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New Boundary Function In Fuzzy Topological Spaces

Sanjivappa K Dembare

Assistant Professor, Department of Mathematics, Government first Grade College, Navanagar, Bagalkot -587103, Karnataka, INDIA.

ABSTRACT: *In this Paper the notion of fuzzy boundary generalized-continuity, fuzzy boundary generalized-irresolute mappings,fuzzy boundary generalized - homeomorphism and fuzzy boundary generalized*-homeomorphism are introduced and some characterizations are obtained. It is shown that f-continuous map is fuzzy boundary generalized-continuous, fuzzy boundary generalized-irresolute map is fuzzy boundary generalized-continuous, fuzzy boundary generalized-continuous map is fgb-continuous and fuzzy boundary generalized-continuous is fspg-continuous, but the converses need not be true in each of the cases which is shown by examples.*

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Key words and Phrases: *Fuzzy boundary closed sets, Fuzzy boundary open sets, fuzzy boundary generalized-continuity, fuzzy boundary generalized homeomorphism.*

INTRODUCTION

N.Levine introduced generalized closed (g-closed) sets in general topology as a generalization` of closed sets. This concept was found to be useful and many results in general topology were improved. For example, it was proved that a g-closed subset of a compact space is compact. Many researchers like S.P.Arya and R.Gupta , K.Balachandran, P.Sundaram and H.Maki , S.G.Crosseley and S.K.Hilderbrand , J.Dontchev ,H.Maki, J.Umehara and T.Noiri , S.R.Malghan , N.Palaniappan and K.Chandrasekhara Rao , T.Noiri , W.Dunham and P.Sundaram have worked on this and related problems in general topology.

This idea of N.Levine motivated us to generalize the concept of closed fuzzy sets in fuzzy topological spaces to a concept called b-closed (boundary-closed) fuzzy sets, using the concept of boundary of a fuzzy set defined by R.H.Warren .

1. Preliminaries

1.1 Definition: A fuzzy subset A in a set X is a function $A : X \to [0, 1]$. A fuzzy subset in X is empty iff its membership function is identically 0 on X and is denoted by 0 or μ_ϕ.The set X can be considered as a fuzzy subset of X whose membership function is identically 1 on X and is denoted by μ_x or I_x. In fact every subset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

1.2 Definition: A fuzzy set on X is ‘Crisp ‘if it takes only the values 0 and 1 on X.

1.3 Definition: Let X be a set and be a family of fuzzy subsets of (X,τ) is called a fuzzy topology on X iff τ satisfies the following conditions.

(i) μ_ϕ ; $\mu_X \in \tau$: That is 0 and $1 \in \tau$

(ii) If $G_i \in \tau$ for $i \in I$ then $\vee_{i \in I} G_i \in \tau$

(iii) If $G,H \in \tau$ then $G \wedge H \in \tau$

The pair (X, τ) is called a fuzzy topological space. The members of τ are called fuzzy open sets and a fuzzy set A in X is said to be closed iff $1 - A$ is an fuzzy open set in X.

1.4 Remark: Every topological space is a fuzzy topological space but not conversely.

1.5 Definition : If A and B are any two fuzzy subsets of a set X , then A is said to be included in B or A is contained in B iff $A(x) \leq B(x)$ for all x in X. Equivalently, $A \leq B$ iff $A(x) \leq B(x)$ for all x in X.

1.6 Definition: Two fuzzy subsets A and B are said to be equal if $A(x) = B(x)$ for every x in X.
Equivalently $A = B$ if $A(x) = B(x)$ for every x in X.

1.7Definition: The complement of a fuzzy subset A in a set X, denoted by A' or $1 - A$, is the fuzzy subset of X defined by $A'(x) = 1 - A(x)$ for all x in X. Note that$(A')' = A$.

1.8 Definition: The union of two fuzzy subsets A and B in X, denoted by $A \vee B$, is a fuzzy subset in X defined by $(A \vee B)(x) = \mathrm{Max}\{\mu_A(x), \mu_B(X)\}$ for all x in X.

1.9 Definition: The intersection of two fuzzy subsets A and B in X, denoted by $A \wedge B$, is a fuzzy subset in X defined by $(A \wedge B)(x) = \mathrm{Min}\{A(x), B(x)\}$ for all x in X.

