1,2,4)(14,15,15)+(7,8,9)(8,10,12)$$
=(197,291,433) i.e. fuzzy solution.

VII. COMPARISON

Methods	Optimal solution(example.1)	Optimal solution(example.2)
Vogel's approximation method	(210,320,360)	(256,310,452)
Proposed Algorithm	(230,320,356)	(197,291,433)

VIII. CONCLUSION

In this paper, we have studied the Fuzzy transportation problem involving fuzzy triangular numbers. We have used the ranking function to obtain the compromise fuzzy optimal solution by a new algorithm. The proposed methods are very easy to understand and to apply for solving the fuzzy transportation problems occurring in real life situations. We can use this method for business purpose and saving purpose. The proposed method is verified with examples and compared with other existing method. The optimal solution is achieved is fuzzy minimum transportation value i.e. solution is an interval .

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