

Five Dimensional Inflationary Universes in the Saez-Ballester Theory

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Abstract: - In the current work our point is to build five dimensional inflationary Universes in the framework of Saez-Ballester Theory of Gravitation (SBTG). For construction of the model of the Universe we have consider a five dimensional Bianchi Type-V space-time. Some physical and kinematical parameters of the models have been graphically described.

Keywords: - Bianchi Type-V space time, Inflationary Universe, Saez-Ballester Theory..

I. INTRODUCTION

The inflationary universe is a cosmological model wherein there is a period during the early universe when the volume of room grows dramatically. A time of expansion may emerge when matter is portrayed by molecule material science (rather than by an ideal gas) in the early universe. Inflationary models give likely answers for a portion of the issues of standard cosmological models. They additionally give an instrument which produces energy thickness changes which can seed systems and different constructions in the universe.

It is notable that self-communicating scalar fields assume an imperative part in the investigation of inflationary cosmology. The solution to horizon and flatness of the universe investigated aside Guth [1]. With new inflationary universe a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems obtained aside Linde [2], Daile and Paul [3] studied extended inflationary universe. Super-inflation is likely to occur in closed cosmologies when the slow-roll approximation is valid and a new super-inflationary solution got Lidsey [4]. Wald [5] devoted to study asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant. String-driven inflationary and deflationary cosmological models obtained by Barrow [6] and Feinstein and Ibanez [7] found exact inhomogeneous scalar field universes.

Exact solutions for the scalar field and the potential function in an exponentially inflationary universe, whose source of the gravitational field is a classical scalar field plus anisotropic fluid, are found Mendez [8]. Exact cosmological solutions of a higher derivative theory in the presence of interacting scalar field obtained aside Paul [9]. Many pedagogical introductions of inflation are effective due to the simplicity of the relevant equations. Analytic solution of the cosmological equations and used as an example to discuss fundamental aspects of the inflationary paradigm Faraoni [10]. A new exact solution to Einstein's equations that describes the evolution of imaginary universe models with the inflation is driven by the evolution of a scalar field with an approximate two loop four dimensional string potential Wang [11]. Einstein's field equations are considered for a locally rotationally symmetric Bianchi Type-II space-time in the presence of a massless scalar field with a scalar potential. To get inflationary solutions, a flat region is considered in which the scalar potential is constant Singh and Kumar [12]. Using the Friedman equation in rainbow Universe, an exact scalar field Inflationary Solution is obtained, which is a modification of the exact scalar field with negative potential Lin and Yang [13]. A simple axially symmetric inflationary universe in the presence of mass less scalar field with a flat potential had been discussed. To get an inflationary universe, had considered a flat region in which potential V is constant Reddy et al. [14] and Reddy [15]. Reheating is an important part of inflationary cosmology. It describes the production of Standard Model particles after the phase of accelerated expansion. The reheating process had focus in depth discussion of the preheating stage, which had characterized by exponential particle production due to a parametric resonance or tachyon instability Allahverdi et al. [16]. Kaluza-Klein inflationary universe in General Relativity (GR) had studied. To obtain the deterministic model of the universe, it has been considered that the energy momentum tensor of particles almost vanishes in the course of the expansion of the universe and thereby total energy-momentum tensor reduces to vacuum stress tensor Adhav [17]. A five-dimensional Bianchi type-I inflationary Universe had investigated in the presence of massless scalar field with a flat potential Katore et al. [18]. Dynamics of induced gravity cosmological models with the sixth degree polynomial potentials that had found using the super potential method Pozdeeva and Vernov [19]. The mechanism of the initial inflationary scenario of the Universe and of its late-time acceleration described by assuming the existence of some