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In this section, fuzzy bg-continuous maps, fuzzy bg-irresolute maps, fuzzy bg-closed maps, fuzzy bg-open maps, fuzzy bg-homeomorphisms and fuzzy bgc-homeomorphisms, in fuzzy topological spaces are introduced and studied.

Definition 4.3.1: A mapping $f : (X, \tau) \longrightarrow (Y,\sigma)$ is said to be fuzzy b-generalized
Continuous (briefly, fuzzy boundary generalized-continuous), if $f^{-1}(A)$ is fuzzy boundary generalized-closed set in X, for every fuzzy-closed set A in Y.

Theorem 4.3.2: $f : (X, \tau) \longrightarrow (Y,\sigma)$ is fuzzy boundary generalized-continuous iff the inverse image of each fuzzy open set in Y is fuzzy boundary generalized-open in X.

Proof: Let B be a fuzzy boundary generalized open set in Y. Then 1 - B is fuzzy boundary generalized-closed in Y. Since f is fuzzy boundary generalized-continuous
$f^{-1}(1 - B) = 1 - f^{-1}(B)$ is fuzzy boundary generalized-closed in X. Thus $f^{-1}(B)$) is fuzzy boundary generalized-open set in X. Converse, is obvious.

Theorem 4.3.3: If $f : (X, \tau) \longrightarrow (Y,\sigma)$ is fuzzy boundary generalized-continuous then

(i) for each fuzzy point x_p of X and each $A \in Y$ such that $f(x_p)\, q\, A$, there exists a fuzzy boundary generalized-open set A of X such that $x_p \in B$ and $f\{B) < A$.

(ii) for each fuzzy point x_p of X and each $A \in Y$ such that $f(x_p)\, q\, A$, there exists a fuzzy boundary generalized-open set B of X such that $x_p\, q\, B$ and $f(B) < A$.

Proof:

(i) Let x_p be a fuzzy point of X. Then f(x_p) is a fuzzy point in Y. Now let A ∈ Y be a fuzzy boundary generalized-open set such that f(x_p) q A. Put B =f^{-1}(A). Since
f : X —> Y is fuzzy boundary generalized-continuous B is fuzzy boundary generalized-open set of X and x_p ∈B. Therefore f(B) =f (f^{-1}(A)) < A.

(ii) Let x_p be a fuzzy point of X, and let A € Y such that f(x_p) q A. Put B = f(A). Then B is fuzzy boundary generalized-open set of X, such that x_p G∈B and f(B)= f(f^{-1}(A)) < A.

Theorem 4.3.4: Every f-continuous map is fuzzy boundary generalized-continuous.

Proof: Let f: (X, τ) —> (Y,σ) be f-continuous function. Let A be an fuzzy open set in Y. Since f is f-continuous, f^{-1}(A) is f-open in X. Since every fuzzy open set is fuzzy boundary generalized-open, f^{-1}(A) is fuzzy boundary generalized-open in X. Therefore
f is fuzzy boundary generalized-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.3.5: Let X = {a, b, c} and Y ={ l,m} Let the fuzzy sets A and B be defined as follows: A = {(a, 1), (b,0), (c, 0)}, B = {(l, 0), (m, 1)}. Consider τ = {0,1,A} and
ρ = {0,1, B} .Define f : (X, τ) —> (Y,σ) as f(a) = f(c) = m and f(b) = l. Then f is fgb-continuous but not f-continuous, since for the f-open set B ∈ ρ, f^{-1} (B)

Definition 4.3.6:A mapping f : (X, τ) —> (Y,σ) is said to be fuzzy b-generalized irresolute (briefly fuzzy boundary generalized-irresolute), if f^{-1}(A) is fuzzy boundary generalized-closed set in X, for every fuzzy boundary generalized-closed set A in Y.

Theorem 4.3.7: Every fuzzy boundary generalized-irresolute map is fuzzy boundary generalized-continuous

Proof.: Let f : X —> Y be fuzzy boundary generalized-irresolute and let A be fuzzy closed set in Y. Since every fuzzy closed set is also fuzzy boundary generalized-closed,
A is fuzzy boundary generalized-closed in Y. Since f : X —> Y is fuzzy boundary generalized-irresolute, f^{-1}(A) is fuzzy boundary generalized-closed. Thus inverse image of every fuzzy closed set in Y is fuzzy boundary generalized-closed in X. Therefore the function f: X —► Y is fuzzy boundary generalized-continuous

The converse is not true.

