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Numerical Solution of Unsteady Magneto Hydro Dynamic Flow Along Vertical Porous Plat Plate

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Abstract:

The present work is confined to obtain the numerical solution of an unsteady flow of viscous incompressible fluid along vertical porous plat plate with Hall current subjected to a time dependant transpiration velocity. A magnetic field is imposed in the direction perpendicular to the flow. Galerkin finite element method is used to solve the non-linear boundary value problem. The parametric analysis is made by the aid of Graphical results for velocity, temperature and concentration. The results obtained are in fine agreement with realistic physical phenomenon.

Keywords

*Galerkin Finite-element Technique***,** *Magnetic field, Effect of Hall current, Diffusion thermo effects;*

I. INTRODUCTION

Owing to the rotation and drift of the charged particles, the conductivity declines parallel to the electric field and current is induced during a line perpendicular to both electric as well as the magnetic field. This phenomenon is called as the "Hall Effect". Hall current effect on heat and mass transfer flow has applications in hydro magnetic power generators and in meteorological field. Because of these applications, the study of effect of Hall current is given by Cowling [1]. Couette heat transfer flow in the presence of magnetic field is discussed by Soundalgekar and Uplekar [2]. Flow between two parallel plates in the presence of magnetic field is studied by Hiroshi Sato [3]. Masakazu Katagiri [4] has made an investigation, a stable, narrow layer through an infinite magnetic flux, taking under consideration the outcomes of the present hole. On the other hand, Hossain [5] discussed the fluid flow that cannot be maintained along a vertical porous plate that is characterized by a similar suction / injection rate $(time)^{-\frac{1}{2}}$. Hossain *et al* [6] studied an unsteady free-convection flow of a viscous incompressible fluid with mass transfer along a vertical porous plate with hall current subjected to a time dependant transpiration velocity in the presence of transverse magnetic field. Several authors [7-14] have dealt the unsteady heat transfer flows on different geometry and considering various flow conditions. Ajay Kumar Singh [15] made an effort to review the steady free-convection and mass transfer flow with Hall current, viscous dissipation and joule heating, taking under consideration the thermal diffusion and magnetic field effects. Sriramulu *etal* [16] analyzed the effect of Hall current on hydro magnetic heat and mass transfer flow along a porous plate. In addition to this, Srihari *etal* [17] analyzed the effects of Source/Sink on free-convective mass transfer flow along an infinite vertical porous plate with hall current. Sharma and Chaudhary [18] studied the effect of Hall current on hydro magnetic unsteady mixed convective mass transfer flow past a vertical porous plate. Satyanarayana et al. [19] studied the steady mass transfer flow past a semi infinite vertical porous plate in the presence of magnetic field by considering the effects of Hall current. Shankar Goud., Raja Shekar [20] has made a Finite element attempt on heat and mass transfer flow through a porous medium taking Soret and radiation effects in to account. Shankar Goud *et al* [21] made a Finite element analysis on unsteady flow of Casson fluid past a vertical oscillating plate in the presence of magnetic field. Effects of thermal diffusion on hydro magnetic Jeffrey fluid flow along a vertical permeable moving plate is analyzed by Pramod Kumar *et al* [22]. Recently Shankar Goud [23] studied flow of a micropolar fluid flow through a porous medium in the presence of variable suction/injection and magnetic field.

Because of the coupled non-linearity of the problem, the present study is confined to obtain the numerical solution of heat and mass transfer flow past an infinite vertical porous plate subjected to choosing a time dependant transpiration velocity condition. Magnetic field is imposed normal to the flow. To get an approximate solution and to know the physics of the problem, the present non-linear boundary value problem is solved using Galerkin finite element method.

II. Mathematical Formulation

An unsteady free-convection flow of an electrically conducting viscous incompressible fluid with mass transfer along an infinite vertical porous plate isconsidered. The flow is assumed to be in x' - direction, which is taken along the plate in upward direction. The y' axis is taken to be perpendicular to the plate. At time $t \geq 0$, the temperature and the species concentration at the plate are raised to $T_w\ (\neq T_\infty)$ *and* $C_w\ (\neq C_\infty)$ and are maintained uniform thereafter. It is also supposed that the species concentration level is very low and hence species thermal diffusion in addition to diffusion thermal energy effects are neglected. A magnetic field of uniform strength is supposed normal to the porous plate. The magnetic Reynolds number of the flow is taken to be small adequate so that the induced magnetic field can be ignored. The equation of conservation of electric charge $\nabla . J = 0$ *gives* j_y =constant, where $J = (j_x,\ j_y,\ j_z)$ .It is further considered that the plate is non-conducting. This gives $j_y = 0$ at the plate and hence zero everywhere. When the strength of magnetic field is very large the generalized Ohm's law, in the absence of electric field takes the following form:

$$J + \frac{\omega_e \tau_e}{B_0} J \times B = \sigma\left(\mu_e V \times B + \frac{1}{en_e} \nabla P_e\right) \tag{1}$$

