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Analysis of An Age-Infection-Structured HIV Model With Impulsive Finding

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Abstract *—This paper develops an impulsive SUI model with infection age for HIV. Firstly the basic reproduction number, which depends on the impulsive HIV-finding period and the HIV-finding proportion, is obtained by mathematical analysis. Secondly there exists a globally asymptotically stable infection- free equilibrium when the basic reproduction number is less than one. Therefore the HIV epidemic is theoretically cleared if we have the suitable HIV-finding proportion and the impulsive HIV-finding period such that the basic reproduction number is less than one.*

Keywords — *Basic reproduction number, Global stablity, HIV/AIDS, Impulsive period, Infection age*

I. INTRODUCTION

As we know, acquired immunodeficiency syndrome (AIDS) is a very harmful infectious disease, caused by human immunodeficiency virus (HIV) which can attack the body's immune system and make the human body lose its immune function.

HIV has a long incubation period (average 8-10 years) and infection period. During the incubation period, people can live and work for many years without any symptoms. We know the infection age is the time passed since a host was infected. Note that the infectivity changes with the infection age, therefore, many scholars have established a large number of mathematical models with age structure to describe the dynamic behavior of AIDS[1]-[4]. These models tend to focus on threshold theory. The threshold of many epidemiological models is the basic reproduction number R_0, which refers to the average number of secondary infections produced by an infected person in a whole susceptible population. Heffernan et al.[5] investigated R_0 for some simple HIV transmission models. May et al.[6] used models with age structure to examine the demographic effects of AIDS in African countries.

Due to economic or social constraints, it is impossible to test all population for HIV. Forthermore, the survey[7]-[9] shows that STD clinic patients have a low awareness rate of HIV knowledge. We should test for low education, HIV related risk behaviors, married or cohabitation and so on. However, considering the practical factors, it is impossible to continuously test whether these high-risk groups have AIDS, consequently, HIV-finding can be carried out once every T month to test HIV in time and persuade them not to spread HIV. It may be more effective and economic to control the HIV epidemic, especially in poor areas. In conclusion, we should consider infection age and impulse effect, which are often ignored in traditional HIV model.

Some models described by impulsive differential equations were proposed to discuss the dynamic behavior of HIV/AIDS [10]-[20]. By capitalizing on a fairly classical mathematical literature [21]-[22], the above impulsive models focused on the drug-treatment strategies at fixed times for HIV/AIDS epidemic. However, taking into account the reality, we establish an age-infection-structured HIV model with impulsive finding.

II. MODEL AND ASSUMPTIONS

If we ignore the recovery population, we divide the total population into three epidemiologically different groups: SUI, where S denotes susceptible individuals uninfected with HIV virus but susceptible to it, U denotes individuals infected with HIV but undiscovered and I denotes individuals with AIDS or with HIV-positive, that is, individuals discovered to be infected with HIV. We assume that individuals in I -group can take effective measures to prevent the transmission of AIDS when they have sexual contact with susceptible individuals, thus in our model only individuals in U -group can transmit the HIV. Because of the mortality, infectivity and the conversion rate from U -group to I -group are related to the time of infection, therefore, the infection age a is introduced. $S(t)$ denotes the density of susceptible populations at time t. Let $U(a,t)$ be the density of undiscovered individuals infected with HIV at infection age a and time t. Let $I(a,t)$ be the density of individuals with HIV-positive or AIDS at infection age a and time t, According to the mechanism of infection, we introduce the following force of infection denoted by $\lambda(t)$:

$$\lambda(t) = \int_0^{a_m} \kappa(a)U(a,t)da,$$

where $\kappa(a)$ denotes the age-infection-specific infectious rate, let a_m be the highest age attained of the individuals. They satisfy the following assumption:

(H0) $\kappa(a)$ is continuous on $[0,+\infty)$ and $\kappa(a)\equiv 0 \quad (a\notin[0,a_m))$.