Example 4.3.8: Let X = {a, b},Y = {x, y}, A = {(a, 0.6), (b, 0.3)}, B = {(x, 0.5),
(y, 0.2)}. Let τ = {0, A, 1}, σ ={0, B, 1}. Then the mapping f : (X, τ) —► (Y,σ) defined by f(a) = x and f(b) = y is fuzzy boundary generalized-continuous but not fuzzy boundary generalized-irresolute.

Theorem 4.3.9: Every fuzzy boundary generalized- continuous map is fspg-continuous.
Proof: .Let f : X —► Y be fuzzy boundary generalized-continuous and Let A be fuzzy closed set in Y. Since f: X —► Y is fuzzy boundary generalized-continuous f^{-1}{A) is fuzzy boundary generalized-closed. Every fuzzy boundary generalized-closed set is fsp-closed, so f^{-1} (A) is fsp-closed. Thus inverse image of every fuzzy closed set in Y is fspg-closed in X. Therefore the function f : X —> Y is fspg-continuous.

The converse is not true.

Example 4.3.10: Let X ={a,b}, Y ={x,y}, A={{a, 0.4), (b, 0.4)}, B= {(x,0 .3), (y, 0.3)}. Let
τ = {0, A, 1}, σ = {0, B, 1}. Then the mapping f : X —> T defined by f(a) = x and f(b) = y is fspg-continuous but not fuzzy boundary generalized-continuous.

Theorem 4.3.11: Every fuzzy boundary generalized- continuous map is fgb -continuous.

Proof: Clear from the fact that fuzzy boundary generalized-closed set is fgb-closed.

Theorem 4.3.12. Let f : X —► Y, g : Y —> Z be two mappings. Then

(i) gof is fuzzy boundary generalized-continuous, if f is fuzzy boundary generalized-continuous and g is f-continuous.
(ii) gof is fuzzy boundary generalized- irresolute, if f and g are fuzzy boundary generalized- irresolute.
(iii) gof is fuzzy boundary generalized-continuous if f is fuzzy boundary generalized-irresolute and g is fuzzy boundary generalized-continuous.

Proof:

(i)Let B be f-closed in Z. Since g : Y —> Z is fuzzy continuous, by definition g^{-1}(B) is
f-closed set of Y. Now f : X —► Y is fuzzy boundary generalized-continuous so
$f^{-1}(g^{-1}(B)) = (gof)^{-1}(B)$ is fuzzy boundary generalized-closed in X. Hence
g • f : X —> Z is fuzzy boundary generalized-continuous.

(ii) Let g: Y —» Z be fuzzy boundary generalized-irresolute and let B be fuzzy boundary generalized-closed subset in Z. Since g is fuzzy boundary generalized-irresolute by definition ,
g^{-1}(B) is fuzzy boundary generalized-closed set in Y. Also f : X —► Y is fuzzy boundary generalized-irresolute, so $f^{-1}(g^{-1}(B)) = (g \bullet f)^{-1}(B)$ is fuzzy boundary generalized-closed.
Thus g • f : X —► Z is fuzzy boundary generalized-irresolute.

(iii) Let B be fb-closed in Z. Since g : Y —> Z is fuzzy boundary generalized-continuous,
g^{-1}(B) is fuzzy boundary generalized-closed in Y. Also f : X —> Y is fuzzy boundary generalized-irresolute, so every fuzzy boundary generalized-closed set in Y is fuzzy boundary generalized-closed in X. Hence $f^{-1}(g^{-1}(B)) = (g \bullet f)^{-1}(B)$ is fuzzy boundary generalized-closed in X. Thus g • f : X —► Z is fuzzy boundary generalized-irresolute.

Theorem 4.3.13: If f : f : (X, τ) —► (Y,σ) is fb*-continuous and g :(Y, σ) —> (Z, ρ) is fuzzy boundary generalized-continuous then g • f : (X, r) —> (Z, p) is fuzzy boundary generalized-continuous if Y is fb$T_{1/2}$-space.