Where V is the velocity vector, σ is the electric conductivity, μ_e is the magnetic permeability, ω_e is the electron frequency, τ_e is the electron collision time, e is the electron charge, n_e is the number density of the electron and P_e is the electron pressure. Under the assumption that the electron pressure (for weakly ionized gas), the thermo-electric pressure and ion-slip are negligible, equation (1) becomes:

$$J_x = \frac{\sigma \mu_e B_0}{1+m^2}(mu - w) \text{ and } j_z = \frac{\sigma \mu_e B_0}{1+m^2}(u + mw) \tag{2}$$

Where u is the x component of V, w is the z component of V and $m(= w_e \tau_e)$ is the Hall parameter.

Within the above framework, the problem is governed by the following non-dimensional equations under the usual Boussinesq approximations;

$$v_{yy} = 0 \tag{3}$$

$$u_t + v u_y = u_{yy} - \frac{M}{1+m^2}(u + mw) + Gr\theta + GmC \tag{4}$$

$$w_t + vw_y = w_{yy} - \frac{M}{1+m^2}(w - mu) \tag{5}$$

$$\theta_t + v\theta_y = \frac{1}{\Pr}\theta_{yy} + DuC_{yy} \tag{6}$$

$$C_t + vC_y = \frac{1}{Sc}C_{yy} \tag{7}$$

with corresponding boundary conditions

$t \le 0$: $u=0,\ w=0,\ T=0, C=0$ for all y

$$t>0: \begin{cases} u=0,\ w=0,\ \theta=1,\ C=1 & at\ y=0 \\ u=0,\ w=0,\ \theta=0,\ C=0 & as\ y\to\infty \end{cases} \tag{8}$$

From equation (3), it is seen that v is either constant or a function of time t. Similarity solutions of equations (4)-(7) with the boundary conditions (8) exist only if we choose $v = -\lambda t^{\frac{-1}{2}}$,

Where, $t = \frac{t' U_0^2}{\upsilon},\ y = \frac{y' U_0}{\upsilon},\ u = \frac{u'}{U_0},\ v = \frac{v'}{U_0},\ w = \frac{w'}{U_0}$, $\theta = \frac{(T - T_\infty)}{(T_w - T_\infty)},\ C = \frac{(C' - C_\infty)}{(C'_w - C_\infty)}$,

$Du = \frac{D_m k_T (C_w - C_\infty)}{\upsilon C_S\, C_P (T_w - T_\infty)}$, $M = \frac{\sigma \mu_e^2 B_0^2 \upsilon}{\rho U^2}$, $\Pr = \frac{\mu c_p}{k}$, $Sc = \frac{\upsilon}{D}$, $Gr = \frac{\upsilon g \beta (T_w - T_\infty)}{U^3}$,

$Gm = \frac{\upsilon g \beta^* (C_w - C_\infty)}{U^3}$

Gm-Modified, Gr-Grashof number, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the coefficient of volume expansion with species concentration, T is the temperature of the fluid within the boundary layer, C' is the species concentration, $\rho,\ \mu,\ \upsilon,\ k,\ c_p$ are respectively density, viscosity, kinematics viscosity, thermal conductivity, specific heat at constant pressure and D is the chemical molecular diffusivity. For suction, $\lambda > 0$ and for blowing $\lambda < 0$.

III. METHOD OF SOLUTION

Implementation of the finite element method

This method has the following steps:

(1) Division of the domain into linear elements, called the finite element mesh.

(2) Creation of the element equations using variational formulations.

(3) Assembly of element equations as obtained in step (2).

(4) Imposition of the boundary conditions to the equation obtained in (3).

(5) Solution of the assembled algebraic equations.