According to the above assumption, this paper considers the following model：

$$\begin{cases} \dfrac{dS(t)}{dt} = N - dS(t) - \lambda(t)S(t), \\ \dfrac{\partial U(a,t)}{\partial a} + \dfrac{\partial U(a,t)}{\partial t} = -\mu(a)U(a,t) - \gamma(a)U(a,t), \\ \dfrac{\partial I(a,t)}{\partial a} + \dfrac{\partial I(a,t)}{\partial t} = -\mu(a)I(a,t) + \gamma(a)U(a,t), \\ U(0,t) = \lambda(t)S(t), I(0,t) = 0, \\ U(a_m,t) = 0, I(a_m,t) = 0, \\ U(a,0) = U^0(a) \in L^1(0,a_m), I(a,0) = I^0(a) \in L^1(0,a_m), \end{cases} \tag{1}$$

where N and d are the birth rate and the death rate of susceptible individuals, $\mu(a)$ and $\gamma(a)$ denote the age-specific mortality rate and the rate of transition from U -group to I -group, respectively.

In order to prevent and control the transmission of HIV/AIDS epidemic, we adopt an impulsive HIV-finding strategy for risk population groups every T months. After each impulsive HIV-finding, we know that U -group has been infected, that is, the HIV-finding rate is $q(a)(0<q(a)\le 1)$, we note that the clinical phenomenon of individuals with different infection age a is different, so the $q(a)$ depends on the infection age. Thus the population in U -group obtains its new initial value, that is when $t=kT(k\in\square_+)$, we know $U(a,kT)=(1-q(a))U(a,kT^-)$. T is the impulsive HIV-finding period and is a positive real number. kT is the time at which we apply the k th pulse, kT^- is the time just before applying the k th pulse. Therefore when $t=kT$ the dynamical behavior is governed by the following problem：

$$\begin{cases} S(kT) = S(kT^-), \\ U(a,kT) = (1-q(a))U(a,kT^-), \\ I(a,kT) = I(a,kT^-) + q(a)U(a,kT^-). \end{cases} \tag{2}$$

We know that the solution to equation (1) is as follows:

$$\begin{aligned} S(t) &= S^0 e^{-\int_0^t (d+\lambda(\tau))d\tau} + \int_0^t N e^{\int_\tau^t (d+\lambda(\eta))d\eta} d\tau, \\ U(a,t) &= \begin{cases} U^0(a-t)e^{-\int_0^t (\mu(\tau+a-t)+\gamma(\tau+a-t))d\tau}, a \ge t, \\ U(0,t-a)e^{-\int_{t-a}^t (\mu(\tau+a-t)+\gamma(\tau+a-t))d\tau}, a < t, \end{cases} \\ I(a,t) &= \begin{cases} I^0(a-t)e^{-\int_0^t \mu(\tau)d\tau} + \int_0^t \gamma(\tau+a-t)U(\tau+a-t,\tau)e^{-\int_\tau^t \mu(\tau)d\tau} d\tau, a \ge t, \\ \int_{t-a}^t \gamma(\tau+a-t)U(\tau+a-t,\tau)e^{-\int_\tau^t \mu(\tau)d\tau} d\tau, a < t. \end{cases} \end{aligned} \tag{3}$$

In order to simplify system (1) - (2), the following functions are introduced

$$S(t) = s(t), \quad U(a,t) = u(a,t)e^{-\int_0^a \mu(\tau)d\tau}, \quad I(a,t) = i(a,t)e^{-\int_0^a \mu(\tau)d\tau}. \tag{4}$$

