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Modelling and Analysis of Queueing System: A Heuristic Approach in an Agricultural Development Bank

Anitha Kiran [#1], Dr. Karabi Sikdar [#2]

[#1, #2] *Department of Mathematics, BMS Institute of Technology & Management, Bengaluru-560054, India.*

Abstract: *Agriculture sector, being the sublime of the Indian economy, has been the second largest contributor to Indian GDP. In the midst of COVID-19, farmers have made a major contribution towards mankind by assuring food security during this situation. To help farmers in advance farming, agricultural loans are provided to farmers to fulfil their requirement. In this view, a field experiment was conducted at Union Bank of India, Kandali, Hassan district, India by using queueing theory technique to improve customer service. Various performance measures were determined to reduce the waiting time and improve efficiency of the server. The data was collected for a period of one and half month through direct observation. In this study, a multi-server queue which follows Poisson process with First in First Out (FIFO) queue discipline was used.*

An analysis of the queueing system at the bank reveals that the existing system can be upgraded to help the farmers in availing loan by adding extra service point to the system.

Keywords: *Queue, Bank, Poisson process, Service delivery.*

I. INTRODUCTION

In India, Agricultural sector plays a vital role in the development of Indian economy. Agriculture is the main source of livelihood for about 58% of the Indian population. Gross value added by agriculture and allied sectors has contributed about 4% in Fiscal year 2020. Indian food market is world's sixth largest where retail firm has contributed for about 70% of the sales. The main crops grown in India are Rice and Wheat. Despite the fact that agriculture has a major contribution to Indian economy, there are various issues in the domain which has resulted in crop loss. To overcome such issues loans are provided by Agricultural Development banks.

Agricultural Development banks are classified into:

a) All India level : NABARD
b) State level : State Land Development Banks
c) Local-level Institutions : Primary Land Development Banks, Branches of State Land Development Banks.

Customers make fixed deposits in banks to avail interests at lower rate and banks will make use of these amounts to provide loans at higher rate of interest. If these loans are paid on time, then banks can make huge profits. In Agricultural Development banks, loans are broadly divided into Short term credit and Long term credit. Farmers are supported with Short term credit loans such as Crop loan, Loan-warehouse receipt(farmers), Kisan credit card and Long term credit loans such as Minor irrigation loan, Land purchase loan, Union Mortgage, Kissan tatkal scheme etc.,

Long queues are common in banking sector, which results in customer complaints, congestion, poor service leading to revenue loss, Customer loss. Customer satisfaction and revenue growth is the key component for achieving long term success in banking sector. In most of the top growing industries, customers opinion is considered with high priority for profit growth.

In this regard, the concept of Queueing theory is applied to the banking system, to reduce the queue length, waiting time of customers and increase service efficiency.

Queueing theory :

Queues are customers or items waiting in a line for getting the service rendered. In every walk of life, we come across these waiting lines. The concept of Queueing theory was discovered by Danish Mathematician, Agner Krarup Erlang, who is considered as the father of Queueing theory published his first paper in 1909. The study by Erlang, A.K. 1909 reveals theory of probabilities and Telephone conversation, in which the probability of a certain number of calls being originated during a certain interval of time, delay in answering the calls were calculated. The study by Shanmugasundaram, S et al., 2015, Sharma, J. K. 2007, reveals applications of Queueing theory in our daily life, Readers can refer to Sundarapandian, V. 2009 to understand the concepts of Queueing theory.

We use the standard Kendall notation to represent different types of Queueing system which is as follows:

Kendall Notation: (A/S/C/K/N/D) where

A : Interarrival time probability distribution is the distribution in which the random variables represents the number of customers arriving in succession during a time interval.

S : Service time probability distribution is the distribution in which the random variables includes the number of customers who avail service at any time, the length and mode of service.

C : Total number of servers which can be either a single server or multiple server. Multiple servers can be arranged either in series or in parallel.

K : Capacity of the queue is the maximum number of customers allowed in queue, which can be finite/infinite.

