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# On Strongly **β**-Generalized c*-Closed Sets in Topological Spaces

S. Malathi[1], J. Maheswari[2]

*[1,2]Assistant Professor, Department of Mathematics, Wavoo Wajeeha Women's College of Arts and Science, Kayalpatnam - 628204, Tamilnadu, India.*

[1]malathis2795@gmail.com, [2]hepzibahdani@gmail.com

***Abstract-*** *The aim of this paper is to introduce the notion of strongly β-generalized c*-closed sets which are stronger than the generalized β-closed sets and discuss their relation with some other nearly closed sets in topological spaces.*

**Key words:** *c*-open sets, gc*-open sets, strongly βgc*-closed sets*

## I. Introduction

In 1963, Norman Levine introduced semi-open sets and in 1970, he introduced the concept of generalized closed sets in topological spaces. In the year 1965, Njastad introduced the concepts of α-sets (known as α-open sets) and β-sets (known as β-open sets) for topological spaces. Andrijevic called β-sets as semi-pre open sets in 1986. Palaniappan and Rao introduced regular generalized closed (briefly, rg-closed) sets in 1993. In this paper we introduce strongly β-generalized c*-closed sets in topological spaces and study their basic properties.

Section 2 deals with the preliminary concepts. In section 3, strongly β-generalized c*-closed sets are introduced and their basic properties are studied.

## II. Preliminaries

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, cl(A) denotes the closure of A, int(A) denotes the interior of A, pcl(A) denotes the pre-closure of A and βcl(A) denotes the β-closure (equivalently, sp-closure) of A. The following definitions are very useful in the subsequent sections.

**Definition: 2.1** A subset A of a topological space X is called

i. a semi-open set [4] if A⊆cl(int(A)) and a semi-closed set if int(cl(A))⊆A.
ii. a pre-open set [10] if A⊆int(cl(A)) and a pre-closed set if cl(int(A))⊆A.
iii. a regular-open set [16] if A=int(cl(A)) and a regular-closed set if A=cl(int(A)).
iv. a π-open set [19] if A is the finite union of regular-open sets and the complement of π-open set is said to be π-closed.
v. a γ-open set [18] (b-open set [3]) if A⊆cl(int(A))∪int(cl(A)) and a γ-closed set (b-closed set) if int(cl(A))∩cl(int(A))⊆A.
vi. a α-open set [12] if A ⊆int(cl(int(A))) and a α-closed set if cl(int(cl(A)))⊆A.
vii. a β-open set [1] (semi-pre open set [2]) if A⊆cl(int(cl(A))) and a β-closed set (semi-pre closed set) if int(cl(int(A)))⊆A.

**Definition: 2.2** [6] A subset A of a topological space X is said to be a c*-open set if int(cl(A))⊆A⊆cl(int(A)).

**Definition: 2.3** [6] A subset A of a topological space X is said to be a generalized c*-closed set (briefly, gc*-closed set) if cl(A)⊆H whenever A⊆H and H is c*-open. The complement of the gc*-closed set is gc*-open [7].

**Definition: 2.4** A subset A of a topological space X is called

i. a generalized closed (briefly, g-closed) set [5] if cl(A)⊆H whenever A⊆H and H is open in X.
ii. a regular-generalized closed (briefly, rg-closed) set [13] if cl(A)⊆H whenever A⊆H and H is regular-open in X.
iii. a generalized β-closed (briefly, gβ-closed) set [17] if βcl(A)⊆H whenever A⊆H and H is open in X.
iv. a π-generalized β-closed (briefly, πgβ-closed) set [14] if βcl(A)⊆H whenever A⊆H and H is π-open in X.
v. a generalized semi pre regular-closed (briefly, gspr-closed) set [11] if spcl(A)⊆H whenever A⊆H and H is regular-open in X.
vi. a regular pre semi-closed (briefly, rps-closed) set [15] if spcl(A)⊆H whenever A⊆H and H is rg-open in X.
vii. a pre-generalized c*-closed (briefly, pgc*-closed) set [8] if pcl(A)⊆H whenever A⊆H and H is c*-open in X.
viii. a α-generalized c*-closed (briefly, αgc*-closed) set [9] if αcl(A)⊆H whenever A⊆H and H is c*-open in X.