From the above transformation, we know

$$t \ne kT \begin{cases} \dfrac{ds(t)}{dt} = N - ds(t) - \lambda(t)s(t), \\ \dfrac{\partial u(a,t)}{\partial a} + \dfrac{\partial u(a,t)}{\partial t} = -\gamma(a)u(a,t), \\ \dfrac{\partial i(a,t)}{\partial a} + \dfrac{\partial i(a,t)}{\partial t} = \gamma(a)u(a,t), \\ u(a,0) = u^0(a), i(a,0) = i^0(a), \\ u(0,t) = \lambda(t)s(t), i(0,t) = 0, \\ u(a_m,t) = 0, i(a_m,t) = 0, \end{cases} \tag{5}$$

and

$$t=kT\begin{cases}s(kT)=s(kT^{-}),\\ u(a,kT)=(1-q(a))u(a,kT^{-}),\\ i(a,kT)=i(a,kT^{-})+q(a)u(a,kT^{-}),\end{cases} \tag{6}$$

where $u^0(a)=U^0(a)e^{-\int_0^a\mu(\tau)d\tau}, i^0(a)=I^0(a)e^{-\int_0^a\mu(\tau)d\tau}$.

Since (4) is an invertible transformation, system (5) - (6) and system (1) - (2) are equivalent, so we only need to discuss the existence and stability of the equilibrium in system (5) - (6).

Theorem 2.1. System (5) - (6) always has an infection-free equilibrium $E_0=(N/d,0,0)$.

According to the biological significance of the epidemic model, the age-specific infection rate $\kappa(a)$ depends on the total population, so it is bounded, there is the following assumption:

(H1) $\sup_{a>0}\kappa(a)=\bar{\kappa}<+\infty$.

The basic reproduction number and stability of the infection-free equilibrium E_0 are discussed following.

III. GLOBAL STABILITY OF INFECTION FREE EQUILIBRIUM

The basic reproduction number is defined as

$$R_0(q(a),T)=N/d\sum_{k=0}^{[a_m/T]}\int_{kT}^{(k+1)T}(1-q(\alpha))^k\kappa(\alpha)e^{-\int_0^\alpha\mu(\tau)d\tau}e^{-\int_0^\alpha\gamma(\tau)d\tau}d\alpha, \tag{7}$$

where $[a_m/T]$ represents the maximum integer not greater than a_m/T, that is $[a_m/T]T\le a_m<[a_m/T]T+T$. $e^{-\int_0^\alpha\mu(\tau)d\tau}$ represents the probability that an individual survival in time α, $e^{-\int_0^\alpha\gamma(\tau)d\tau}$ denotes the probability that an individual in a U -group is still in the U -group in time α, N/d indicates the initial number of susceptible individuals. $R_0(q(a),T)$ is when only one individual in U -group is introduced into susceptible population, the average number of secondary infection in U -group.

Theorem 3.1. Under assumptions (H0)- (H1), the infection-free equilibrium $E_0=(N/d,0,0)$ of system (5) - (6) is globally asymptotically stable if $R_0(q(a),T)<1$.

Proof. The nonnegative right continuous solutions of system (5) - (6) depend continuously on the results of initial data, we know that for any given positive constant $\Gamma\ge a_m$, E_0 is stable when $t\in[0,\Gamma]$, that is for any $\varepsilon>0$ there exists $\delta>0$, if $\left\|\left(s^0(\cdot)-N/d,u^0(\cdot,\cdot),i^0(\cdot,\cdot)\right)\right\|<\delta$, then we know $\left\|(s(t)-N/d,u(\cdot,t),i(\cdot,t))\right\|<\varepsilon$ when $t\in[0,\Gamma]$.

Let's now discuss E_0 is globally attractive. First, using the characteristic method and according to equation (4), we can know that $u(a,t)$ satisfies the following equation:

$$u(a,t)=\begin{cases}\omega(a)\Phi(t-a)s(t-a),(t-kT>a\ge 0),\\ (1-q(a))\omega(a)\Phi(t-a)s(t-a),(t-(k-1)T>a\ge t-kT),\\ (1-q(a))^2\omega(a)\Phi(t-a)s(t-a),(t-(k-2)T>a\ge t-(k-1)T),\\ \vdots\\ (1-q(a))^n\omega(a)\Phi(t-a)s(t-a),(t>a\ge t-T),\\ (1-q(a))^nu^0(a-t)e^{-\int_0^t\gamma(\tau+a-t)d\tau},(a\ge t),\end{cases} \tag{8}$$

where $\omega(a)=e^{-\int_0^a\gamma(\tau)d\tau},\Phi(t)=\int_0^{a_m}\varphi(\alpha)u(\alpha,t)d\alpha,\varphi(\alpha)=\kappa(\alpha)e^{-\int_0^\alpha\mu(\tau)d\tau}$. (9)