N : Calling Population, is the size of the population from which the customers come.

D : Queue Discipline, is the order in which the customers in queue are being served. The most common queue discipline used is FIFO (first in, first out) system. Some of the customers will be served based on priority called as priority queue. The other queue discipline are Last-Come, First-Served (LCFS), Random Selection for Service (RS).

Utilization of the system is the probability that the server is busy denoted by $\rho = \lambda/(c\mu)$ where λ is the mean arrival rate, μ is the mean service rate, and c is the number of servers. If $\rho > 1$, then the queue will grow based on time. If $\rho < 1$, then the queue length is small, which indicates that the system is more ideal.
In this study, we have adopted Multi Server Queue / ***(M/M/c)*** Model to analyse the banking system.

Multi Server Queue / (M/M/c) Model:

Multi server queue has two or more service points, where the service points provide identical service in parallel. The arrival time distribution follows Poisson distribution whereas the service time distribution follows exponential distribution. Akinnuli, B. O. et al., 2014, Ezeliora, et al., 2014, Shanmugasundaram, et al., 2016 have studied about multiple channel, multi servers system. The Performance measures of the model is listed below:

Notation/ Meaning	***Formula***
λ / Mean Arrival rate	$\lambda = \frac{1}{E(Inter\ arrival\ time)}$ $where\ E\ is\ Expectation\ operator$
μ / Mean Service rate	$\mu = \frac{1}{E(service\ time)}$ $where\ E\ is\ Expectation\ operator$
c / Number of servers in the system	-
ρ / Server utilization factor	$\rho = \frac{\lambda}{c\mu}$
p_0 / Probability that the system is idle	$p_0 = \frac{1}{\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)}$
ℓ_s / Average number of customers in system	$\ell_s = \frac{\lambda\mu(\frac{\lambda}{\mu})^c p_0}{(c-1)!\,(c\mu-\lambda)^2} + (\frac{\lambda}{\mu})$
ℓ_q / Average number of customers in queue	$\ell_q = \ell_s - (\frac{\lambda}{\mu})$
w_q / Average waiting time of a customer in queue	$w_q = \frac{\ell_q}{\lambda}$
w_s / Average waiting time of a customer in system	$w_s = w_q + \frac{1}{\mu}$
p_w / Probability that server is busy and a customer has to wait for service	$p_w = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)p_0$
S_J / Probability that server is idle and a customer can be served	$S_J = 1 - p_w$

II. LITERATURE REVIEW AND DATA COLLECTION

A. LITERATURE REVIEW:

A study conducted at the Union Bank of India, Kandali branch reveals that there is a positive correlation between the customers arrival rate and service rates in which the customers/farmers are served by the bank employees. The objective of the bank is to contribute best services to the community by supporting various socio-economic, educational and health initiatives. In addition to providing the best service, the bank also provides better policies to farmers in order to adopt unconventional farming practices. Also, the farmers club is a new initiative to help farmers from rural background. The bank holds around 20,000 SB accounts among which currently 10,000 accounts are active, also it holds approximately 400 agricultural loan accounts. Due to which the bank encounters long queues, to resolve this issue and improve customer satisfaction queueing analysis is carried out to reduce the queue length and increase service efficiency. To resolve this issue, the concept of Queueing theory is applied in the banking sector in order to meet customer's requirement and to help the bank achieve maximum profit. The Application of queuing theory in Banking sector has been discussed and various methods for system improvement and service optimization is suggested by Adedoyin et al. 2014, Adedoyin et al., 2020, Bannerman 2014, Chuka et al. 2014, Xiao et al. 2010, Oluwaseun, X. Pei 2008.

B. BRIEF DESCRIPTION OF THE BANK AND DATA COLLECTION:

A field experiment was conducted in Union Bank of India, Kandali, Hassan district India, which is a Primary Land Development Bank.