The complements of the above mentioned closed sets are their respective open sets.

## III. Strongly β-generalized c*-closed sets

In this section, we introduce strongly β-generalized c*-closed sets and study their basic properties. We begin this section with the definition of a strongly β-generalized c*-closed set.

**Definition: 3.1** A subset A of a topological space X is said to be strongly β-generalized c*-closed (briefly, strongly βgc*-closed) if βcl(A)⊆H whenever A⊆H and H is gc*-open in X.

**Example: 3.2** Let X={a,b,c,d} with topology τ={∅,{a},{c},{a,c},{b,c,d},X}. Then the subsets ∅,{a},{b},{d},{a,b},{a,d},{b,d},{a,b,d},{b,c,d},X are strongly βgc*-closed in X.

**Proposition: 3.3** Let X be a topological space. Then every closed subset of X is strongly βgc*-closed in X.

**Proof:** Let A be a closed subset of X. Then A = cl(A). Let U be a gc*-open set in X containing A. Then βcl(A)⊆cl(A)=A⊆U. Therefore, A is strongly βgc*-closed in X.

**Proposition: 3.4** Let X be a topological space. Then every π-closed subset of X is strongly βgc*-closed in X.

**Proof:** Let A be a π-closed subset of X. Then A is closed. Therefore, by Proposition 3.3, A is strongly βgc*-closed in X.

**Proposition: 3.5** Let X be a topological space. Then every regular closed subset of X is strongly βgc*-closed in X.

**Proof:** Let A be a regular closed subset of X. Then A is closed. Therefore, by Proposition 3.3, A is strongly βgc*-closed in X.

**Proposition: 3.6** Let X be a topological space. Then every β-closed subset of X is strongly βgc*-closed in X.

**Proof:** Let A be a β-closed subset of X. Then A=βcl(A). Let U be a gc*-open set in X containing A. Then βcl(A)⊆U. Therefore, A is strongly βgc*-closed in X.

**Proposition: 3.7** Let X be a topological space. Then every semi-closed subset of X is strongly βgc*-closed in X.

**Proof:** Let A be a semi-closed subset of X. Then A is β-closed. Therefore, by Proposition 3.6, A is strongly βgc*-closed in X.

**Proposition: 3.8** Let X be a topological space. Then every pre-closed subset of X is strongly βgc*-closed in X.

**Proof:** Let A be a pre-closed subset of X. Then A=pcl(A). Let U be a gc*-open set in X containing A. Then pcl(A)⊆U. This implies, βcl(A)⊆pcl(A)⊆U. Therefore, A is strongly βgc*-closed in X.

**Proposition: 3.9** Let X be a topological space. Then every γ-closed subset of X is strongly βgc*-closed in X.

**Proof:** Let A be a γ-closed subset of X. Then A=γcl(A). Let U be a gc*-open set in X containing A. Then γcl(A)⊆U. This implies, βcl(A)⊆γcl(A)⊆U. Therefore, A is strongly βgc*-closed in X.

**Proposition: 3.10** Let X be a topological space. Then every rps-closed subset of X is strongly βgc*-closed in X.

**Proof:** Let A be a rps-closed subset of X. Let U be a gc*-open set in X containing A. Then U is the rg-open set in X containing A. Therefore, spcl(A)⊆U. This implies, βcl(A)=spcl(A)⊆U. Therefore, A is strongly βgc*-closed in X.

**Proposition: 3.11** Let X be a topological space. Then every strongly βgc*-closed subset of X is gβ-closed in X.

**Proof:** Let A be a strongly βgc*-closed subset of X. Let U be an open set in X containing A. Then U is the gc*-open set in X containing A. Therefore, βcl(A)⊆U. Hence, A is gβ-closed in X.

**Proposition: 3.12** Let X be a topological space. Then every strongly βgc*-closed subset of X is πgβ-closed in X.

**Proof:** Let A be a strongly βgc*-closed subset of X. Let U be a π-open set in X containing A. Then U is the gc*-open set in X containing A. This implies, βcl(A)⊆U. Therefore, A is πgβ-closed in X.