Proof: Suppose A is fb-closed subset of Z. Since g : Y —> Z is fuzzy boundary generalized continuous, g^{-1}(B) is fuzzy boundary generalized-closed subset of Y. Now since Y is fb$T_{1/2}$-space, g^{-1}{B) is fb-closed subset of Y. Also since f : X —► Y is fb*-continuous $f^{-1}(g^{-1}(B)) = (gof)^{-1}(B)$ is fb-closed. Thus gof : X —Z is fuzzy boundary generalized-continuous.

Theorem 4.3.14: Let f : (X, τ) —► (Y,σ) be fuzzy boundary generalized-continuous. Then f is fb-continuous if X is fb$T_{1/2}$-space.

Proof: Let B be fuzzy closed set in Y. Since f **:** X **—>** Y is fuzzy boundary generalized-continuous, f^{-1}{B) is fuzzy boundary generalized-closed subset in X. Since X is fb$T_{1/2}$-space, by hypothesis every fuzzy boundary generalized-closed set is fb-closed . Hence
f^{-1}{A) is fb-closed subset in X. Therefore f : X —> Y is fb-continuous.

Theorem 4.3.15: Let f **:** (X, τ) —► (Y,σ) be onto fuzzy boundary generalized-irresolute and fuzzy b*-closed. If X is fb$T_{1/2}$ -space, then is fb$T_{1/2}$ -space.

Proof: Let A be a fuzzy boundary generalized-closed set in Y**.** Since f : X —» Y is fuzzy boundary generalizedirresolute, f^{-1} (A) is fuzzy boundary generalized-closed set in X.
As X is fb$T_{1/2}$ -space, f^{-1}(A) is fuzzy b-closed set in X. Also f : X —►Y is fb* closed,
f(f^{-1}(A)) is fuzzy b-closed in Y. Since f : X **—**> Y is onto, f(f^{-1}(A)) =A. Thus A is fb-closed in Y. Hence (Y,σ) is also fb$T_{1/2}$ -space.

Theorem 4.3.16: Let f : (X, τ) —► (Y,σ) be fuzzy boundary generalized-continuous and
g : (Y,σ) —►(Z,ρ) be fg-continuous. Then g • f is fuzzy boundary generalized- continuous if Y is f$T_{1/2}$ space.

Proof: Let A be fuzzy closed set in Z. Since g is fg-continuous, g^{-1} {A) is fg-closed in Y**.** But Y is f$T_{1/2}$ space and so g^{-1}{A) is fuzzy closed in Y. Since f is fuzzy boundary generalized-continuous is fuzzy boundary generalized-closed in X. Hence g o f fuzzy boundary generalized-continuous.

Theorem 4.3.17: If the bijective map f : (X, τ) —► (Y,σ) is fb*-open and
fb*-irresolute, then f : (X, τ) —► (Y,σ) is fuzzy boundary generalized-irresolute.

Proof.: Let A be a fuzzy boundary generalized-closed set in Y and let f^{-1}(A) < B where B is a fb-open set in X . Clearly, A < f(B). Since f: X —► Y is fb*-open map, f(B) is fb-open in Y and A is fuzzy boundary generalized-closed set in Y. Then bCl(A) < f(B), and thus f^{-1}(bCl{A)) < B. Also f : X —► Y is irresolute and bCl(A) is a fb-closed set in Y, then f^{-1} bCl(A)) is fb-closed set in X. Thus bCl(f^{-1} (A)) < bCl{f^{-1}bCl{A))) < B. So f^{-1}(A) is fuzzy boundary generalized-closed set in X. Hence f : X —►Y is fuzzy boundary generalized-irresolute map.

Definition 4.3.18: A mapping f : (X, τ) —► (Y,σ) is said to be fuzzy bg-open(briefly fuzzy boundary generalized-open) if the image of every f-open set in X, is fuzzy boundary generalized-open inY.

Definition 4.3.19: A mapping f : (X, τ) —► (Y,σ) is said to be fuzzy bg-closed(briefly fuzzy boundary generalized-dosed) if the image of every f-closed set in X is fuzzy boundary generalized-closed in Y.