Details of the Union Bank of India, Kandali branch is as follows:

1. Year of Establishment : 1970.
2. Account type : Savings, Current, Fixed deposit account, Loan account.
3. Working Hours: 10 am to 4 pm
4. Number of Counters : Three
5. Holiday : Second and Fourth Saturday
6. Number of Savings Bank Account : 20,000 SB with 10,000 SB running account.
7. Number of Agricultural loan Account : 400
8. Agricultural loan interest rate : 7.30 %

The bank works six days per week except on holidays. It has six counters in which three counters are meant for customer service and other three counters operates with back end process. The bank offers various services which includes Account opening, Customer service information, Deposits, Amount Withdrawal, Loan payment, Western bank Union, etc., Apart from home loan, vehicle loan, personal loan, education loan, credit card, business loan, mortgage loan, the bank offers Agricultural loans.

The bank has well developed Corporate Social Responsibility (CSR) inititatives. Initiatives such as Village Knowledge Centres and Farmers' Clubs etc., meant for rural sectors. Girl child education expenses schemes, drinking water for schools, setting up bus shelters etc. These CSR initiatives has made the bank to contribute in nation growth and development. Due to the pandemic situation, the bank is providing Union SHG Covid Suvidha Loan for COVID victims.

Various loans are provided to the customers with reasonable rate of interest. To help customers who are farmers in availing agricultural loans, the verification of loan documents is taken care of by a field officer. The Bank Manager is responsible for maintenance of the entire team, manage the work of all banking staff and improve the bank performance. Customers who arrive to the bank avail various services such as account opening, service information, deposits, amount withdrawal results in smaller queues, whereas services chosen such as availing loans in which customers need to get the documents verified results in longer queues, due to which service time gets delayed.

Assumptions made in the Existing Model:

1. Customers remain in the system until they are served.
2. Customers who are farmers are directed to the field officer for document verification initially.
3. Field officer visit to the branch is on every Wednesday and Thursday.
4. Amount withdrawal is done through ATM Machine.
5. One of the three working counters works until 5 to 6 p.m, until all customers in the system are served.

The main objective of the study is to increase the bank service efficiency, minimize operating costs and help customers/farmers to avail loans, best services from the banking system.

III. CONCEPTUAL DESIGN OF THE MODEL:

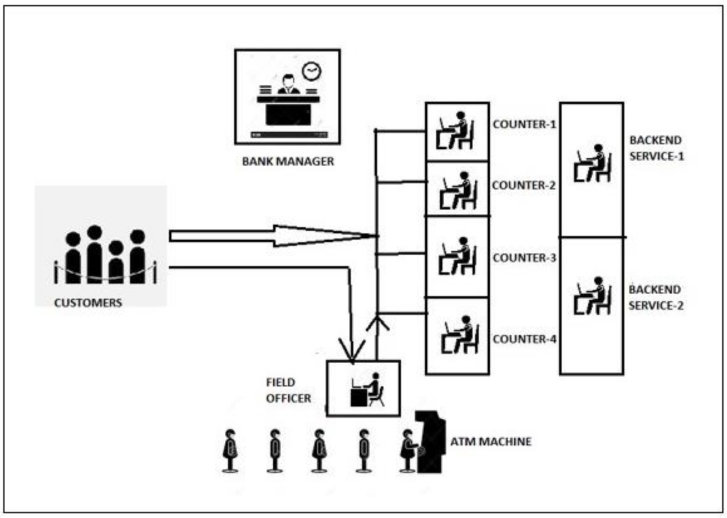


Fig.1 Concept design of the model

IV. RESULT AND DISCUSSIONS:

In the current scenario, bank has three customer service counters in which one of them is the cash counter, other three counters are meant for backend service. The bank manager maintains the bank by monitoring the entire team. The field officer visits the bank for verification of documents related to agricultural loans. Farmers who stay in the vicinity of Kandali branch avail agricultural loans from the bank.

