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Relative Efficiency of Split-Plot Design To Randomized Designs

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Abstract *--- The aim of this study is to compare the efficiency of split plot design relative to completely randomized design, randomized complete block design and Latin square design using their error variances. The data collected on the soil variability and methods of soil preparation were analysed using SPSS software. The results from the analyses showed that split plot design was more efficient than completely randomized design, randomized completely block design and Latin square design for split-plot comparison and less efficient than completely randomized design and randomized completely block design but more efficient than Latin square design for whole-plot comparison. It was therefore concluded that for maximum leave area of African yam bean, the experiment should be performed using split-plot design instead of completely randomized design, randomized completely block design or Latin square design by assigning the levels of poultry manure to the split-plot and the levels of phosphorus application to the whole-plot.*

Keywords: *Experimental designs, Split plot, Whole plot, variability, Relative efficiency, Precision*.

I. INTRODUCTION

Experimental designs have been widely used for the purpose of controlling experimental errors: See [1]. Some of the natural variations among the set of experimental units are physically handled in these designs so as to contribute minimum to differences among treatment means: See [2]. There are various experimental designs available to meet the experimenters' requirements for different practical situations in order to control variability among experimental treatments: See [3]. The best design to use in any given situation is the one which provides the estimate of desired effects and contrasts with maximum precision (efficiency), and has a simple layout and analysis: See [4] and [2].

Good experimentation requires accurate planning, data collection, data analysis and interpretation. In agricultural field experimentation, the use of proper design plays an important role in attaining precision of results: See [5]. In order to minimize the experimental error, suitable experimental design has to be selected from many available designs to meet experimenter's requirements under different circumstances: See also [1]. The choice of any experimental design as well as of statistical analysis are of great importance in field experiments as it helps to obtain valid and reliable conclusion from field experiments: See [6]. Hence, the need to determine the efficiency of one design relative to another design in terms of time, cost and precision of the results of the experiment. Efficiencies are measures of goodness of the design of an experiment: See [1] and [7].

A split plot design is a crossed factor three-dimensional design that comprises the sub-plots called the replications or blocks, the whole-plots and the split plot treatments: See [8] and [9]. It is a design most adequate for situations where large experimental materials are required for the whole-plot unit and relatively small experimental materials are required for the sub-plot (split-plot) units. In constructing a split plot design, the whole plot can either be arranged in randomized complete block design RCBD form where both the replications and the whole plots are arranged as in the RCBD thereafter the split plots are randomized within the whole plot units or arranged in Latin square form where the number of replications is equal to the number of whole-plot treatments: See [10] and [11].

In this work, we are to compare the efficiency of split-plot design and some selected randomized designs such as completely randomized design (CRD), randomized complete block design (RCBD) and latin square design (LSD) in order to ascertain whether SPD is preferred over CRD, RCBD and LSD in determining the leave area of African yam bean using the error variances of the designs.

II. MATERIALS AND METHODS

The experiment was designed and conducted as a split-plot design (SPD) with the whole-plot treatments A (method of phosphorus application) arranged according to the Latin square design (LSD) and treatment B (poultry manure application) assigned to the sub-plot to determine the effectiveness of African yam bean. The leave area of the African yam bean was used as the response variable. The split-plot design was chosen due to the fact that poultry manure and phosphorus were in various levels and land preparation varies too.

The experiment was performed on a large portion of plot. Factor A (method of phosphorus application) which is the whole plot treatment was assigned such that each level appeared once in each row and column, forming the Latin square. Rows and columns are blocking variables representing methods of land preparation and three different types of soil (clay, sand and loam) respectively. These three blocks are due to different methods of land preparation which were done with potassium, urea and control. Then, each whole-plot is split to form sub-plot experimental units for the B treatments factor.

