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Effect of Lipoprotein Concentration on MHD Blood flow through Parallel Plate with Heat source and Magnetic Intensity

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Abstract - *A mathematical model on the effect of Lipoprotein concentration on MHD blood flow through parallel plate under the action of magnetic field intensity with heat source was formulated. The system of partial differential equations governing the flow has been resolved to a system of ordinary differential equations and analytical expressions have been obtained under the conditions defined in the model. It is observed that parameters like decay parameter, Schmidt number, Hartmann number, heat source parameter and Prandtl number affect the flow profiles. Solutions for the axial velocity, mass concentration, normal velocity and temperature distribution are shown graphically to better understand the effect of the various involved parameters.*

Keywords — *Blood flow, Parallel plate channel, Mass concentration, Heat source*

I. INTRODUCTION

Many biological systems are multi-component dispersions such that their overall properties are non-Newtonian even if some of the individual components are Newtonian; blood often exhibits non-Newtonian behaviour, although its plasma component is Newtonian. There are a number of experimental as well as theoretical studies on the pattern of blood flow through vessels having non-uniform cross section or having constriction with reference to atherosclerosis, but most studies have been carried out by considering blood as a Newtonian fluid.

Magneto-hydrodynamics (MHD), as a field of science, involves the flow of electrically conducting physiological fluids under the action of exerted magnetic field. Physiological fluids are body fluids such as blood, semen, saliva etc. The magnetic fluids are a kind of physiological fluids, the flows of which are affected by magnetic fields.

Bio-magnetic fluid dynamics has many major applications such as Magnetic drug targeting, adjusting blood flow during surgery and transporting complex bio-waste fluids, cancer tumour treatment etc. Extensive research has been undertaken on the fluid dynamics of bio-magnetic fluids under the presence of an external magnetic field. The application of magneto-hydrodynamics in physiological flow is of growing interest as many researchers have reported that blood is an electrically conducting fluid [4].

The cardiovascular system is made up of blood cells, blood vessels and the heart. The main function of the heart is to pump blood into circulation, to the tissues and organs of the human body through the blood vessels. According to Bunonyo and Amos [17], blood an essential ingredient of the vitality of the body system, and the major constituents are the red blood cells (erythrocytes), the platelet and the plasma fluid. Blood contains haemoglobin which has magnetic properties that are different depending on the oxidation state of haemoglobin. The body contains Proteins, LDL cholesterol, which for increasing quantity in the cardiovascular system can build up in various arteries, clogging and reducing their flexibility. Hardening of the arteries results to an atherosclerosis and that restrict normal blood flow because of the loss of the vessel flexibility, so the heart works harder to push blood through to the downstream [18].

Das and Saha [1] studied the Arterial MHD pulsatile flow of blood under periodic body acceleration and observed a mathematical model for pulsatile flow of blood through a stenosed porous medium with periodic body acceleration under the influence of a uniform transverse magnetic field. Using finite Henkel and Laplace transforms, they obtained analytical expressions for velocity profile, volumetric flow rate and wall stress, their study is useful in various field of biomedical engineering. A mathematical model for blood flow in narrow capillaries under the effect of transverse magnetic field has been investigated by Jain *et al*. [2]. Rathod and Tanveer [3] studied a pulsatile flow of blood which is considered as a couple stress fluids through a porous medium under the influence of periodic body acceleration in the presence of magnetic field. They obtained the exact solution of the equation, analytical expressions for axial velocity, flow rate, fluid acceleration and shear stress by applying the Laplace and finite Hankel transforms.

An analytical solution of two-dimensional model of blood flow with variable viscosity through an indented artery due to low density lipoprotein effect in the presence of magnetic field was studied by Singh and Rathee [4]. Their approach offers an understanding of practical problem of blood flow through stenosed artery. The analytical expressions for radial velocity, axial velocity, shear stress and pressure gradient were obtained. The investigation shows that hypertensive patients are more adequate to have heart circulatory problems. Sanyal and Biswas [5] investigated the effect of uniform transverse magnetic field on pulsatile motion of blood through an Axi-symmetric Artery. They implored perturbation technique with small amplitude of pulsation. Zamir and Margot [6], studied Blood flow downstream of a two-dimensional bifurcation with a symmetrical steady flow. Their analysis considers the flow in a two-dimensional bifurcation with a symmetrical flow divider perfused with steady flow at variable Reynolds numbers. Eldesoky [7], analysed a mathematical model of unsteady blood flow through parallel plate channel under the action of an applied constant transverse magnetic field. Their model was analysed to find the effects of various parameters on the axial velocity, temperature distribution and the normal velocity.

Midya *et al*. [8] investigated Magnetohydrodynamic viscous flow separation in a channel with constrictions. They developed a solution technique for governing magnetohydrodynamic (MHD) equation in primitive variable formulation. They observed that with increase in the magnetic field, the flow separation zone is affected. Misra *et al*. [9] studied hydromagnetic flow of a second-grade fluid in a channel and developed an asymptotic series solution for steady flow of an incompressible, second grade electrically conducting fluid in a channel permeated by a uniform transverse magnetic field. Misra et al. [10] investigated the Flow of a bio magnetic viscoelastic fluid: application to estimation of blood flow in arteries during electromagnetic hyperthermia. Their study pertains to a situation where magnetization of the fluid varies with temperature. they considered blood as a bio magnetic fluid and attempted to analyse some parameters of blood flow by developing suitable numerical method and devising an appropriate finite difference scheme.

Mathematical modelling of blood flow in a porous vessel having double stenoses in the presence of an external magnetic field was investigated by Misra *et al*. [11]. They obtained expressions for the velocity profile, volumetric flow rate, wall shear stress and pressure gradient analytically under the purview of their model. Shit and Roy [12] investigated the effect of externally imposed body acceleration and magnetic field on pulsatile flow of blood through an arterial segment having stenosis. They solved numerically nonlinear equation that governs the flow using finite difference technique by employing a suitable coordinate transformation. The flow and heat transfer of MHD viscoelastic fluid in a channel with stretching walls was investigated by Misra *et al*. [13]. Their study showed that with the increase in the strength of the magnetic field, the fluid velocity decreases but the temperature increases. Maranhão [14], published an article on Lipoprotein (a): structure, pathophysiology, and clinical implications. From their research they observed that Sex and age have little influence on lipoprotein (a) levels and direct deposition of lipoprotein (a) on arterial wall is prone to oxidation than LDL. Sharma *et al*. [15] investigated the entropy generation in a natural convective flow of a viscous incompressible electrically conducting fluid between two infinite non-conducting inclined parallel plates channel filled with a porous medium in the presence of transverse magnetic field and heat source. Their investigation shows that an increment in the strength of the magnetic field declines the velocity of the fluid and prop-ups the rate of entropy generation. Murtaza *et al*. [16] worked on numerical solution of three-dimensional unsteady bio magnetic flow and heat transfer through stretching/shrinking sheet using temperature dependent magnetization. Their study constitutes an initial inside for all kinds of applications that deal with blood flow aiming to control the flow rate and rate of heat transfer such as magnetic drug targeting or/and magnetic hyperthermia.

Currently a theoretical mathematical model of blood flow through parallel plate with mass concentration has been carried out under the effect of external transverse magnetic field and heat source. This work is an extensive study of Eldesoky [7] with heat transfer under the conditions defined in our model. The main aim of this work is to obtain analytical expressions for axial velocity, temperature distribution, normal velocity, and concentration of mass lipid concentration using the same boundary conditions in [7] and with converting the system of partial differential equation into system of ordinary differential equation.

II. MATHEMATICAL FORMULATION

Eldosky [7] used a mathematical model to study the unsteady blood flow through a very narrow parallel plate channel with heat source and external transverse magnetic field is presented. His work is an extensive study of Madhu *et al*. [8] with heat transfer under the conditions defined in our model. His aim was to obtain analytical expressions for axial velocity, temperature distribution and normal velocity using new boundary conditions and with converting the system of partial differential equations into system of ordinary differential equations. However, in Eldesoky and Madhu research concentration was never considered.