In this perspective the unstable, nonlinear, coupled, partial differential equations (4-7) are numerically derived from the limit conditions (8), with the aid of Galerkin finite element method (5) over element (e). $(y_j \le y \le y_k)$:

Element (e) $(y_j \le y \le y_k)$ is:

$$\int_{y_j}^{y_k} N^{(e)}\left[\omega_{yy}^{(e)} - \omega_t^{(e)} - M_1\omega^{(e)} + R\right] \qquad \ldots (9)$$

Where $M_1 = \left(M + \frac{1}{m^2}\right), R = \left(\frac{Mu}{1+m^2}\right)$ and applying bi-parts to the equation (9) and ignoring the first term, neglecting the initial term by taking the linear approach to the solutions with the aid of the some basis functions. Then after the applying the element equations assembled for the previous two successive components $(y_{i-1} \le y \le y_i)$ and $(y_i \le y \le y_{i+1})$: one gets the following

$$\frac{1}{l^{(e)2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}\omega_{i-1}\\ \omega_i\\ \omega_{i+1}\end{bmatrix} + \frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{\omega}_{i-1}\\ \dot{\omega}_i\\ \dot{\omega}_{i+1}\end{bmatrix} - \frac{M_1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\omega_{i-1}\\ \omega_i\\ \omega_{i+1}\end{bmatrix} = \frac{R}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \qquad \ldots (10)$$

Now set row related to the node i to zero, from equation (10) the various models with $l^{(e)} = h$ is:

$$\frac{1}{6}\left[\dot{\omega}_{i-1} + 4\dot{\omega}_i + \dot{\omega}_{i+1}\right] + \left[\frac{M_1}{6} - \frac{1}{h^2}\right]\omega_{i-1} + \left[\frac{4M_1}{6} + \frac{2}{h^2}\right]\omega_i + \left[\frac{M_1}{6} - \frac{1}{h^2}\right]\omega_{i+1} = R \qquad \ldots (11)$$

After using the Crank – Nicholson scheme to the equations (11), the following is got

$$B_1\omega_{i-1}^{j+1} + B_2\omega_i^{j+1} + B_3\omega_{i+1}^{j+1} = B_4\omega_{i-1}^{j} + B_5\omega_i^{j} + B_6\omega_{i+1}^{j} + R^{**} \qquad \ldots (12)$$

The following is got, after using similar method on the equation (4,6&7),:

$$A_1u_{i-1}^{j+1} + A_2u_i^{j+1} + A_3u_{i+1}^{j+1} = A_4u_{i-1}^{j} + A_5u_i^{j} + A_6u_{i+1}^{j} + R^{*} \qquad \ldots (13)$$

$$\xi_1\theta_{i-1}^{j+1} + \xi_2\theta_i^{j+1} + \xi_3\theta_{i+1}^{j+1} = B_4\xi_{i-1}^{j} + C_5\xi_i^{j} + C_6\xi_{i+1}^{j} + R^{**}$$

$$D_1C_{i-1}^{j+1} + D_2C_i^{j+1} + D_3C_{i+1}^{j+1} = D_4C_{i-1}^{j} + C_5C_i^{j} + D_6C_{i+1}^{j}$$

Where

$B1 = 2 - (6*r) + (F*k) + (3*B*r*h)$; $B2 = 8 + (12*r) + (4*F*k)$;
$B3 = 2 - (6*r) + (F*k) - (3*B*r*h)$; $B4 = 2 + (6*r) - (F*k) - (3*B*r*h)$;
$B5 = 8 - (12*r) - (4*F*k)$; $B6 = 2 + (6*r) - (F*k) + (3*B*r*h)$;
$A1 = 2 - (6*r) + (F*k) + (3*B*r*h)$; $A2 = 8 + (12*r) + (4*F*k)$;
$A3 = 2 - (6*r) + (F*k) - (3*B*r*h)$; $A4 = 2 + (6*r) - (F*k) - (3*B*r*h)$;
$A5 = 8 - (12*r) - (4*F*k)$; $A6 = 2 + (6*r) - (F*k) + (3*B*r*h)$;

$\xi 1 = (2*Pr) - (6*r) + (3*B*r*h*Pr)$; $\xi 2 = (8*Pr) + (12*r)$;
$\xi 3 = (2*Pr) - (6*r) + (3*B*r*h*Pr)$; $\xi 4 = (2*Pr) + (6*r) - (3*B*r*h*Pr)$;
$\xi 5 = (8*Pr) - (12*r)$; $\xi 6 = (2*Pr) + (6*r) + (3*B*r*h*Pr)$;