Table 1: Leave area of African yam bean

Soil Variability for whole plot (column) / Methods of final land preparation Split-plot (row)		
A	B	C
1=56	3=55	1=54
3=65	1=34	2=62
2=63	4=58	4=41
4=78	2=75	3=52
3=54	3=47	1=34
2=54	1=32	2=64
1=33	2=70	3=54
4=69	4=41	4=64
2=56	4=68	2=85
3=43	2=94	1=36
4=25	1=47	3=59
1=48	3=69	4=56

III. THE SPLIT-PLOT DESIGN (SPD) ARRANGED IN LATIN SQUARE DESIGN (LSD)

In this experiment and analysis, the whole-plot treatment is replicated three (3) times and the sub-plot treatment is not replicated. In SPD, we usually refer to 'with replication' when the sub-plot treatment is replicated. Consequently, this work assumes without replication.

The parameters involved are M and S, where M is the size of the Latin square and S is the number of levels of the sub-plot treatments. The construction is done as in the usual Latin square design where any suitable method can be used such as the cyclic rotation method, etc. The result of this construction is a three-stage randomization. First, the row is randomized, second the column. Third, the whole-plot treatment is randomly assigned to the whole-plots and finally, the sub-plots treatments are randomly assigned to the sub-plots.

IV. MODEL AND DEFINITION OF VARIABLES

$Y_{ijkl} = \mu + a_i + b_j + c_k + d_{ijk} + t_l + m_{kl} + \varepsilon_{ijk\,l;\; i=1,2,3, j=1,2,3, k=1,2,3, \; l=1,2,3,4}$

where:

Y_{ijkl} is the observed yam leave area from the k^{th} level of method of phosphorus of the i^{th} level of land preparation, j^{th} soil variability and l^{th} level of poultry manure;

μ is the overall mean or the universal constant;

a_i is the effect of i^{th} methods of land preparation (row);

b_j is the effect of the j^{th} soil variability (column);

c_k is the effect of k^{th} method of phosphorus application (whole-plot);

d_{ijk} is the whole-plot error

t_l is effect of the l^{th} level of poultry manure (sub-plot);

m_{kl} is the interaction between the k^{th} method of phosphorus application and the l^{th} poultry manure;

ε_{ijkl} is the random error associated with the observed response Y_{ijkl} (split-plot error)

Table 2: ANOVA table of SPD arranged in Latin square

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-ratio	P-value
Land preparation (a_i)	m-1	SS_a	MS_a	$\frac{MS_a}{MS_d}$	
Soil variability (b_j)	m-1	SS_b	MS_b	$\frac{MS_b}{MS_d}$	
Phosphorus (c_k)	m-1	SS_c	MS_c	$\frac{MS_c}{MS_d}$	

Whole-plot error (d_{ijk})	(m-1)(m-2)	SS_d	MS_d		
Poultry manure (t_l)	(s-1)	SS_t	MS_t	$\frac{MS_t}{MS_e}$	
Whole-plot & sub-plot interaction (m_{kl})	(s-1)(m-1)	SS_m	MS_m	$\frac{MS_m}{MS_e}$	
Error (e_{ijkl})	fe=m(m-1)(s-1)	SS_e	MS_e		
Total	$sm^2 - 1$	SS_T			

V. COMPARISON OF EFFICIENCIES OF SPD TO SOME RANDOMIZED DESIGNS

The information in any design is given by $\frac{1}{MS_{error}}$. The relative efficiency of a design say A to design B denoted as RE (A : B) is defined as the ratio of the information in design A to the information in design B

$$RE\ (A:B) = \frac{1}{MS_{error(A)}} \div \frac{1}{MS_{error(B)}} = \frac{MS_{error(B)}}{MS_{error(A)}}$$

In this comparison, the ratio of error variances will be adopted as a measure of relative efficiencies. Consequently, the ratio of MSE of SPD to MSE of CRD, RCBD and LSD will be used in obtaining the relative efficiency of SPD to CRD, RCBD AND LSD respectively. A correction factor (k) will be used in case where the error degrees of freedom for both designs are less than 20.

The relative efficiency of the sub-plot and whole-plot of the SPD to CRD, RCBD, LSD will be calculated.