The models formulated by Eldesoky to study the flow through a parallel channel are given in equations (1-2) using the diagram below (see Figure 1).

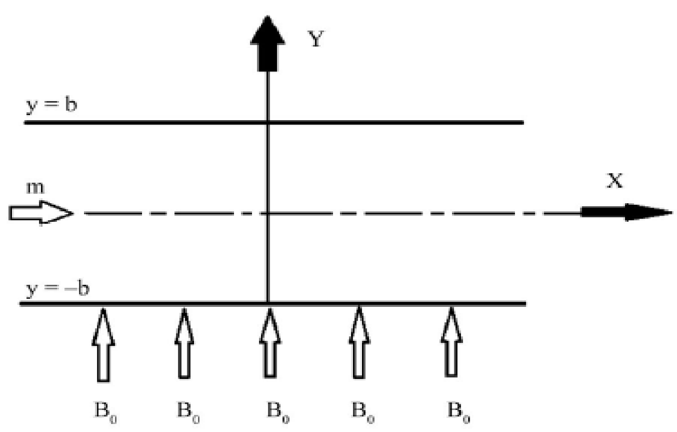

Figure 1 : Schematic diagram showing geometry of the model
Source: Eldesoky [7]

Blood Momentum Equation

$$\rho\frac{\partial\vec{w}}{\partial\vec{t}}=-\frac{\partial\vec{P}}{\partial\vec{x}}+\mu\frac{\partial^{2}\vec{w}}{\partial\vec{y}^{2}}-\sigma B_{0}^{2}\vec{w}+\rho g\beta_{T}(T-T_{0}) \quad (1)$$

Continuity Equation

$$\frac{\partial w}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2)$$

Heat Equation

$$\frac{\partial T}{\partial t}=\frac{k_{T}^{'}}{\rho c_{p}}\frac{\partial^{2}T}{\partial y^{2}}+\frac{Q}{\rho c_{p}}(T-T_{0}) \quad (3)$$

III. Proposed Formulation of the Problem

The significance of lipoprotein (lipid) in the blood can't be overemphasized, hence the need for this research. In this paper, we formulate mathematical model to study the effect of lipid concentration on an unsteady blood flow caused by the pumping rate of the heart through a parallel channel with heat and magnetic field intensity. In this study we aim to achieve mathematical expression for the blood velocity, temperature and lipid concentration within the specified boundary set out by Eldesoky. The research also aims to achieve the following objectives:

- To study the effect of magnetic field Hartmann number Ha and Schmidt number on blood velocity.
- Obtain analytical expressions for angular velocity, temperature distribution, concentration, and normal velocity.
- To study the effect of Prandtl number (Pr), Hartmann's number (Ha) and decay parameter λ on the axial velocity, normal velocity and temperature distribution in horizontal and vertical directions respectively.
- To study the effect of Schmidt number on concentration on the lipid and the blood velocity
- Validate the results by comparing it with that obtained by Eldesoky [7]

In formulating mathematical models to study the effect of lipid concentration on an unsteady blood flow through a parallel channel with heat and magnetic field intensity, we consider the following assumptions:

A. Research assumptions

1. Blood is Newtonian, incompressible, homogeneous, viscous and hybrid fluid, [18].
2. There is a constant viscosity of the fluid (blood containing lipoprotein)
3. Magnetic field is applied perpendicularly with intensity $\vec{B}=\left(0,0,B_{0}\right)$
4. Heat is applied through source to warm the area in order to encourage blood thining
5. Electromotive force is assumed to be zero Bunonyo *et al*. [17].

Consider w and v as the velocity components the axial and normal directions respectively at time t with initial and final temperature are T_0 and T and with initial and final concentration are C_0 and C.

Based on the aforementioned assumptions and using Figure 1, we present the modified set of models as follows:

B. Volumetric Expansion due to Lipid Concentration

$$\beta_C = -\frac{1}{\rho}\left(\frac{\rho - \rho_\infty}{C - C_\infty}\right) \tag{4}$$

Continuity Equation

$$\frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

Substituting equation (4) into equation (2), we obtain the momentum equation as:

Blood momentum Equation

$$\rho\frac{\partial \vec{w}}{\partial \vec{t}} = -\frac{\partial \vec{P}}{\partial \vec{x}} + \mu\frac{\partial^2 \vec{w}}{\partial \vec{y}^2} - \sigma B_0^2 \vec{w} + \rho g \beta_T (T - T_0) + \rho g \beta_C (\mathrm{C} - C_0) \tag{6}$$

Heat Equation

$$\frac{\partial T}{\partial t} = \frac{k_T^{'}}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_0) \tag{7}$$

Lipid Mass Concentration Equation

$$\frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} \tag{8}$$

Introducing the following non-dimensionless variables, Eldesoky [7]:

$$\left.\begin{aligned} & x^* = \frac{\vec{x}}{b},\ y = \frac{\vec{y}}{b},\ w^* = \frac{\vec{w}}{(m/2\rho b)},\ v^* = \frac{\vec{v}}{(m/2\rho b)},\ t^* = \frac{\vec{t}}{(\rho b^2/\mu)},\ h^*(x,t) = \frac{\partial p/\partial x}{(m/2\rho b)}, \\ & \theta^* = \frac{T(2\rho^2 b^3)}{\mu m},\ \phi^* = \frac{C(2\rho^2 b^3)}{\mu m},\ \upsilon = \frac{\mu}{\rho},\ Pr = \frac{\mu c_p}{k_T},\ N = \frac{Qb^2}{k_T},\ Ha = \sqrt{\frac{\sigma B_0^2 b^2}{\rho \upsilon}},\ Sc = \frac{\upsilon}{D_m} \end{aligned}\right\} \tag{9}$$

Using equation (9) in simplifying equations (5-8) after dropping the asterisks for the sake of mathematical simplification, we have the following

$$\frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$\frac{\partial w}{\partial t} = -h + \frac{\partial^2 w}{\partial y^2} - Ha^2 w + g\beta_T\theta + g\beta_C\phi \tag{11}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\upsilon Pr}\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\upsilon Pr}N\theta \tag{12}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \tag{13}$$

The corresponding boundary conditions are as follows:

$$\left.\begin{array}{ll} w=1,\ \theta=1,\ \phi=1, & \text{at } y=-1 \\ w=0,\ \theta=0,\ \phi=0, & \text{at } y=1 \end{array}\right\} \tag{14}$$

IV. Method of solution

Following Eldesoky [7], the solutions to equations (11)-(13) it can be written in the following form:

$$\left.\begin{array}{l} w = w_0(y)e^{-\lambda^2 t} \\ \phi = \phi_0(y)e^{-\lambda^2 t} \\ \theta = \theta_0(y)e^{-\lambda^2 t} \\ v = v_0(y)e^{-\lambda^2 t} \end{array}\right\} \tag{15}$$

Now, differentiate equation (15) according to the order in equation (31) to (35), we have

$$\frac{\partial w}{\partial t} = -\lambda^2 w_0 e^{-\lambda^2 t},\ \frac{\partial v}{\partial y} = v_0 e^{-\lambda^2 t},\ \frac{\partial \phi}{\partial t} = -\lambda^2 \phi_0 e^{-\lambda^2 t},\ \frac{\partial \theta}{\partial t} = -\lambda^2 \theta_0 e^{-\lambda^2 t} \tag{16}$$

$$\frac{\partial^2 w}{\partial y^2} = e^{-\lambda^2 t}\frac{\partial^2 w_0}{\partial y^2},\ \frac{\partial \phi}{\partial y} = e^{-\lambda^2 t}\frac{\partial^2 \phi_0}{\partial y^2},\ \frac{\partial \theta}{\partial t} = e^{-\lambda^2 t}\frac{\partial^2 \theta_0}{\partial y^2} \tag{17}$$