$D1 = (2*Sc) - (6*r) + (3*B*r*h*Sc)$; $D2 = (8*Sc) + (12*r)$
$D3 = (2*Sc) - (6*r) - (3*B*r*h*Sc)$; $D4 = (2*Sc) + (6*r) - (3*B*r*h*Sc)$
$D5 = (8*Sc) - (12*r)$; $D6 = (2*Sc) + (6*r) + (3*B*r*h*Sc)$

$$R = 12*k*\left(\theta_i^i + C_i^j - \frac{M*m\omega_i^j}{1+m^2}\right), F = \frac{M*m}{1+m^2}; B = \frac{\lambda}{\sqrt{t}}, R^{*} = \frac{M*mu_i^j}{1+m^2}$$

$$R^{**} = 12*k*Du*\frac{\partial^2 C}{\partial y^2}.$$

Here, h, k are grid size along the y direction and the time direction respectively. Index i refers to space and j refers to the time. The following are obtained, taking $i = 1....n$ and by means of boundary conditions (11) in the equations (12) – (13),

$$A_i \zeta_i = B_i \;\; i = 1(1)n \qquad \ldots (14)$$

Where A_i's are nth order a matrix and ζ_i, B_i's column matrices. Using Thomas algorithm, the above results are got for the velocity, temperature concentration, by operating the MATLAB program with smaller changed values of *h* and *k*. The numerical simulation is carried out with the aid of Galerkín finite element method, which is stable and convergent. No considerable change was found in the results of velocity, temperature and concentration.

IV. Results and Discussion

Figure (1) shows that the main flow velocity reduces for the growing values magnetic parameter M by the reason of the magnetic pull of Lorentz force. This sort of magnetic full of Lorentz force trims down the flow velocity. But figure (4), reveals that an increase in M, the secondary velocity boosts up because the resulting Lorentz force proceeds as supporting body force on the secondary flow. From figures (2) and (5) it is seen that for the rising values of *m* velocity profiles u and w enhance as an increase in hall current causes a deflection.

Figure (3) reveals that that increase in the value of transpiration parameter leads to enhance in the velocity of the flow because the transpiration velocity is inversely proportional to the square-root of time. From figures (6), (7) and (8) it is noted that the growing values of Du enhances the main, secondary flow velocities and temperature of the fluid because the thermal acceleration causes the enrichment of fluid temperature and velocity.

V. Conclusions

Finite element (FEM) analysis is made on unsteady free-convection flow of viscous incompressible fluid along vertical porous plat plate. From this study following conclusions are drawn.

1. Velocities of the fluid raise for growing values of Du and *m*.
2. Magnetic parameter suppresses the main flow velocity while reverse effect is observed in the case of secondary flow velocity. The results obtained are in good agreement with realistic physical phenomenon
3. Obtained results showed that Galerkin finite element method is more efficient

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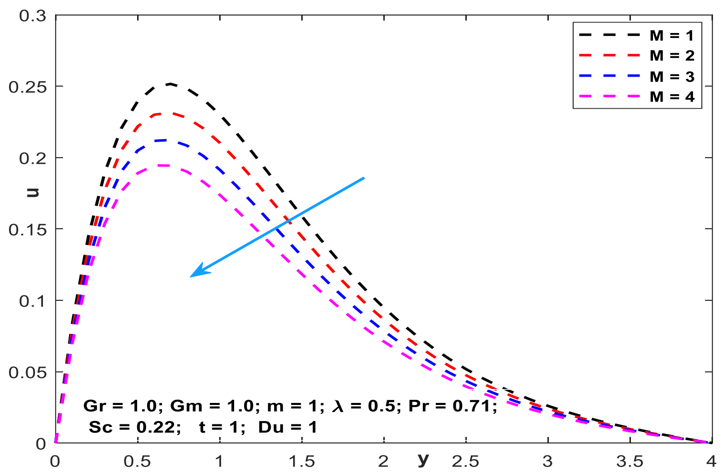