(i). Relative efficiency of SPD to CRD denoted by RE (SPD : CRD) is defined as

$$RE\ (SPD:CRD)_{(s-p)} = \frac{\hat{\sigma}^2_{CRD}}{\hat{\sigma}^2_{SPD_{(s-p)}}} = \frac{MSE_{CRD}}{MSE_{SPD_{(s-p)}}} \quad \text{(split-plot)}$$

$$RE\ (SPD:CRD)_{(w-p)} = \frac{\hat{\sigma}^2_{CRD}}{\hat{\sigma}^2_{SPD_{(w-p)}}} = \frac{MSE_{CRD}}{MSE_{SPD_{(w-p)}}} \quad \text{(whole-plot)}$$

(ii). Relative efficiency of SPD to RCBD denoted by RE (SPD : RCBD) is defined as

$$RE\ (SPD:RCBD)_{(s-p)} = \frac{\hat{\sigma}^2_{RCBD}}{\hat{\sigma}^2_{SPD_{(s-p)}}} = \frac{MSE_{RCBD}}{MSE_{SPD_{(s-p)}}} \quad \text{(split-plot)}$$

$$RE\ (SPD:RCBD)_{(w-p)} = \frac{\hat{\sigma}^2_{RCBD}}{\hat{\sigma}^2_{SPD_{(w-p)}}} = \frac{MSE_{RCBD}}{MSE_{SPD_{(w-p)}}} \quad \text{(whole-plot)}$$

(iii). Relative efficiency of SPD to LSD denoted by RE (SPD : LSD) is defined as

$$RE\ (SPD:LSD)_{(s-p)} = \frac{\hat{\sigma}^2_{LSD}}{\hat{\sigma}^2_{SPD_{(s-p)}}} = \frac{MSE_{LSD}}{MSE_{SPD_{(s-p)}}} \quad \text{(split-plot)}$$

$$RE\ (SPD:LSD)_{(w-p)} = \frac{\hat{\sigma}^2_{LSD}}{\hat{\sigma}^2_{SPD_{(w-p)}}} = \frac{MSE_{LSD}}{MSE_{SPD_{(w-p)}}} \quad \text{(whole-plot)}$$

where,

$\hat{\sigma}^2_{CRD}$ is the error variance of CRD;

$\hat{\sigma}^2_{RCBD}$ is the error variance of RCBD;

$\hat{\sigma}^2_{LSD}$ is the error variance of LSD;

$\hat{\sigma}^2_{SPD_{(s-p)}}$ is the error variance of SPD with spilt-plot comparison;

$\hat{\sigma}^2_{SPD_{(w-p)}}$ is the error variance of SPD with whole-plot comparison;

MSE_{CRD} is an estimate of the error variance of CRD;

MSE_{RCBD} is an estimate of the error variance of RCBD;

MSE_{LSD} is an estimate of the error variance of LSD;

$MSE_{SPD_{(s-p)}}$ is an estimate of the error variance of SPD with split-plot comparison;

$MSE_{SPD_{(w-p)}}$ is an estimate of the error variance of SPD with whole-plot comparison;

An estimate of the error variance of CRD, RCBD and LSD will be obtained as follows; since experiments were not performed on these designs.

(i). **For CRD**

$$MSE_{CRD} = \frac{(m-2)MSE_{(W-P)} + m(s-1)MSE_{(S-P)}}{ms} \quad \text{(split-plot comparison)}$$

$$MSE_{CRD} = \frac{(m-2)MSE_{(S-P)} + m(m-1)(s-1)MSE_{(W-P)}}{ms} \quad \text{(whole-plot comparison}$$

$$k = \frac{(n_{SPD}+1)(n_{CRD}+3)}{(n_{SPD}+3)((n_{CRD}+1))};$$

where,

n_{CRD} = error degree of freedom for CRD;

n_{SPD} = error degree of freedom for SPD (ii). **For RCBD**

$MSE_{RCBD} = \frac{(m-2)MSE_{(W-P)}+m(s-1)MSE_{(S-P)}}{ms-1}$ (split-plot comparison)

$MSE_{RCBD} = \frac{(m-2)MSE_{(S-P)}+m(m-1)(s-1)MSE_{(W-P)}}{ms-1}$ (whole-plot comparison)