**Proposition: 3.13** Let X be a topological space. Then every strongly βgc*-closed subset of X is gspr-closed in X.

**Proof:** Let A be a strongly βgc*-closed subset of X. Let U be a regular open set in X containing A. Then U is the gc*-open set in X containing A. This implies, βcl(A)⊆U. Therefore, A is gspr-closed in X.

The converse of the above Propositions need not be true, which can be verified from the following example.

**Example: 3.14**

1. Let X={a,b,c,d,e} with topology τ={∅,{a},{b},{a,b},{a,b,c},{a,b,c,d},{a,b,c,e},X}. Then the subset {a,b,d,e} is strongly βgc*-closed but not closed (regular closed, π-closed, β-closed, rps-closed, semi-closed, pre-closed , γ-closed).
2. Let X={a,b,c,d} with topology τ={∅,{a},{b},{a,b},{a,c},{a,b,c},X}. Then the subset {a,d} is gβ-closed(πgβ-closed , gspr-closed) but not strongly βgc*-closed.

The following example shows that strongly βgc*-closed sets and g-closed (gc*-closed, pgc*-closed, αgc*-closed)sets are independent with each other.

**Example: 3.15** Let X={a,b,c,d} with topology τ={∅,{a},{b},{a,b},{a,c},{a,b,c},X}. Then

i. the subset {b} is strongly βgc*-closed but not gc*-closed and the subset {a,b} is gc*-closed but not strongly βgc*-closed
ii. the subset {b,c} is strongly βgc*-closed but not g-closed and the subset {a,d} is g-closed but not strongly βgc*-closed
iii. the subset {b} is strongly βgc*-closed but not pgc*-closed( αgc*-closed) and the subset {a,b} is pgc*-closed ( αgc*-closed) but not strongly βgc*-closed.

**Proposition: 3.16** If all the subsets of a topological space X are gc*-closed, then the subset A of X is β-closed if and only if A is strongly βgc*-closed.

**Proof:** Assume that all the subsets of X are gc*-closed. Let A be a β-closed set. Then by Proposition 3.4, A is strongly βgc*-closed. Conversely, Assume that A is strongly βgc*-closed. Since all the subsets of X are gc*-closed, we have all the subsets of X are gc*-open. In particular, A is the gc*-open set containing A. Therefore, by our assumption, βcl(A)⊆A. Always, A⊆βcl(A). Therefore, A=βcl(A). Hence A is β-closed.

**Proposition: 3.17** If all the subsets of a topological space X are gc*-open, then the subset A of X is β-closed if and only if A is strongly βgc*-closed.

**Proof:** Assume that all the subsets of X are gc*-open. Let A be a β-closed set. Then by Proposition 3.4, A is strongly βgc*-closed. Conversely, Assume that A is strongly βgc*-closed. Since all the subsets of X are gc*-open, we have A is the gc*-open set containing A. Therefore, by our assumption, βcl(A)⊆A. Always, A⊆βcl(A). Therefore, A=βcl(A). Hence A is β-closed.

**Proposition: 3.18** If ∅ and X are the only c*-open sets, then a subset A of X is β-closed if and only if A is strongly βgc*-closed.

**Proof:** Assume that ∅ and X are the only c*-open sets. Let A be a β-closed set. Then by Proposition 3.4, A is strongly βgc*-closed. Conversely, Assume that A is strongly βgc*-closed. Since ∅ and X are the only c*-open sets, by Proposition 4.21[6], all the subsets of X are gc*-closed. This implies, all the subsets of X are gc*-open. In particular, A is the gc*-open set containing A. Therefore, by our assumption, βcl(A)⊆A. Always, A⊆βcl(A). Therefore, A=βcl(A). Hence A is β-closed.

**Remark: 3.19** The collection of strongly βgc*-closed sets are not closed under union and intersection. The following examples prove this.

1. Let X={a,b,c} with topology τ={∅,{b},{c},{b,c},X}. Then the subsets {b} and {c} are strongly βgc*-closed but their union {b,c} is not strongly βgc*-closed.
2. Let X={a,b,c,d} with topology τ={∅,{a},{b},{a,b},{a,c},{a,b,c},X}. Then the subsets {a,c} and {a,b,d} are strongly βgc*-closed but their intersection {a} is not strongly βgc*-closed.

**Proposition: 3.20** If a subset A of a topological space X is strongly βgc*-closed in X, then βcl(A)\A does not contain any non-empty gc*-closed set in X.