Using the following boundary conditions

$$\left.\begin{array}{ll} w_0=1,\ \theta_0=1,\ \phi_0=1, & \text{at } y=-1 \\ w_0=0,\ \theta_0=0,\ \phi_0=0, & \text{at } y=1 \end{array}\right\} \tag{18}$$

A. Analytical Solutions for Concentration Profile

Using equation (16) and equation (17) into the concentration equation (13), and after simplification, we obtain the follows ordinary differential equation:

$$\frac{\partial^2 \phi_0}{\partial y^2} + \lambda^2 Sc\phi_0 = 0 \quad (19)$$

Let $K^2 = \lambda^2 Sc$, so that equation (19), it becomes:

$$\frac{\partial^2 \phi_0}{\partial y^2} + K^2 \phi_0 = 0 \quad (20)$$

Solving the homogenous ODE in equation (20) by letting $\phi_0 = e^{m_1 y}$ so that the solution becomes

$$\phi_0(y) = C_3 \cos Ky + C_4 \sin Ky \quad (21)$$

Using the boundary conditions in equation (18) in order to obtain the constant coefficients in equation (21), we obtain the following:

$$\phi_0(y) = \frac{1}{2\cos K}\cos Ky - \frac{1}{2\sin K}\sin Ky \quad (22)$$

Thus the concentration profile in the fluid is obtained by substituting equation (22) into equation (15), we have:

$$\phi(y,t) = \left(\frac{1}{2\cos K}\cos Ky - \frac{1}{2\sin K}\sin Ky\right)e^{-\lambda^2 t}. \quad (23)$$

B. Analytical Solutions for Temperature Profile

In a similar vein, using equation (16) and equation (17) into the temperature equation (12), and after simplification, we obtain the follows ordinary differential equation:

$$\frac{\partial^2 \theta_o}{\partial y^2} + (\lambda^2 \upsilon Pr + N)\theta_o = 0 \quad (24)$$

Let $Q^2 = (\lambda^2 \upsilon Pr + N)$ then equation (24) becomes:

$$\frac{\partial^2 \theta_o}{\partial y^2} + Q^2 \theta_o = 0. \quad (25)$$

Solving equation (25), we let $\theta_o = e^{m_1 y}$ so that the solution to equation (25) becomes:

$$\theta_0(y) = C_1 \cos Qy + C_2 \sin Qy \quad (26)$$

Using the boundary conditions in equation (18) in order to obtain the constant coefficients in equation (26), we obtain the following:

$$\theta_0(y)=\frac{1}{2\cos Q}\cos Qy-\frac{1}{2\sin Q}\sin Qy \tag{27}$$

Hence, the temperature profile of the fluid is obtained by substituting equation (27) into equation (15), we have:

$$\theta(y,t)=\left(\frac{1}{2\cos Q}\cos Qy-\frac{1}{2\sin Q}\sin Qy\right)e^{-\lambda^2 t} \tag{28}$$

C. Analytical Solutions for Velocity Profile

In a like manner, using equation (16) and equation (17) into the blood velocity equation (11), and after simplification, we obtain the follows ordinary differential equation:

$$\frac{\partial^2 w_0}{\partial y^2}+w_0\xi^2=\hbar-g\beta_T\theta_0-g\beta_C\phi_0. \tag{30}$$

where $\xi=\sqrt{(\lambda^2-Ha^2)}$ and $\hbar=\frac{h}{e^{-\lambda^2 t}}$

In order to study the effect of lipoprotein concentration and temperature profiles on blood velocity, we substitute equation (22) and equation (27) into equation (30), so we obtain the following:

$$\frac{\partial^2 w_0}{\partial y^2}+w_0\xi^2=\hbar-g\beta_T\left(\frac{1}{2\cos Q}\cos Qy-\frac{1}{2\sin Q}\sin Qy\right)-g\beta_C\left(\frac{1}{2\cos K}\cos Ky-\frac{1}{2\sin K}\sin Ky\right) \tag{31}$$

Let $m_1=\frac{g\beta_T}{2\cos Q}$, $m_2=\frac{g\beta_T}{2\sin Q}$, $m_3=\frac{g\beta_C}{2\cos K}$, $m_4=\frac{g\beta_C}{2\sin K}$ so that equation (31) becomes:

$$\frac{\partial^2 w_0}{\partial y^2}+w_0\xi^2=\hbar-m_1\cos Qy+m_2\sin Qy-m_3\cos Ky+m_4\sin Ky \tag{32}$$

The complimentary solution of equation (32) is

$$w_{0c}=C_5\cos\xi y+C_6\sin\xi y \tag{33}$$

And the particular solution is:

$$w_{0p}=\frac{\hbar}{\xi^2}-\frac{m_1}{\xi^2-Q^2}\cos Qy+\frac{m_2}{\xi^2-Q^2}\sin Qy-\frac{m_3}{\xi^2-K^2}\cos Ky+\frac{m_4}{\xi^2-K^2}\sin Ky. \tag{34}$$

So that the solutions to equation (32) is

$$w_0(y)=\left\{C_5\cos\xi y+C_6\sin\xi y+\frac{\hbar}{\xi^2}-\frac{m_1}{\xi^2-Q^2}\cos Qy+\frac{m_2}{\xi^2-Q^2}\sin Qy-\frac{m_3}{\xi^2-K^2}\cos Ky+\frac{m_4}{\xi^2-K^2}\sin Ky\right\} \tag{35}$$

Solving for the constant coefficients in equation (35) using the boundary condition in equation (18), we have:

$$w_0(y)=\left(\begin{array}{l}\dfrac{1-2\left(\dfrac{\hbar}{\xi^2}\right)+\dfrac{g\beta_T}{\xi^2-Q^2}+\dfrac{g\beta_C}{\xi^2-K^2}}{2\cos\xi}\cos\xi y-\dfrac{1+\dfrac{g\beta_T}{\xi^2-Q^2}+\dfrac{g\beta_C}{\xi^2-K^2}}{2\sin\xi}\sin\xi y+\dfrac{\hbar}{\xi^2}\\ -\dfrac{g\beta_T}{2\cos Q\left(\xi^2-Q^2\right)}\cos Qy+\dfrac{g\beta_T}{2\sin Q\left(\xi^2-Q^2\right)}\sin Qy-\dfrac{g\beta_C}{2\cos K\left(\xi^2-K^2\right)}\cos Ky\\ +\dfrac{g\beta_C}{2\sin K\left(\xi^2-K^2\right)}\sin Ky\end{array}\right) \quad (36)$$

Thus, the blood velocity profile with the effect of lipid concentration and temperature is obtained after we substitute equation (360 into equation (15), which is:

$$w(y,t)=\left[\begin{array}{l}\dfrac{\hbar}{\xi^2}-\left(\dfrac{g\beta_T}{\xi^2-Q^2}\right)\dfrac{\cos(Qy)}{2\cos Q}+\left(\dfrac{g\beta_T}{\xi^2-Q^2}\right)\dfrac{\sin(Qy)}{2\sin Q}\\ -\left(\dfrac{g\beta_C}{\xi^2-K^2}\right)\dfrac{\cos Ky}{2\cos K}+\left(\dfrac{g\beta_C}{\xi^2-K^2}\right)\dfrac{\sin Ky}{2\sin K}+\left(1-2\left(\dfrac{\hbar}{\xi^2}\right)+\dfrac{g\beta_T}{\xi^2-Q^2}+\dfrac{g\beta_C}{\xi^2-K^2}\right)\dfrac{\cos\xi y}{2\cos\xi}\\ -\left(1+\dfrac{g\beta_T}{\xi^2-Q^2}+\dfrac{g\beta_C}{\xi^2-K^2}\right)\dfrac{\sin\xi y}{2\sin\xi}\end{array}\right]e^{-\lambda^2 t} \quad (37)$$