If a customer is involved with Account opening, Customer service information, Deposits, Amount Withdrawal then the service counters requires less time compared to customers who avail various loans. Among them, farmers availing Agricultural loans form longer queues. As the bank is situated in Kandali, a village in Hassan taluk, rural sector customers are large in number. As, the literacy rate in Kandali village is around 67%, the service time required for each customer who are farmers is more compared to that of normal customers.

In this regard, data related to inter arrival time of the customers and service time of three customer service counters were collected for period of 45 days and the average was estimated for every five days. The data was collected such that the average of the data reflects the period when the bank is very busy or less busy. The arrival of customers and service time of the customers in every one hour period was computed by monitoring. The average arrival rate (λ), average service rate (μ) and the number of servers (c) are the basic operators used in computing the performance measures of the existing queueing system.

The performance of the existing system reveals that there are long queues on Monday and Wednesday. Also, customers who are involved in Deposits, Amount withdrawal are large in number due to which the customers must wait for long time which results in customer frustration and withdrawal of accounts. To avoid such issues, a model is proposed, which results in better performance of the banking system. As the field officer visits the bank twice in a week, documents verification of Agricultural loans are done on these days. Due to which long queues resulting from farmer customers are expected on these days.

The services offered by the bank is divided into following:
1: Amount Withdrawal, deposits, Non- Agricultural Loan payment
2: Customer Service Information
3: Salary, Prime account
4: Account Opening
5: Western Union
6: Agriculture related customers

Based on the type of service chosen, Customers are divided into following three groups:
Customer type A: 1,4,5
Customer type B: 2,3
Customer type C: 6

Table 1- Data related to Customer Arrival and Service in every five days

Date	Average number of Customers of type A,B per hour in every 5 days (a)	Average number of Customers of type A,B served per hour in every 5 days (b)	Average number of Customers of type C per hour in every 5 days (c)	Average number of Customers of type C served per hour in every 5 days (d)	Average number of Customers per hour in every 5 days (a) + (c)	Average number of Customers served per hour in every 5 days (b) + (d)
02 Nov to 06 Nov 2020	29	14	16	5	45	19
07 Nov to 12 Nov 2020	28	10	18	4	46	14
16 Nov to 20 Nov 2020	32	11	15	4	47	15
21 Nov to 26 Nov 2020	35	12	20	5	55	17
27 Nov to 03 Dec 2020	32	11	17	4	49	15
04 Dec to 09 Dec 2020	32	12	15	4	47	16
10 Dec to 16 Dec 2020	30	15	15	5	45	20
17 Dec to 22 Dec 2020	32	14	19	6	51	20
23 Dec to 30 Dec 2020	35	12	21	5	56	17
Average	**32**	**12**	**17**	**5**	**49**	**17**

From Table 1. We can observe that, data related to type A, B customers are grouped together and for type C customer the data is presented separately. The average number of customer arrival and service of type A, B per hour in every five days ranges from 28 to 35 customers, 10 to 14 customers respectively, whereas the average number of customers arrival and service of type C per hour in every five days ranges from 15 to 21 customers, 4 to 6 customers respectively.
Average number of customers of type A, B per hour in every five days is observed to be 65% whereas
Average number of customers of type C per hour in every five days is observed to be 35% which is depicted in Fig.2.