No loss of generality, if $t>a_m$ and $t\in[kT,(k+1)T)$, there exists $k_0\in\mathbb{Z}_+$ such that $1\le k_0\le k$ and $a_m\in(t-k_0T,t-k_0T+T]$. Therefore we know $k_0=k+1-[a_m/T]$ and

$$\Phi(t)=\int_0^{a_m}\varphi(\alpha)u(\alpha,t)d\alpha=\int_0^{t-kT}\varphi(\alpha)\omega(\alpha)\Phi(t-\alpha)s(t-\alpha)d\alpha+\int_{t-kT}^{t-(k-1)T}(1-q(\alpha))\varphi(\alpha)\omega(\alpha)\Phi(t-\alpha)s(t-\alpha)d\alpha$$
$$+\cdots+\int_{t-k_0T}^{a_m}(1-q(\alpha))^{k-k_0+1}\varphi(\alpha)\omega(\alpha)\Phi(t-\alpha)s(t-\alpha)d\alpha.$$

Note that the parameters in the above equation are nonnegative and $0 \le s(t) \le N/d$. For all $t > a_m, t \in [kT,(k+1)T)$ and $a_m \in \left(t-k_0T, t-k_0T+T\right]$, we know

$$\begin{aligned}\Phi(t) &\le N/d \int_0^{t-kT} \varphi(\alpha)\omega(\alpha)\Phi(t-\alpha)d\alpha + N/d \int_{t-kT}^{t-(k-1)T} (1-q(\alpha))\varphi(\alpha)\omega(\alpha)\Phi(t-\alpha)d\alpha + \cdots \\ &+ N/d \int_{t-k_0T}^{a_m} (1-q(\alpha))^{k-k_0+1}\varphi(\alpha)\omega(\alpha)\Phi(t-\alpha)d\alpha.\end{aligned} \tag{10}$$

For all $t > a_m, t \in [kT,(k+1)T)$ and $a_m \in \left(t-k_0T, t-k_0T+T\right]$, let

$$h(\alpha) = \begin{cases} N/d\ \varphi(\alpha)\omega(\alpha), \alpha \in [0, t-kT), \\ N/d\ (1-q(\alpha))\varphi(\alpha)\omega(\alpha), \alpha \in [t-kT, t-(k-1)T), \\ \vdots \\ N/d\ (1-q(\alpha))^{k-k_0+1}\varphi(\alpha)\omega(\alpha), \alpha \in \left[t-k_0T, a_m\right), \\ 0, \alpha \ge a_m. \end{cases} \tag{11}$$

We know

$$\begin{aligned}\int_0^t h(\alpha)d\alpha = \int_0^{a_m} h(\alpha)d\alpha &\le N/d \int_0^T \varphi(\alpha)\omega(\alpha)d\alpha + N/d \int_T^{2T} (1-q(\alpha))\varphi(\alpha)\omega(\alpha)d\alpha + \cdots \\ &+ N/d \int_{(k+1)T-k_0T}^{a_m} (1-q(\alpha))^{k-k_0+1}\varphi(\alpha)\omega(\alpha)d\alpha.\end{aligned}$$

According to the definition of basic reproduction number $R_0(q(a),T)$, it's easy to get

$$\int_0^t h(\alpha)d\alpha \le R_0(q(a),T) < +\infty, t \in \left(a_m, +\infty\right). \tag{12}$$

By using equations (10) and (11), we can get the following inequality

$$\Phi(t) \le \int_0^t h(\alpha)\Phi(t-\alpha)d\alpha \quad \forall t \in [kT,(k+1)T) \text{ and } t > a_m. \tag{13}$$

It can be seen that $\limsup_{t\to+\infty} \Phi(t) \le \bar{\kappa} a_m\ N/d < +\infty$ from the equation (9) and assumption (H1).