Definition 4.3.20: A mapping f : (X, τ) —► (Y,σ) is said to be fuzzy bg*-open(briefly fuzzy boundary generalized*-open) if the image of every fuzzy boundary generalized-open set in X is fuzzy boundary generalized-open inY.

Definition 4.3.21: A mapping f : (X, τ) —► (Y,σ) is said to be fuzzy bg-closed(briefly fuzzy boundary generalized*-closed) if the image of every fuzzy boundary generalized-closed set in X is fuzzy boundary generalized-closed in Y.

Definition 4.3.21: A mapping f : (X, τ) —► (Y,σ) is said to be fuzzy bg-closed(briefly fuzzy boundary generalized*-closed) if the image of every fuzzy boundary generalized-closed set in X is fuzzy boundary generalized-closed in Y.

Remark 4.3.22:

(i)Every fuzzy boundary generalized*-closed mapping is fuzzy boundary generalized-closed.
(ii)Every fuzzy boundary generalized*-closed mapping is fgb* -closed.

Theorem 4.3.23: If f : (X, τ) —► (Y,σ) is f-closed and g : (Y,σ) —►(Z,ρ) is
fuzzy boundary generalized-closed, then g • f is fuzzy boundary generalized-closed.

Proof. Let A be a f-closed set in X. Then f(A) is f-closed in Y. Since g : $(Y,\sigma) \longrightarrow (Z,\rho)$ is fuzzy boundary generalized-closed, g(f(A)) = (g • f)(A) is fuzzy boundary generalized-closed in Z. Therefore g • f is fuzzy boundary generalized-closed.

Theorem 4.3.24: If f : $(X, \tau) \longrightarrow (Y,\sigma)$ is a fuzzy boundary generalized-closed map and Y is $fbT^*_{1/2}$ -space, then f is a f-closed.

Proof: Let A be a f-closed set in X. Then f(A) is fgb-closed in Y, since f is fgb- closed. Again since Y is $fbT_{1/2}$ space, f(A) is fuzzy closed in Y. Hence f : $(X, \tau) \longrightarrow (Y,\sigma)$ is a f-closed.

Theorem 4.3.25: If f : $(X, \tau) \longrightarrow (Y,\sigma)$ is a fuzzy boundary generalized-closed map and Y is $fbT_{1/2}$ space, then f is a fb-closed map.

Theorem 4.3.26: A mapping f : $(X, \tau) \longrightarrow (Y,\sigma)$ is fuzzy boundary generalized-closed iff for each fuzzy set A in Y and f-open set B such that $f^{-1}(A) < B$, there is a fuzzy boundary generalized-open set C of Y such that A < C and $f^{-1}(C) < B$.

Proof.: Suppose f is fuzzy boundary generalized-closed map. Let A be a fuzzy set of Y ,and B be an f-open set of X, such that $f^{-1}(A) < B$. Then C = 1 – f(1-B) is a fuzzy boundary generalized-open in Y such that A < C and $f^{-1}(C) < B$.

Conversely, suppose that F is a f-closed set of X. Then $f^{-1}(1 — f((F)) < 1 — F$, and
1 — F is f-open set. By hypothesis, there is a fuzzy boundary generalized-open set C of Y such that 1 — f(I — F) < C and $f^{-1}(C) < 1 — F$. Therefore $F < 1 – f^{-1}(C)$. Hence 1-C < f(C) <
$f(1 – f^{-1}(C)) < 1 — C$, which implies f(F) = 1 — C. Since 1 — C is fgb-closed set, f(F) is fuzzy boundary generalized -closed set and thus f is a fuzzy boundary generalized-closed map.

Theorem 4.3.27: If f : $(X, \tau) \longrightarrow (Y,\sigma)$ and g : $(Y,\sigma) \longrightarrow (Z,\rho)$ are fuzzy boundary generalized-closed maps and Y is $fbT_{1/2}$ space, then go f : X —> Z is fuzzy boundary generalized-closed.

Proof: Let A be a fuzzy closed set in X. Since f : $(X, \tau) \longrightarrow (Y,\sigma)$ is fuzzy boundary generalized-closed, f(A) is fuzzy boundary generalized-closed in F. Now F is $fbT^*_{1/2}$ space, so f(A) is fuzzy closed in Y. Also g : $(Y,\sigma) \longrightarrow (Z,\rho)$ is fuzzy boundary generalized-closed, g(f(A)) = (g • f)(A) is fuzzy boundary generalized-closed in Z. Therefore g • f is fuzzy boundary generalized-closed.