The normal velocity profile is obtained as:

$$v(t)=Ce^{-\lambda^2 t} \quad (38)$$

V. Numerical Results and Discussions

To study the effect of the various entering parameters on the velocity of the fluid, temperature of the fluid and lipid concentration of the fluid, we code the analytically solutions and simulate by varying the pertinent parameters effect. The parameters in focus are: heat source and magnetic field, Prandtl number Pr, decay parameter λ, the flow investigation was carried out. The objective of this project research was to study the effect of magnetic field Ha, heat source parameter, Prandtl number Pr, decay parameter and Schmidt number Sc on the normal velocity, axial velocity, temperature distribution and mass lipid concentration. We use Wolfram Mathematica to do the coding and simulation, and the results are presented as follows:

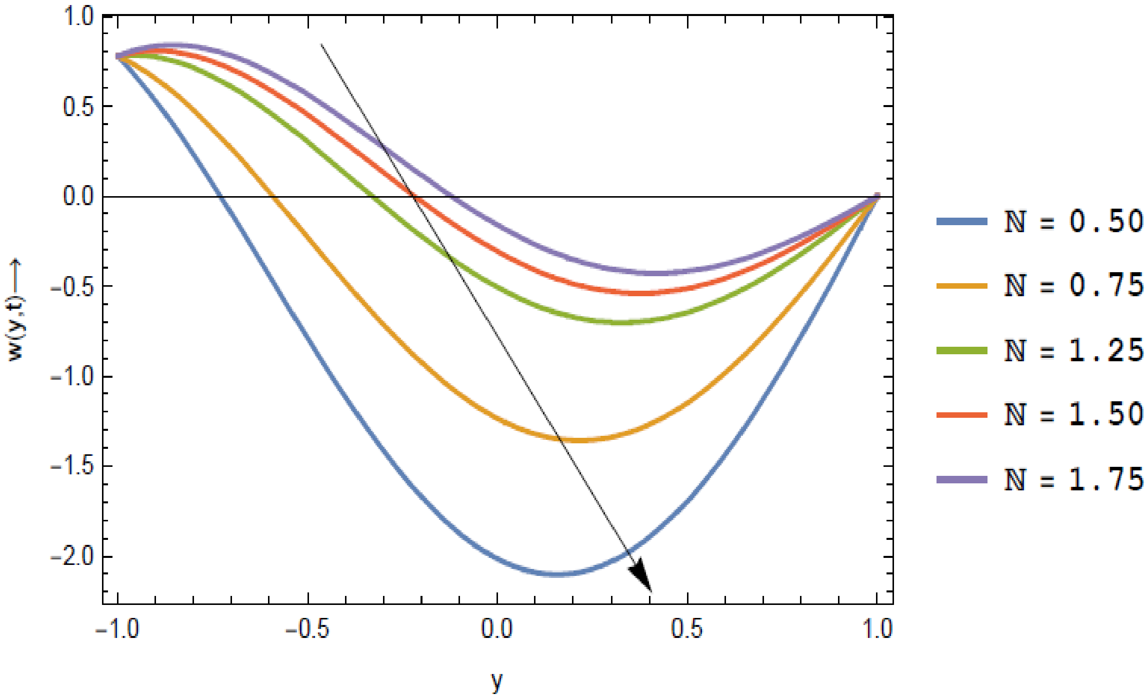

Figure 2: **Effect of Heat source parameter on axial Velocity**

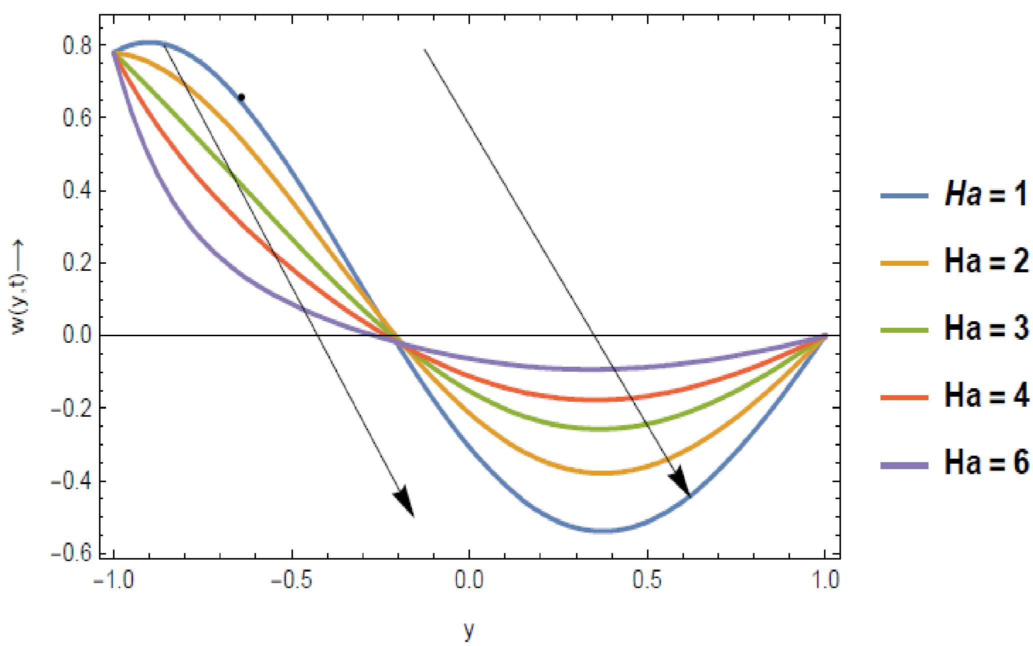

Figure 3: **Effect of Hartmann number on axial velocity**

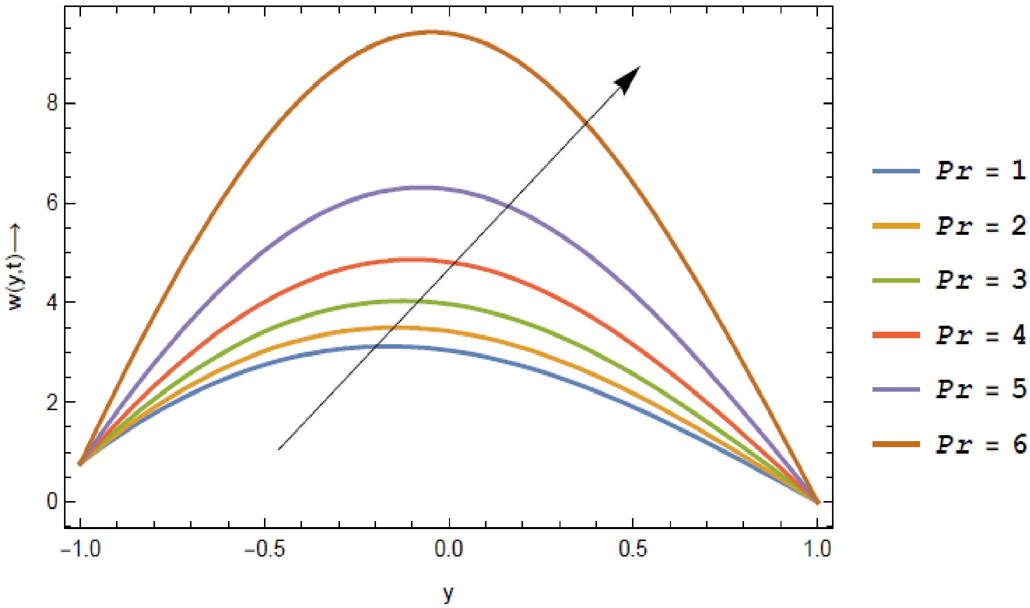

Figure 4: **Effect of different Prandtl number on axial velocity**

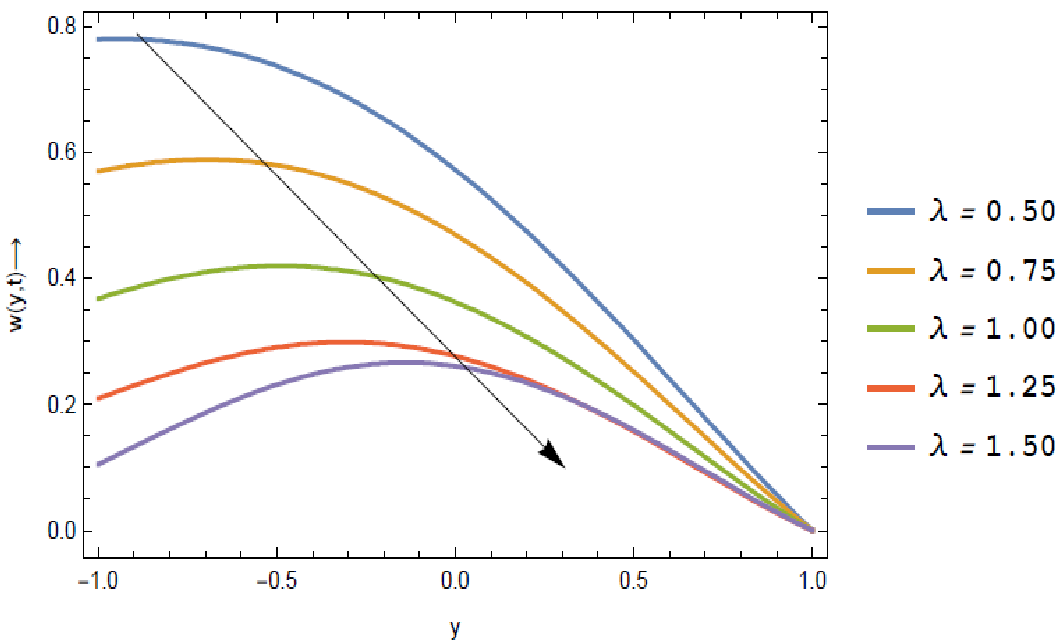

Figure 5: **Effect of different Decay parameter on axial velocity**

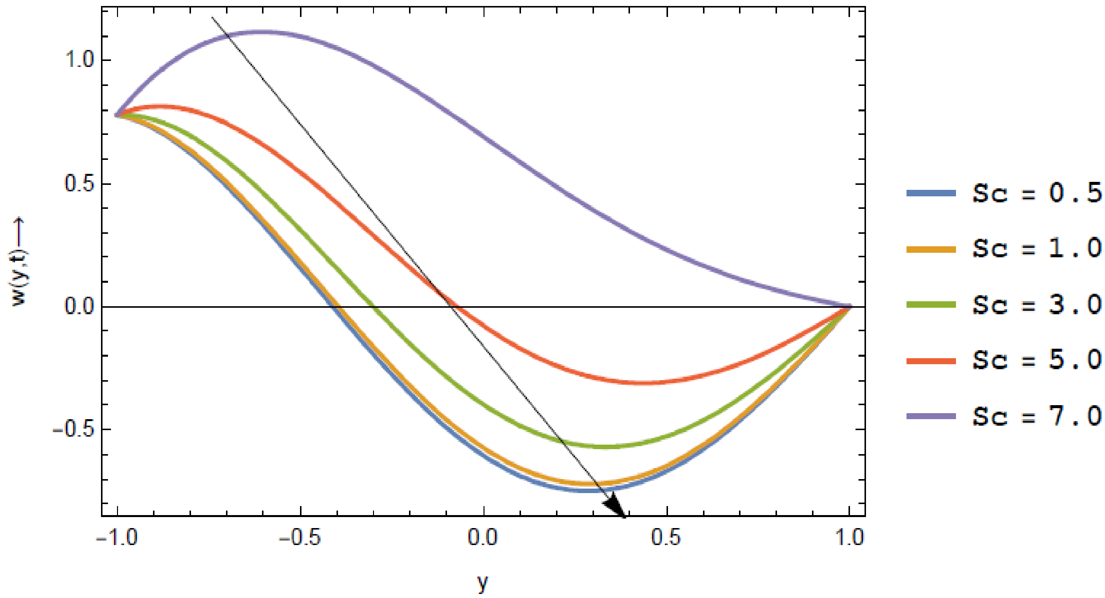

Figure 6: **Effect of Schmidt number on axial velocity**

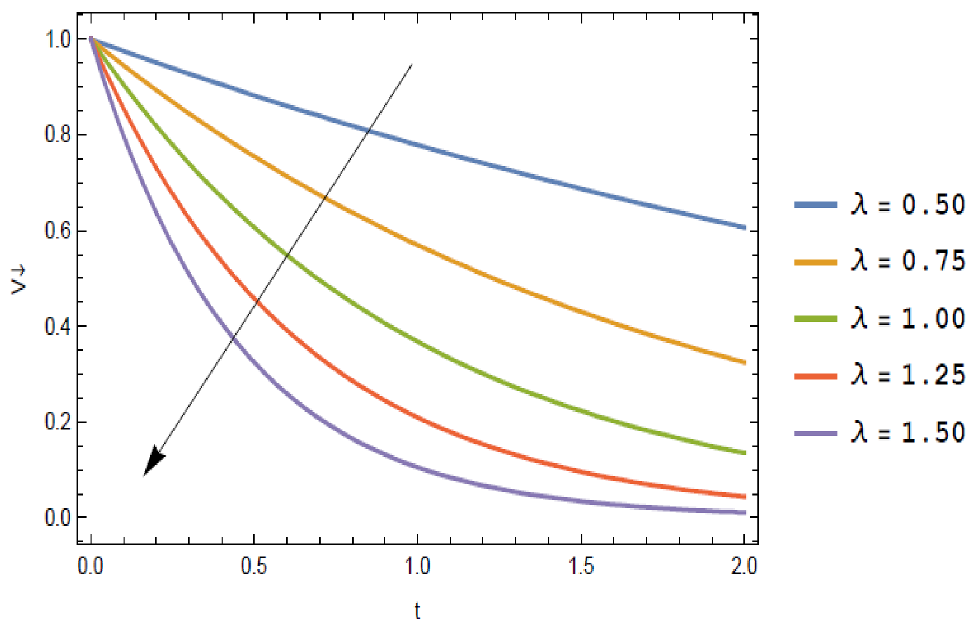

Figure 7: **Effect of different Decay parameter on Normal velocity**

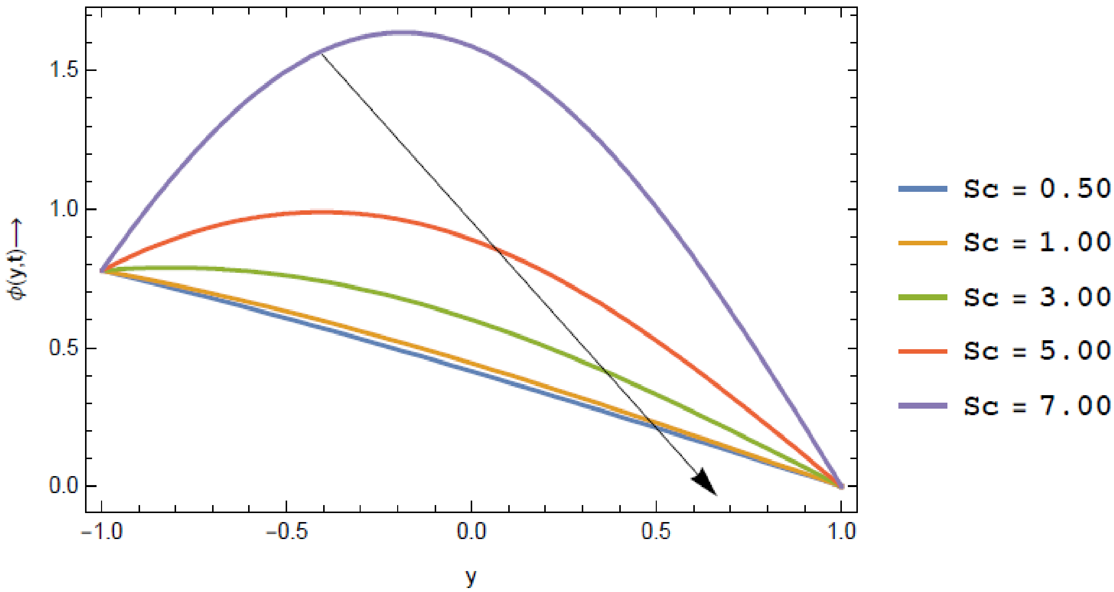

Figure 8: **Effect of different Schmidt number on Mass concentration**

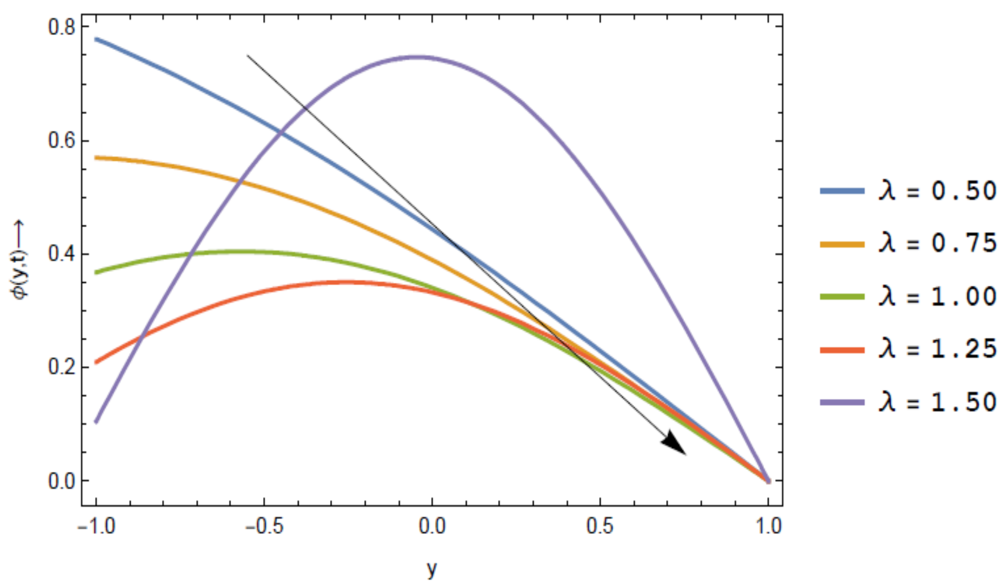

Figure 9: **Effect of different Decay parameter on Mass concentration**

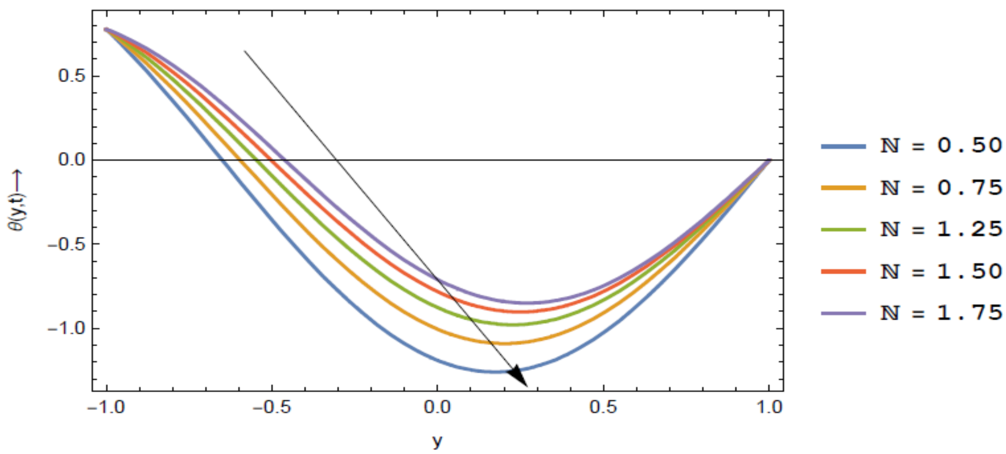

Figure 10 **Effect of Heat source parameter on temperature profile**

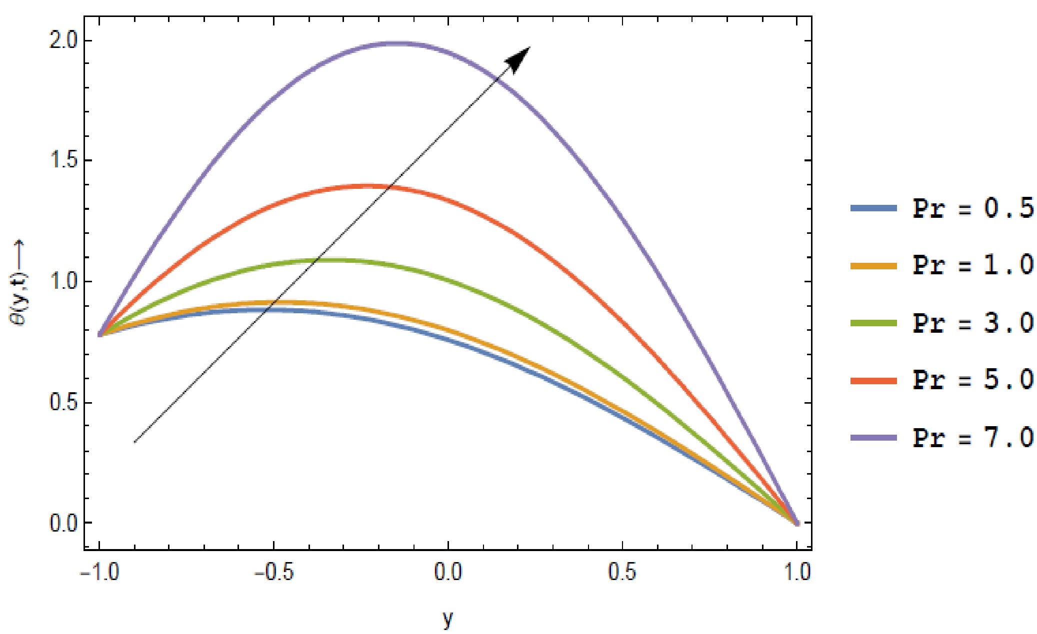

Figure 11 **Effect of Prandtl number on temperature profile**

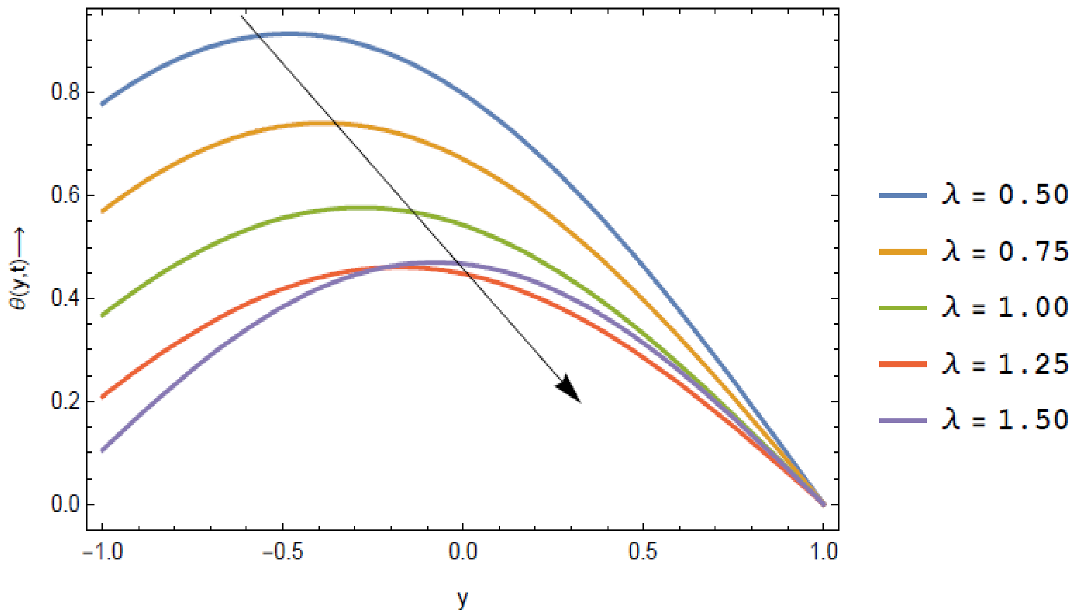