For any given $\varepsilon > 0$, according to equations (12) and (13), we can find $N_0 \in \mathbb{Z}_+$ such that $|\Phi(N_0T)| < \varepsilon$. We know

$$|\Phi(N_0T)| \le \int_{N_0T}^t h(\alpha)\Phi(t-\alpha)d\alpha < \int_{N_0T}^{+\infty} h(\alpha)\Phi(t-\alpha)d\alpha < \varepsilon, \qquad t \ge N_0T > a_m,$$

and

$$\Phi(t-\alpha) < \limsup_{t\to+\infty} \Phi(t) + \varepsilon, \qquad t > N_0T > a_m, \alpha < N_0T.$$

Hence when $t > N_0T$, we derive

$$\begin{aligned}\int_0^t h(\alpha)\Phi(t-\alpha)d\alpha &= \int_0^{N_0T} h(\alpha)\Phi(t-\alpha)d\alpha + \int_{N_0T}^t h(\alpha)\Phi(t-\alpha)d\alpha \\ &< \left(\limsup_{t\to+\infty} \Phi(t) + \varepsilon\right) \cdot \int_0^{N_0T} h(\alpha)d\alpha + \varepsilon \le \left(\limsup_{t\to+\infty} \Phi(t) + \varepsilon\right) \cdot R_0(q(a),T) + \varepsilon.\end{aligned}$$

Because $\varepsilon > 0$ is arbitrary, we can get

$$\limsup_{t\to+\infty} \Phi(t) \le \limsup_{t\to+\infty} \int_0^t h(\alpha)\Phi(t-\alpha)d\alpha \le \limsup_{t\to+\infty} \Phi(t) \cdot R_0(q(a),T). \tag{14}$$

If $R_0(q(a),T) < 1$, we derive $\limsup_{t\to+\infty} \Phi(t) = 0$. Because of $\Phi(t) \ge 0 (t \ge 0)$, we can get

$$\lim_{t\to+\infty} \Phi(t) = 0. \tag{15}$$

For all $t > a_m, t \in [kT,(k+1)T)$ and $a_m \in \left(t-k_0T, t-k_0T+T\right]$, we know

$$\|u(\cdot,t)\|_{L^1(D)} = \int_0^{a_m} u(a,t)da = \int_0^{t-kT} \omega(\alpha)\Phi(t-\alpha)s(t-\alpha)d\alpha + \int_{t-kT}^{t-(k-1)T} (1-q(\alpha))\omega(\alpha)\Phi(t-\alpha)s(t-\alpha)d\alpha$$
$$+\cdots+\int_{t-k_0T}^{a_m} (1-q(\alpha))^{k-k_0+1}\omega(\alpha)\Phi(t-\alpha)s(t-\alpha)d\alpha.$$

Let

$$h_1(\alpha) = \begin{cases} N/d\ \omega(\alpha), \alpha \in [0, t-kT), \\ N/d\ (1-q(\alpha))\omega(\alpha), \alpha \in [t-kT, t-(k-1)T), \\ \vdots \\ N/d\ (1-q(\alpha))^{k-k_0+1}\omega(\alpha), \alpha \in \left[t-k_0T, a_m\right), \\ 0, \alpha \geq a_m. \end{cases}$$

According to the above proof method, it can be known that $\lim_{t\to+\infty} \| u(\cdot,t)\|_{L^1(D)} = 0.$