Theorem 4.3.28: If A is fuzzy boundary generalized-closed in X and f : X $\longrightarrow$*Y is bijective, fb-irresolute and fuzzy boundary generalized-closed, then f(A) is fuzzy boundary generalized-closed in Y.

Proof. Let f(A) < B where B is fb-open in F. Since f is fbirresolute, $f^{-1}(B)$ is fb-open containing A. Hence $bCl(A) < f^{-1}\{B)$ as A is fuzzy boundary generalized-closed. Since
f is fuzzy boundary generalized-closed, f(bCl(A)) is fuzzy boundary generalized-closed set contained in the fb-open set B, which implies bCl(f(bCl(A))) < B and
hence bCl(f(A)) < B. So f(A) is fuzzy boundary generalized-closed in F.

Theorem 4.3.29: If f : $(X, \tau) \longrightarrow (Y,\sigma)$ is fuzzy boundary generalized-closed and
g : $(Y,\sigma) \longrightarrow (Z,\rho)$ is fuzzy boundary generalized*-closed, then g • f is fuzzy boundary generalized -closed.

Proof. Let A be f-closed set in X. Then f(A) is fuzzy boundary generalized-closed in Y. Since
g : $(Y,\sigma) \longrightarrow (Z,\rho)$ is fuzzy boundary generalized*-closed. Thus g(f(A)) = (g • f){A) is fuzzy boundary generalized-closed in Z. Therefore g • f is fuzzy boundary generalized-closed.

Theorem 4.3.30: If f : (X, τ) —► (Y,σ) and g : (Y,σ) —►(Z,ρ) are fuzzy boundary generalized*- closed maps, then g • f : X —► Z is fuzzy boundary generalized*-closed.

Theorem 4.3.31: Let f : (X, τ) —► (Y,σ) , g : (Y,σ) —►(Z,ρ) be two maps such that g • f : X —> Z is fuzzy boundary generalized-closed.

(i) If f is f-continuous and surjective, then g is fuzzy boundary generalized-closed.
(ii) If g is fuzzy boundary generalized-irresolute and injective, then f is fuzzy boundary generalized-closed.

Proof.(i) Let F be f-closed in Y. Then $f^{-1}(F)$ is f-closed in X, as f is f-continuous. Since gof is fuzzy boundary generalized-closed map and f is surjective, **(g** • f)($f^{-1}(F)$) = g(F) is fuzzy boundary generalized-closed in Z. Hence g : Y —> Z is fuzzy boundary generalized closed

(ii)Let F be a f-closed in X. Then (g • f)(F) is fuzzy boundary generalized-closed in Z. Since g is fuzzy boundary generalized-irresolute and injective g^{-1}{g • f)(F) = f(F) is fuzzy boundary generalized-closed in Y. Hence f is a fuzzy boundary generalized-closed.

Theorem 4.3.32: Let f : (X, τ) —► (Y,σ) , g : (Y,σ) —►(Z,ρ) be two maps such that g • f : X —> Z is fuzzy boundary generalized*-closed map.

(i) If f is fuzzy boundary generalized-continuous and surjective, then g is fuzzy boundary generalized-closed.
(ii) If g is fuzzy boundary generalized-irresolute and injective, then f is fuzzy boundary generalized*-closed.

Definition 4.3.34: A mapping f : (X, τ) —► (Y,σ **)** is called fuzzy bg-homeomorphism (briefly fb-homeomorphism) if f and f^{-1} are fb-gcontinuous.

Definition 4.3.35: A mapping f : (X, τ) —► (Y,σ **)** is called fuzzy bg*-homeomorphism (briefly fuzzy boundary generalized*-homeomorphism) if f and f^{-1} are fuzzy boundary generalized-irresolute.