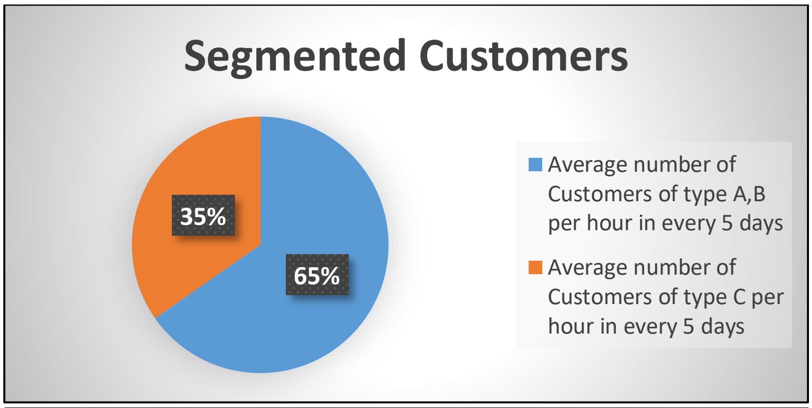


Fig.2 Percentage of Segmented customers in the existing system

In the proposed model, one extra service counter is added to the existing customer service counters, by removing one backend service point, which can speed up the service, in turn the performance of the banking system.
In the current scenario, the number of service counters are ***C* = 3**, in which the server utilization is found to be 96%, average number of customers in system is 26, average waiting time is 31.32 mins, average busy time in system is 92.7% whereas in the proposed model the number of service counters is considered as ***C* = 4**, in which the server utilization reduces to 72%, average number of customers in system reduces to 5, average waiting time reduces to 4.98 mins.
The performance measures for the existing system and proposed system is calculated and compared by using ***Multi Server Queue / (M/M/c) Model*** and is presented in the Table.2.

Table 2: Comparison of Performance analysis of the queueing system in the bank with number of servers, c =3, c=4

Performance measures	*Observed values in current scenario:*	*Observed values in proposed model:*
λ / Mean Arrival rate	**49**	**49**
μ / Mean Service rate	**17**	**17**
c / Number of servers in the system	**3**	**4**
ρ / Server utilization factor	**96%**	**72%**
p_0 / Probability that the system is idle	**0.911%**	**4.48%**
ℓ_s / Average number of customers in system	**25.589 ~ 26**	**4.072 ~ 5**
ℓ_q / Average number of customers in queue	**22.707 ~ 23**	**1.189 ~ 2**
w_q / Average waiting time of a customer in queue	**0.4634 hours / 27.8 mins**	**0.024 hours / 1.44 mins**
w_s / Average waiting time of a customer in system	**0.522 hours / 31.32 mins**	**0.083 hours / 4.98 mins**
p_w / Probability that server is busy and a customer has to wait for service	**92.7%**	**46.11%**
S_J / Probability that server is idle and a customer can be served	**7.28%**	**53.88%**

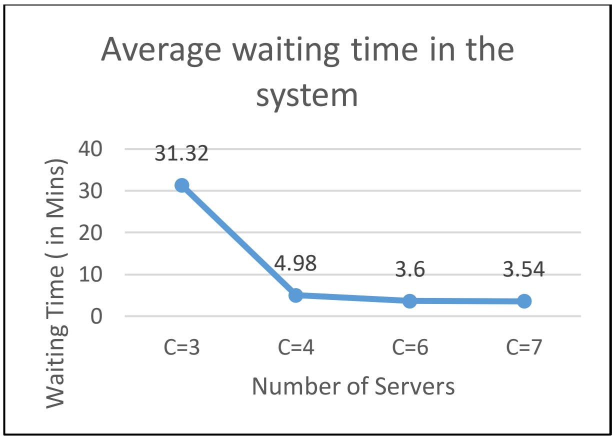
In order to minimize the cost and maximize server efficiency, we have used standard normal distribution to arrive at optimal number of servers in the banking system. Here Table.3 indicates that 6 and 7 servers are required to attain the optimal condition with 95% and 99% service rate. The average waiting time and busy period with number of servers is presented in Table.4 and depicted in Fig.2 and Fig.3 respectively. The cost minimization and efficiency are the two aspects which should be taken care of. Hence in the proposed model, there is no cost variation, but still there is a drastic decrease in the average waiting time, busy time in the system. Although, we find the best results from optimal servers with 95% and 99% service rate, but by introducing 6 and 7 servers, the operating cost increases. Hence in order to reduce the operating costs we propose that the system is efficient with number of servers ***C* = 4.**

Table 3: Optimal Number of Servers with 95% and 99% Service Rate

Customer Type	**λ**	**μ**	**$\rho = \lambda/\mu$**	**k**	**$C = \rho + k\sqrt{\rho}$**
A,B,C	49	17	2.88	$z_{0.05}$= 1.645	6
A,B,C	49	17	2.88	$z_{0.01}$= 2.326	7