Let's discuss the limit of $s(t)$ when $t \to +\infty$. Let $\bar{s}(t) = s(t) - N/d$, then $\bar{s}(t)$ satisifies the following equation

$$\frac{d\bar{s}(t)}{dt} = -(d+\lambda(t))\bar{s}(t) - \lambda(t)\,N/d\ .$$

So we get

$$\bar{s}(t) = \bar{s}(0)e^{-dt}e^{-\int_0^t \lambda(\tau)d\tau} - N/d\ e^{-dt}\int_0^t \lambda(\tau)e^{d\tau}e^{\int_t^\tau \lambda(\eta)d\eta}d\tau$$
$$= N/d\ e^{-dt}\left(e^{-\int_0^t \lambda(\tau)d\tau} - \int_0^t \lambda(\tau)e^{d\tau}e^{\int_t^\tau \lambda(\eta)d\eta}d\tau \right) .$$

Hence it proves that $\lim_{t\to+\infty} \|s(t) - N/d\| = \lim_{t\to+\infty} \| \bar{s}(t)\| = 0$.

We know $\lim_{t\to+\infty} \| i(\cdot,t)\|_{L^1(D)} = \lim_{t\to+\infty} \|s(t) - N/d\| - \lim_{t\to+\infty} \|u(\cdot,t)\|_{L^1(D)} = 0$. Therefore, the infection-free equilibrium E_0 is globally asymptotically stable, we have completed the proof of Theorem 3.1.

According to theorem 3.1, we can know the infection free equilibrium E_0 of the system (5) - (6) is globally asymptotically stable if $R_0(q(a),T) < 1$. E_0 is also the infection-free equilibrium of the initial system (1) - (2). According to the invertible transformation functions (4), we can know that the infection-free equilibrium E_0 of the system (1) - (2) is globally asymptotically stable when $R_0(q(a),T) < 1$.

Remark 1. If we consider that HIV-finding rate has nothing to do with the infection age a, e.g. $q(a) = q$, so we can define the basic reproduction number:

$$R_0(q,T) = N/d \sum_{k=0}^{\lceil a_m/T \rceil} (1-q)^k \int_{kT}^{(k+1)T} \kappa(\alpha) e^{-\int_0^\alpha \mu(\tau)d\tau} e^{-\int_0^\alpha \gamma(\tau)d\tau} d\alpha.$$

It can also be proved that the infection free equilibrium is globally asymptotically stable when $R_0(q,T) < 1$.

IV. CONCLUSIONS

As mentioned in Section II and III, our results provide a global stability analysis for the infection-free equilibrium of a HIV epidemic model. In this model, ignoring the recovery population, the HIV infected individuals are divided into two groups: HIV undetected group U and HIV positive I -group. In group I, there are no recovered individuals, and the individuals are all patients in need of treatment. We hypothesize that patients in group I could take protective measures to prevent the transmission of HIV when they have sexual contact with susceptible. We believe that only individuals in U -group can infect susceptible individuals through effective contact.

Secondly, the infectivity and mortality of infected people vary with the infection age. Because infection age plays an important role in HIV epidemic, we introduce infection age into U -group and I -group. It is helpful to analyze the transmission law of HIV accurately. Our model fully considers the differences brought by different infection age a, and the mortality of individuals with different infection age is different, which is in line with the practical significance, and brings some difficulties to our mathematical analysis.

On the other hand, some continuous models have been used to study the prevalence of HIV/AIDS, but the corresponding

impulsive partial differential equation models are few. In order to find the key factors of HIV/AIDS transmission and provide useful suggestions for the prevention and control of HIV/AIDS epidemic, we use impulsive partial differential equation to explain HIV-finding rate, which is related to the infection age, so as to find the best impulsive period T.

Through mathematical analysis, we calculate the basic replication number $R_0(q(a),T)$, which is not only related to HIV-finding rate $q(a)$, but also related to impulsive period T. Our results show that if we choose the appropriate HIV-finding rate $q(a)$ and impulsive period T, such as $R_0(q(a),T)<1$, in theory, the HIV can be cleared. However, the shortening of HIV pulse period T will increase the cost of preventing the epidemic. We believe that it is more economical and effective to ensure the proportion of HIV-finding $q(a)$. Although the current HIV epidemic is relatively fast, the global stability of the infection-free equilibrium E_0 we have obtained implies that the prevalence of HIV in real life will be greatly reduced.

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