Figure 12 **Effect of Decay parameter on temperature profile**

In **Figure 2** we studied using different values of heat source on the axial velocity at $\lambda = 0.50, Pr = 21, \mu = 0.50, t = 1.00, Ha = 1.00, \beta_t = 0.50, \beta_c = 0.50, h = 0.50, Sc = 0.50, g = 9.81$. The values for Heat source used were $N = 0.50,\ 0.75,\ 1.25,\ 1.50, 1.75$. The axial velocity decreases for increasing values increase and reaches its minimum value at then it increases. In **Figure 3** we studied the axial velocity with different values of Hartmann number at $\lambda = 0.50, Pr = 21, \mu = 0.50, t = 1.00, \mathbb{N} = 1.50, \beta_t = 0.50, \beta_c = 0.50, h = 0.50, Sc = 0.50, g = 9.81$ and $Ha = 1,\ 2,\ 3,\ 4, 6$. It is observed that increasing the magnetic field decreases the axial velocity. At the axial velocity continues decreasing and at the axial velocity increases. **Figure 4** shows the effect of different values of Prandtl number at $\Pr = 1,\ 2,\ 3,\ 4,\ 5,\ 6$ at $\lambda = 0.50,\ Ha = 1.00, \mu = 0.50, t = 1.00, \mathbb{N} = 1.50, \beta_t = 0.50, \beta_c = 0.50, h = 0.50, Sc = 0.50, g = 9.81$.Itis observed that the axial velocity increases with increase in Prandtl number, reaches a maximum at $y = 0$ and then decreases.

Figure 5 explains the effect of decay parameter at $r = 1.00,\ Ha = 1.00, \mu = 0.50, t = 1.00, \mathbb{N} = 1.50, \beta_t = 0.50, \beta_c = 0.50, h = 0.50, Sc = 0.50, g = 9.81$. The maximum effect of the axial velocity with different values of decay parameter is at y=-1.0 and slowly decreases to y=1.0.The effect of Schmidt number is observed in **Figure 6** at using different values of $Pr = 1.00,\ Ha = 1.00, \mu = 0.50, t = 1.00, \mathbb{N} = 1.50, \beta_t = 0.50, \beta_c = 0.50, h = 0.50, \lambda = 0.50, g = 9.81$. The axial velocity slightly increases then decreases. The increase in the mass concentration leads to a slight increase of the axial velocity at start before decreasing. **Figure 7** shows the effect of different values of decay parameter on the normal velocity distribution. When C is arbitrary (C=1). The normal velocity decreases with increasing the decay parameter. Increase in decay parameter leads to a fast decrease in the normal velocity distribution.