Theorem 4.3.36: Every f-homeomorphism is fuzzy boundary generalized homeomorphism.
The converse of the above theorem need not be true as seen from
the following example.
Example 4.3.37: Let X = Y = {a, b, c} and the fuzzy sets A**,** B and C be defined as follows. A = {(a, 1), (b,0.8)}, B **=** {(a, 0.3), (b,0.6)}, C = {(a, 0.4), (b,0.6)}.
Consider τ = {0,1, A} and ρ = {0,1, B}. Then (X, τ) and (Y, ρ) are fts.
Define f: X —► Y by f(a) = a and f(b) = b. Then f is fuzzy boundary generalized-homeomorphism but not f-homeomorphism as A is open in X. f(A) = A is not open in Y. Hence f^{-1} : Y —► X is not f-continuous.

Theorem 4.3.38: Every fuzzy boundary generalized*-homeomorphism is fuzzy boundary generalized- homeomorphism.
Proof:Let f : (X, τ) —► (Y,σ **)** is fgb*-homeomorphism. Then f and f^{-1} are fuzzy boundary generalized-irresolute mappings. By theorem 4.3.7 f and f^{-1} are fuzzy boundary generalized-continuous. Hence f^{-1} is fuzzy boundary generalized-homeomorphism.

The characterization for fuzzy boundary generalized **-** homeomorphism is as follows

Theorem 4.3.39. If f : (X, τ) —► (Y,σ) is fuzzy boundary generalized-homeomorphism and g : (Y,σ) —► (Z,ρ) is fuzzy boundary generalized-homeomorphism and Y is $fbT_{1/2}$ space, then g • f : X —►* Z is fuzzy boundary generalized-homeomorphism.

Proof. To show that g • f and $(g \bullet f)^{-1}$ are fuzzy boundary generalized- continuous. Let A be a f-open set in Z. Since g : (Y,σ) —► (Z,ρ) is fuzzy boundary generalized- continuous, $g^{-1}\{A)$ is fuzzy boundary generalized-open in Y. Then $g^{-1}\{A)$ is a f-open in Y as Y is $fbT_{1/2}$ space. Also since f: X —> y is fuzzy boundary generalized- continuous, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is fuzzy boundary generalized-open in X. Therefore gof is fuzzy boundary generalized - continuous.

Again, let A be a f-open set in X. Since g^{-1} : Y —> X is fuzzy boundary generalized- continuous, $(f^{-1})^{-1}(A)) = f(A)$ is fuzzy boundary generalized-open in y. And so f(A) is f-open in Y since Y is $fbT_{1/2}$ space. Also since g^{-1} : Z -> y is fuzzy boundary generalized-continuous,
$(g^{-1})^{-1}$ (f (A)) = g(f{A)) = (gof)(A) is fuzzy boundary generalized-open in Z. Therefore $((gof)^{-1})^{-1}$ (A) = (gof)(A) is fuzzy boundary generalized-open in Z. Hence
$(gof)^{-1}$ is fuzzy boundary generalized - continuous. Thus gof is fuzzy boundary generalized - homeomorphism.

Theorem 4.3.40: . If f : (X, τ) —► (Y,σ), g : (Y,σ) —► (Z,ρ) are fuzzy boundary generalized* homeomorphism then g • f : X —► Z is fuzzy boundary generalized* homeomorphism.

Proof: Let A be a fuzzy boundary generalized-open set in Z. Then since g : Y —► Z is fuzzy boundary generalized- irresolute $g^{-1}(A)$ is fuzzy boundary generalized-open in Y. Also since
f: X —► Y is fuzzy boundary generalized-irresolute, $(f^{-1}$(g]mhj $^{-1}(A)) = (g$ —> $f)^{-1}(A)$ is fuzzy boundary generalized-open in X. Therefore g —► f : X —> Z is fuzzy boundary generalized-irresolute.

Again, let A be a fuzzy boundary generalized-open set in X. Then since f^{-1} : Y —» X is fuzzy boundary generalized-irresolute, $(f^{-1})^{-1}(A) = f(A)$ is fuzzy boundary generalized-open in Y. Also g^{-1} : Z —► Y is fuzzy boundary generalized-irresolute, $(g^{-1})^{-1}$(f(A) =g(f{A) = (g —> f)(A) is fuzzy boundary generalized-open in Z. Therefore (g —> f) $^{-1}$: Z —> X is fuzzy boundary generalized-irresolute. Hence g —► f : X —► Z is fuzzy boundary generalized* homeomorphism.

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