**Proof:** Assume that A is a strongly βgc*-closed set in X. Suppose H is a gc*-closed set such that H⊆βcl(A)\A and H≠Ø. Then H⊆X\A. This implies, A⊆X\H. Since H is the gc*-closed set, we have X\H is the gc*-open set in X. Then βcl(A)⊆X\H. This implies, H⊆X\βcl(A). Also, H⊆βcl(A). Hence H⊆βcl(A)∩(X\βcl(A))=Ø, which contradicts H≠Ø. Hence βcl(A)\A does not contain any non-empty gc*-closed set in X..

**Proposition: 3.21** Let X be a topological space. Then for any element p∈X, the set X\{p} is either strongly βgc*-closed or gc*-open.

**Proof:** Suppose for any p∈X, X\{p} is not a gc*-open set. Then X is the only gc*-open set containing X\{p}. Also, βcl(X\{p})⊆X. Hence X\{p} is the strongly βgc*-closed set in X.

**Proposition: 3.22** Let A be a strongly βgc*-closed set in a topological space X. Then A is β- closed if and only if βcl(A)\A is gc*-closed.

**Proof:** Suppose A is β-closed. Then βcl(A)=A. This implies, βcl(A)\A=Ø, which is gc*-closed. Conversely, suppose βcl(A)\A is a gc*-closed set in X. Since A is strongly βgc*-closed, we have βcl(A)\A does not contain any non-empty gc*-closed set in X. Then βcl(A)\A=Ø. This implies, A=βcl(A). Hence A is β-closed.

**Proposition: 3.23** Let X be a topological space. If A is a strongly βgc*-closed subset of X such that A⊆B⊆βcl(A), then B is strongly βgc*-closed in X.

**Proof:** Let H be a gc*-open set containing B. Then A⊆H. Since A is strongly βgc*-closed, we have βcl(A)⊆H. This implies, βcl(B)⊆H. Hence B is strongly βgc*-closed in X.

**Proposition: 3.24** Let X be a topological space. If X and Ø are the only gc*-open sets then all the subsets of X are strongly βgc*-closed.

**Proof:** Let A be a subset of X. If A=Ø, then A is strongly βgc*-closed. If A≠Ø, then X is the only gc*-open set containing A. Also, βcl(A)⊆X. Hence A is strongly βgc*-closed.

**Proposition: 3.25** A subset A of a topological space X is strongly βgc*-closed if and only if for each A⊆H and H is gc*-open, there exists a β- closed set F such that A⊆F⊆H.

**Proof:** Suppose A is a strongly βgc*-closed set. Let A⊆H and H be gc*-open. Then βcl(A)⊆H. If we put F=βcl(A), then A⊆F⊆H. Conversely, assume that H is a gc*-open set containing A. Then there exists a β-closed set F such that A⊆F⊆H. Since βcl(A) is the smallest β-closed set containing A, we have A⊆βcl(A)⊆F. Also, since F⊆H, we have βcl(A)⊆H. Hence A is strongly βgc*-closed.

**Proposition: 3.26** If a subset A of a topological space X is strongly βgc*-closed in X, then βcl(A)\A does not contain any non-empty regular closed set in X.

**Proof:** Suppose H is a regular closed set contained in βcl(A)\A and H≠Ø. Since every regular-closed set is gc*-closed, we have H is gc*-closed. Thus, H is the gc*-closed set contained in βcl(A)\A. Therefore, by Proposition 3.19, H=Ø. This is a contradiction. Therefore, βcl(A)\A does not contain any non-empty regular closed set in X.

**Proposition: 3.27** Let X be a topological space and A be a subset of X. If A is regular open and strongly βgc*-closed, then A is β-clopen.

**Proof:** Assume that A is regular open and strongly βgc*-closed. Since every regular open set is gc*-open, we have βcl(A)⊆A. Then A is β-closed. Since A is regular open, we have A is open. Therefore, A is β-open. Thus, A is both β-open and β- closed. Hence A is β-clopen.

## IV. CONCLUSIONS

In this paper we have introduced strongly βgc*-closed sets in topological spaces and studied some of their basic properties. Also, we have discussed their relation with some other nearly closed sets in topological spaces.

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