Table 4: Average Waiting time, Average Busy time for various servers

Number of Servers	**$C = 3$ (Existing Model)**	**$C = 4$ (Proposed Model)**	**$C = 6$ (Optimal servers with 95% LOS)**	**$C = 7$ (Optimal servers with 99% LOS)**
Average waiting time in the system	**0.522 hours / 31.32 Mins**	**0.083 hours / 4.98 Mins**	**0.06 hours / 3.6 Mins**	**0.059 hours/ 3.54 Mins**
Average busy time	**92.7 %**	**46.11 %**	**8.47 %**	**3.24 %**



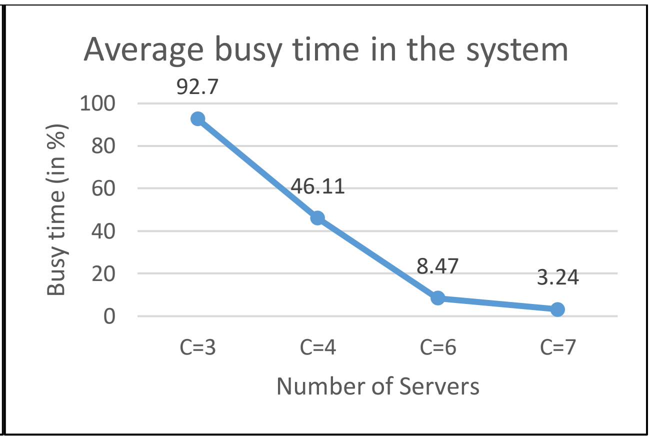


Fig.3 Average waiting time in the system for various servers

Fig.4 Average Busy time for various servers

Optimal Service rate:

Here we have considered ***M/M/c*** Model with ***c*** number of servers. The main aim is to determine the service which minimizes the total operating cost, which includes service cost and waiting cost.

If W_c is the waiting cost per customer per unit time, S_c is the service cost per customer, ℓ_s is the average number of customers in the system. Then total cost $T_c = W_c\ell_s + \mu S_c$ where μS_c represents service cost per unit time. The condition for minimizing the total cost is $\frac{dT_c}{d\mu} = 0$.

We have, $\frac{S_c}{W_c} = \frac{\wp_0 \lambda^{c+1}(c+1+2\lambda)}{\mu^c(c\mu-\lambda)^2} + \frac{\lambda}{\mu^2}$ ……….(1) ,

(1) Provides us the ratio of service cost per customer by waiting cost per customer per unit time. If we solve for μ in (1) we arrive at the optimal service rate for various values of ***c*** (where ***c*** is the number of servers).

V. CONCLUSION

The study reveals that in the existing banking system there are six counters in which three counters are meant for customer service and other three counters are meant for back end process. In the existing system the bank encounters longer queues due to which there is congestion in the system. Hence we have proposed a system in which by removing one backend service and introducing an extra service point to the existing system, the average number of customers, average waiting time in the system, probability that the server is busy, server utilization has reduced from 26 customers to 5 customers, 31.32mins to 4.98 mins, 92.7% to 46.1%, 96% to 72% respectively. By introducing optimal number of servers with 95% and 99% service rate, we can observe that the system becomes more efficient, but in turn maximizes the operating cost. In view of economics of efficient utilization of resources we would recommend the bank to have ***C = 4*** servers in order to have well organized banking system. The condition for minimizing the total cost is $\frac{dT_c}{d\mu} = 0$.

Also Customers of type A, related to Amount withdrawal and cheque deposits should be diverted towards ATM machine. Most of the Customers of type C are illiterates, hence the bank should take extra care in resolving their issues and educating them about the loan payments. In the proposed banking system among four service counters, at least two counters are required to work beyond the working hours until all customers are served in the system.

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Conflict of Interest: The authors declare that they have no conflict of interest.

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