Fig1: Effect of Magnetic parameter on velocity field u

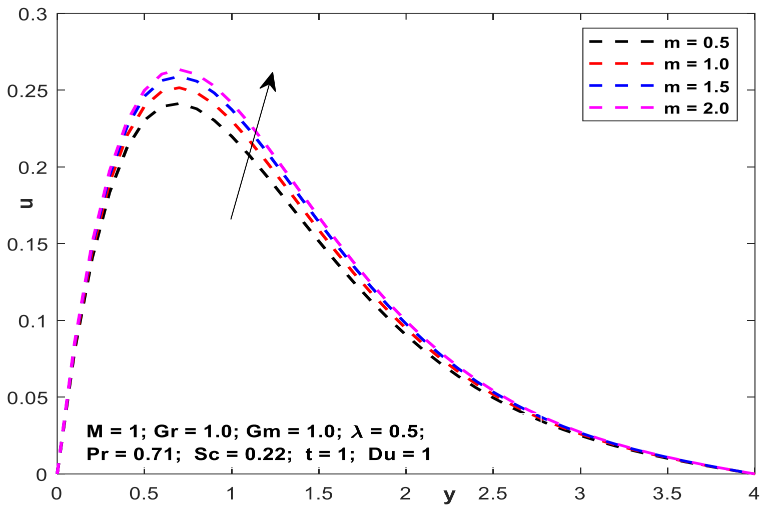

Fig2: Halleffect on velocity field u.

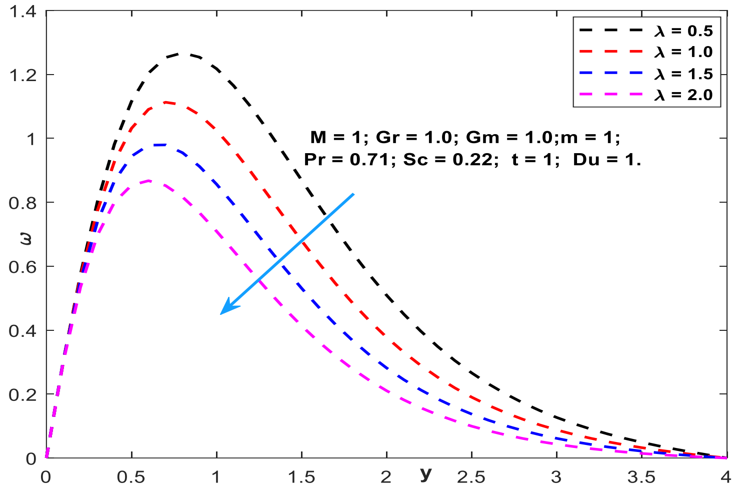

Fig3: Effect of transpiration parameter on velocity field u.

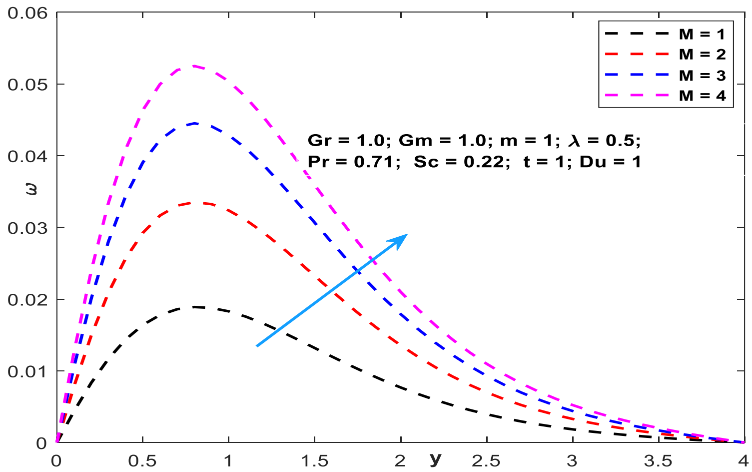

Fig4: Effect of Magnetic parameter on velocity field w

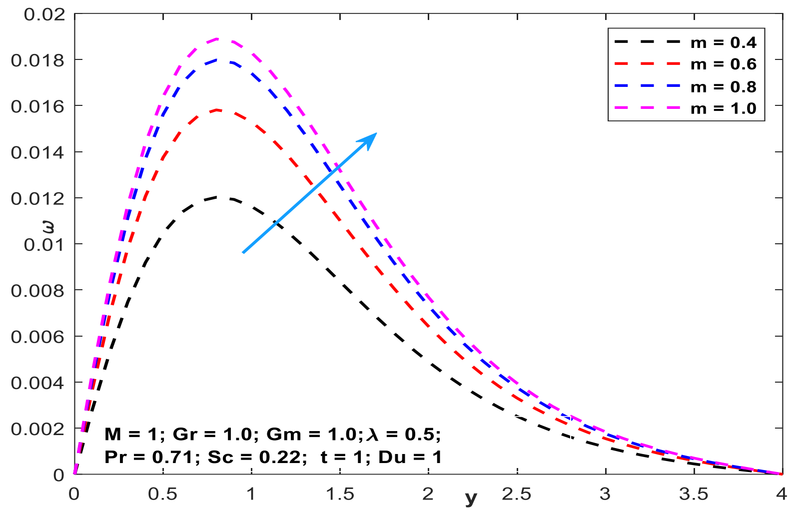

Fig5: Hall effect on velocity field w.