Figure 8 shows the effect of Schmidt number on mass concentration for $Sc = 0.5,\ 1.0,\ 3.0,\ 5.0, 7.0$ at$\lambda = 0.50, t = 1.00$. For lower values of Schmidt number, we observe a decrease in mass concentration for the region defined on y and for higher values of Schmidt number we observe increase in mass concentration reaching maximum at y=-0.8 and then decreases to y=1.0. **Figure 9** then shows the effect of various values of decay parameter on mass concentration at $Sc = 1.00, t = 1.00$ where for $\lambda = 0.50$ we observe steady decrease in mass concentration and for higher values for λ we observe significant increase in mass concentration precisely at $\lambda = 1.50$ with maximum at y=0. In **Figure 10** we studied the temperature distribution with different values for heat source at $\lambda = 0.50, Pr = 21, \mu = 0.50, t = 1.00$ with $N = 1.00,\ 1.25,\ 1.50,\ 1.75,\ 2.00$. We observed a fall in temperature distribution with decreasing values of heat source.

Figure 11 gives the temperature distribution for different values of decay parameter on the temperature field at $\mathbb{N} = 1.00, Pr = 1.00, \mu = 0.50, t = 1.00$ with $\lambda = 0.50,\ 0.75, 1.00, 1.25, 1.50$. The temperature distribution reduces with increasing decay parameter and tends to zero at $\lambda = 2.50$. **Figure 12** gives the effect of Prandtl number on the temperature distribution at $\mathbb{N} = 1.00, \lambda = 0.50, \mu = 0.50, t = 1.00$with $\Pr = 0.50,\ 1.00,\ 3.00,\ 5.00,\ 7.00$ .Increasing Prandtl number increases the temperature distribution and achieves maximum at $y = 0$.

V. Conclusions

We have formulated mathematical models by modifying those in Eldesoky [7], to study the effect of lipid concentration effect on MHD blood flow through parallel plates with external magnetic field and heat source. In the study, the simulations for different profiles was done by varying the pertinent entering parameters, we found the following observations:

1. The study has been able to come up with an analytical expression for the various flow profiles such as the blood velocity, lipid concentration and temperature profiles using the specific boundary conditions.
2. Under close observation, the axial velocity increases with increasing values for Prandtl number but decreases with increasing heat source, decay parameter, Hartmann number and Schmidt number.
3. The normal velocity decrease with increasing decay parameter and tends to zero at higher values for the decay parameter.
4. The mass concentration increases with increasing Schmidt number and decay parameter.
5. The temperature field decreases with decreasing heat source, increasing decay parameter which tends to zero at high values of decay parameter and increases with increasing Prandtl number.

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Nomenclature

ρ : Density of blood

μ : Viscosity of the blood (constant)

σ : Electrical conductivity of the blood

B_0 :Intensity of the magnetic field

g : Gravitational acceleration

β : Coefficient of volume expansion

C : Concentration of the lipid in blood

C_0 : Concentration of the lipid at the wall

T : Temperature of blood

T_0 :Temperature of the wall (fixed temperature)

λ :Decay Parameter

Hartmann's Number $Ha = B_0 b\sqrt{\dfrac{\sigma}{\mu}}$

Heat Source Parameter $N = \dfrac{Qb^2}{k_T}$

Prandtl Number $Pr = \dfrac{\rho c_p}{k_T}$

Kinematic Viscosity $\upsilon = \dfrac{\mu}{\rho}$

Schmidt number $Sc = \dfrac{\upsilon}{D_m}$

Q : Quantity of heat

k_T : Coefficient of the thermal conductivity

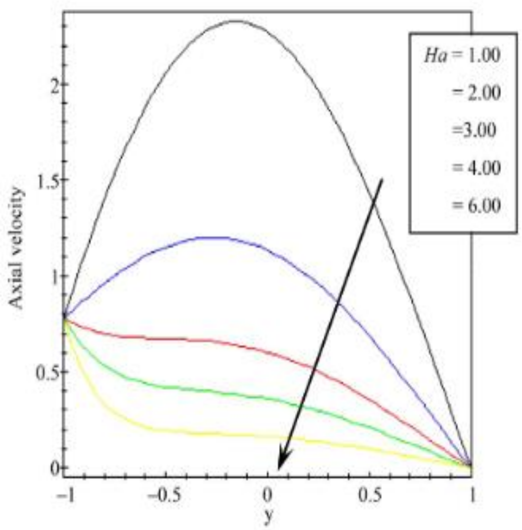
p : pressure gradient

D_m : molecular diffusion

θ : Dimensionless temperature distribution

ϕ : Dimensionless lipid concentration

Appendix



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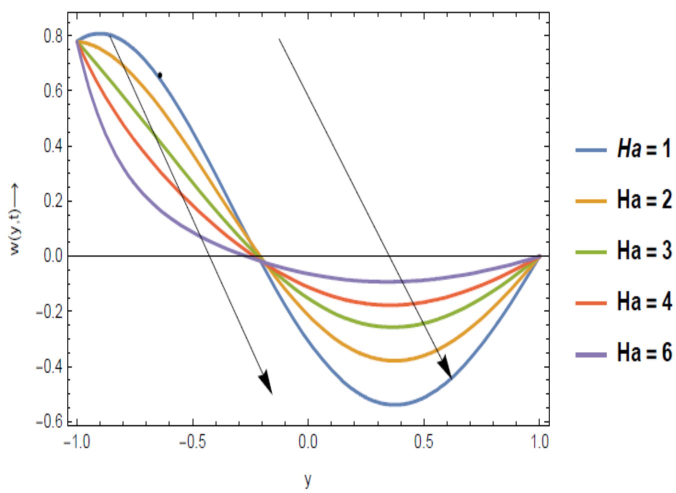