$k = \frac{(n_{SPD}+1)(n_{RCBD}+3)}{(n_{SPD}+3)((n_{RCBD}+1))}$; n_{RCBD} = error degree of freedom for RCBD

(iii). **For LSD**

$MSE_{LSD} = \frac{(m-2)MSE_{W-P}+m(s-1)MSE_{S-P}}{ms}$ (split-plot comparison)

$MSE_{LSD} = \frac{(m-2)MSE_{S-P}+m(m-1)(s-1)MSE_{W-P}}{m(s-1)}$ (whole-plot comparison)

The correction is given by

$k = \frac{(n_{SPD}+1)(n_{LSD}+3)}{(n_{SPD}+3)((n_{LSD}+1))}$; n_{LSD} = error degree of freedom for LSD

VI. RESULTS OF DATA ANALYSES

Table 3: Analysis of variance table

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-ratio	P-value
Land preparation (a_i)	2	302.1667	151.0834	0.2117	0.8253
Soil variability (b_j)	2	90.1667	45.0834	0.0632	0.9406
Phosphorus (c_k)	2	381.5	190.74	0.2673	0.7891
Whole-plot error (d_{ijk})	2	1427.1666	713.5833		
Poultry manure (t_l)	3	914.9722	304.9907	1.5728	0.2306
Whole-plot & sub-plot interaction (m_{kl})	6	2108.2778	351.3796	1.8120	0.1532
Error (e_{ijkl})	18	3490.5	193.9167		
Total	35	8414.75			

The following table shows the summary of the results obtained in the analyses of this experiment.

Table 4: Summary table for RE (SPD: CRD/RCBD/LSD)

Designs	**Split-plot comparison**	**Whole-plot comparison**
CRD	123%	60%
RCBD	146%	71%
LSD	212%	103%

VII. DISCUSSION OF RESULTS

The results of the relative efficiencies computed in this work revealed the following:

(1) RE (SPD_{s-p}: CRD) equals 123% and RE (SPD_{w-p}: CRD) equals 60% meaning that SPD is more efficient than completely randomized design (CRD) for sub-plot comparison but less efficient than CRD for whole plot comparison.

(2) RE (SPD_{s-p}: RCBD) equals 146% and RE (SPD_{w-p}: RCBD) equals 71% meaning that SPD is more efficient than randomized complete block design (RCBD) for sub-plot comparison but less efficient than RCBD for whole-plot comparison,

(3) RE (SPD_{s-p}: LSD) was 212% and RE (SPD_{w-p}: RCBD) was 103% meaning that SPD is more efficient than Latin square design (LSD) for both sub-plot comparison and whole-plot comparison.

In summary, Split Plot Design (SPD) is more efficient than CRD, RCBD and LSD based on split-plot comparison and less efficient than CRD and RCBD while being more efficient than LSD for whole-plot comparison.

VIII. Conclusion

One of the factors to be considered by any researcher is the simplicity and time availability for an experiment. Another yet more important factor considered by experienced researchers is the precision of results obtained from such experiment. A reasonable thing to do is to adopt a design that will lead to the best precision of result, hence efficient design

In computing the relative efficiency of SPD to CRD, RCBD or LSD, interest is on the number of replicates required by CRD, RCBD or LSD to achieve the same result as one replicate of SPD. Relative efficiency can be expressed in terms of percentage of replicates required by the second design to achieve the same result as one replicate of the first design by multiplying it by 100%. If the RE (SPD: CRD/RCBD/LSD) is greater than 100%, it implies that SPD is more efficient than CRD, RCBD or LSD and vice versa. Hence, the experiment could have been performed using CRD, RCBD or LSD.

Based on the results of the experiment, we therefore conclude that for maximum leave area of African yam bean, the experiment should be performed using SPD instead of CRD, RCBD or LSD by assigning the levels of poultry manure to the split-plot and the levels of phosphorus application to the whole-plot.

It is therefore recommended that in future study of the effect of phosphorus application on the leave area of African yam bean, CRD or RCBD should be preferred over SPD in a study where the levels of phosphorus application have to be applied to the whole-plot of a SPD.

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