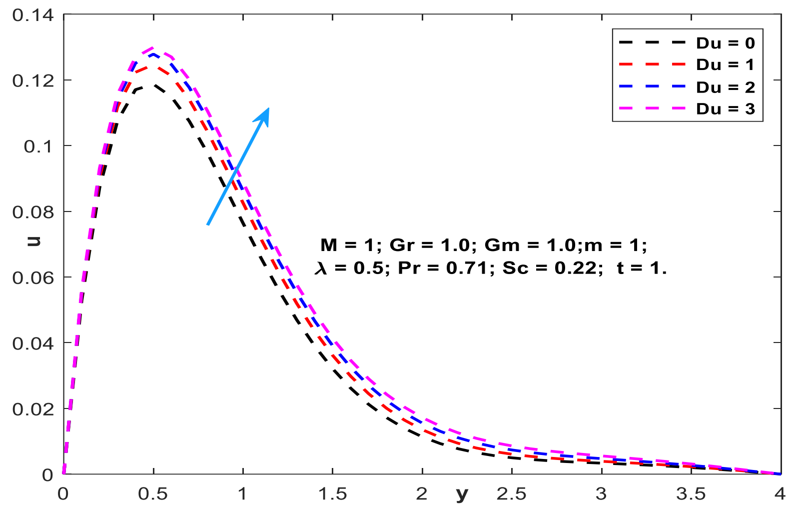

Fig6: Effect of Dufour number Du on velocity field u.

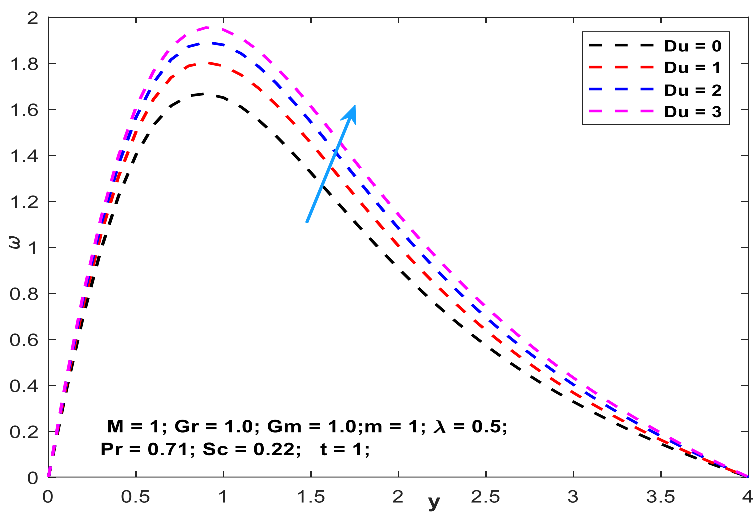

Fig7: Effect of Dufour number on velocity field w.

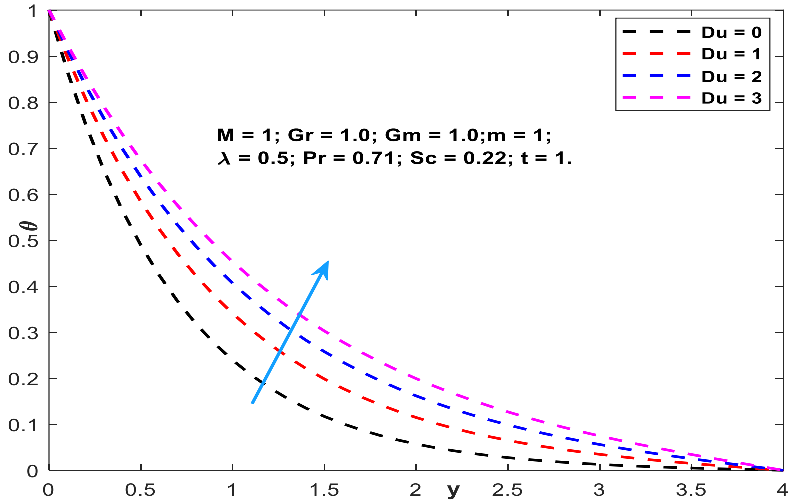

Fig8: Effect of Dufour on temperature field

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