Figure B (Davies and Bunonyo)

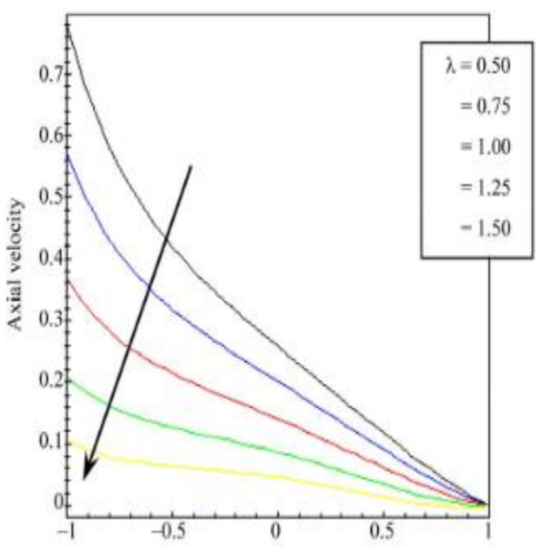
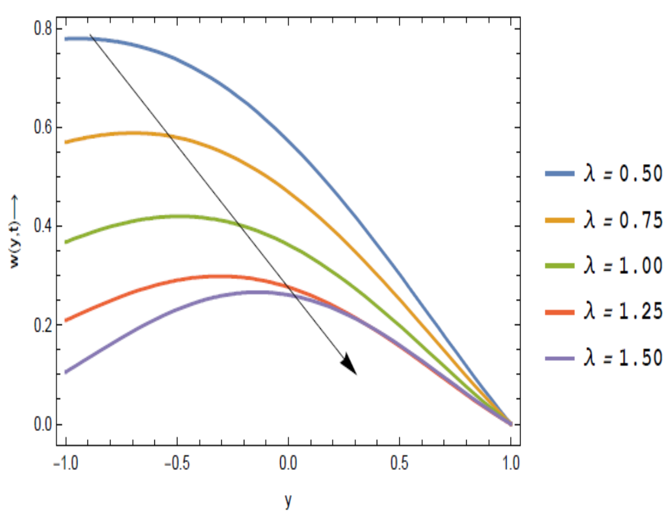


Figure A (Eldesoky [7])



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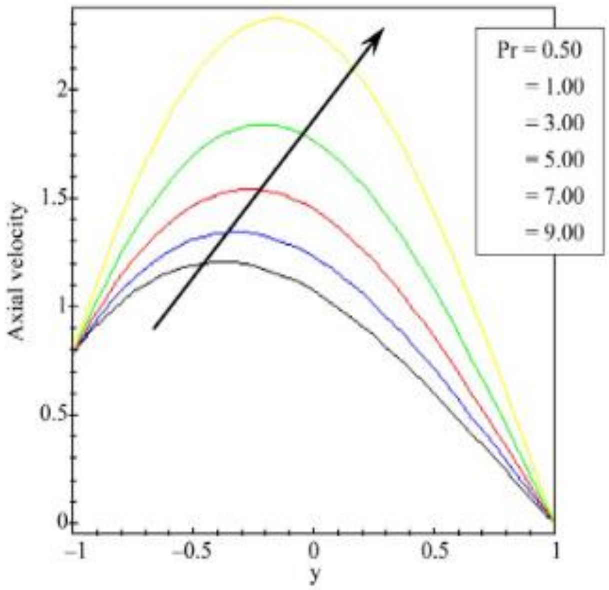

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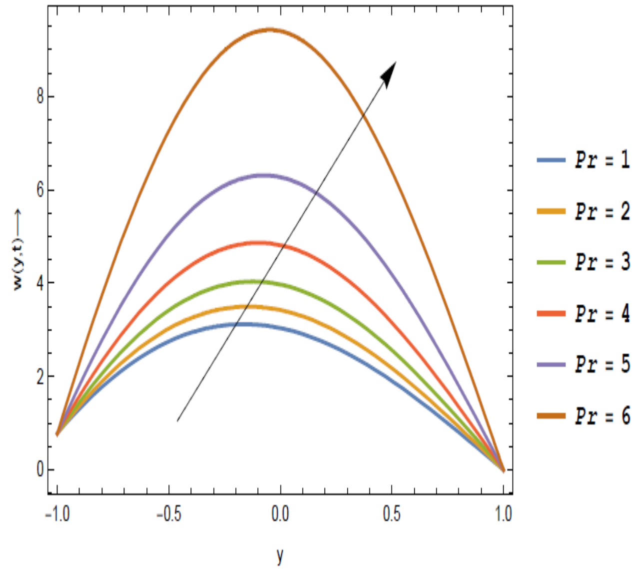
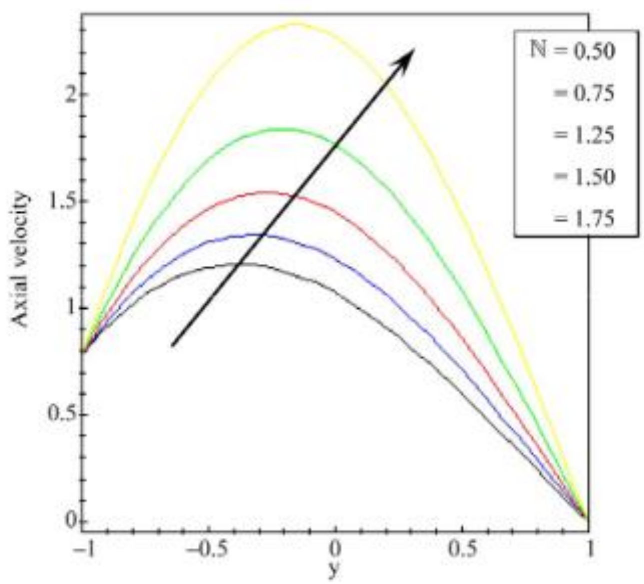


Figure B (Davies and Bunonyo)



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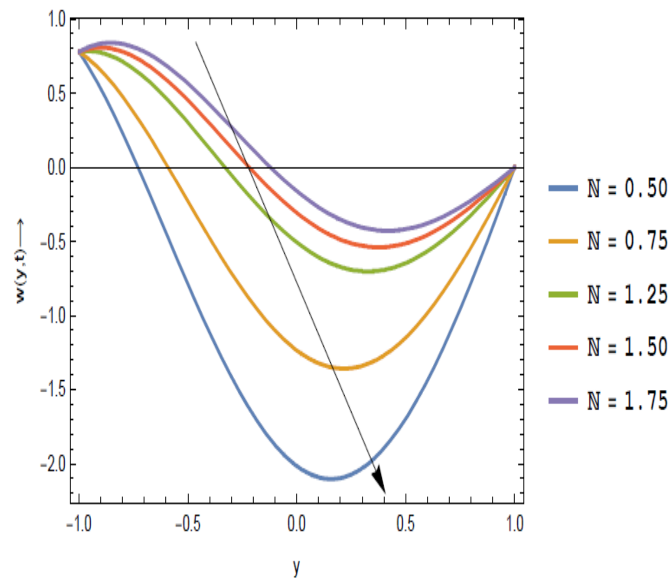

Figure B (Davies and Bunonyo)