gravitationally coupled scalar field's ϕ , with the inflation field generating inflation and the quintessence field being responsible for the late accelerated expansion. Various inflationary and late time accelerated scenarios are distinguished by the choice of an effective self-interaction potential $V(\phi)$, which simulates a temporarily non-vanishing cosmological term. In this work, we present a new formalism for the analysis of scalar fields in flat isotropic and homogeneous cosmological models. The basic evolution equation of the models can be reduced to a first-order non-linear differential equation Harko et al. [20]. The inflationary paradigm is now part of the standard cosmological model as a description of its primordial phase. While its original motivation was to solve the standard problems of the hot big bang model, it was soon understood that it offers a natural theory for the origin of the large-scale structure of the universe. Most models rely on a slow-rolling scalar field and enjoy very generic predictions. Besides, all the matter of the universe is produced by the decay of the inflation field at the end of inflation during a phase of reheating. These predictions had tested from their imprint of the large-scale structure and in particular the cosmic microwave background. Inflation stands as a window in physics where both GR and quantum field theory are at work and which can be observationally studied Uzan[21]. Non-static plane symmetric model in $f(R, T)$ theory of gravity with scalar field (quintessence or phantom) had investigated, where R is Ricci scalar and T is the trace of energy momentum tensor Bhoyar[22]. Had considered $f(R)$ modified gravity with a scalar field and do not specify the form of the $f(R)$ function. Friedmann universe had assumed that acceleration of the scalar curvature had negligible Maharaj et al. [23]. The method of exact analysis of cosmological dynamics at the early inflation stage of the evolution of the Friedman Universe, which is determined by the dynamics of the scalar field for the case of minimal and non-minimal interaction of the field and curvature, had considered aside Fomin[24]. Exact cosmological solution of a scalar field of type $+cosh$ in anisotropic Space-Time of Petrov Type D obtained aside Alvarado [25]. The construction of exact solutions in scalar field inflationary cosmology had growing interest. The method of generating functions for the construction of exact solutions in scalar field cosmology got by Chervon et al.[26]. The geometry of the universe is described by the spatially flat homogeneous and isotropic line element and the scalar fields may interact in their kinetic or potential terms. Within this set up, for a specific geometry in the kinetic part of the scalar fields and specific potential form, the gravitational field equations for the class of N-scalar field models had been exactly solved by Paliathanasis et al. [27]. The ability of the exponential power law inflation had phenomenologically correct model of the early universe. In GR the scalar cosmology equations in Ivanov–Salopek–Bond representation where the Hubble parameter H is the function of a scalar field ϕ . Such approach admits calculation of the potential for given $H(\phi)$ and consequently reconstruction of $f(R)$ gravity in parametric form Fomin and Chevron [28]. Bulk viscous inflationary model with flat potential under framework of LRS Bianchi type II metric discussed and got solution of the field equations Poonia and Sharma [29]. Thus, inspired by the above probe, in this current work, we think about the five dimensional inflationary universes in the Saez-Ballester theory of gravitation (SBTG). Throughout the most couple of many years, Five Dimensional Bianchi Type-V universe acquired interest in hypothetical cosmology. There exist a few models, introduced by different writers both in GR and in altered hypotheses of attraction. A large group of creators have researched with the five dimensional Bianchi Type-V metric Adhav et al. [30], Biswal et al.[31], Reddy and Ramesh [32]. The subsequent classification considers scalar field as dimensionless as opposed to assuming the part of variable G furthermore, the hypothesis was first proposed by Saez-Ballester[33] which likewise sufficiently depicts the feeble fields. This hypothesis has its own significance as it is fit for tending to the question of missing mass in Friedmann–Robertson Walker level universe. A few creators contemplated the elements of the universe by developing cosmological model in SBTG such as, Adhav et al. [34], Katore and Shaikh[35], and Mishra and Chand[36]. Spurred by the above examinations and conversation, in this work, we consider Five Dimensional Bianchi Type-V Cosmological Models with inflation of Universe in the SBTG. The work in this paper is designed as follows: In the section of ‘Model and The Field Equations’, we have considered a Five Dimensional Bianchi Type-V Cosmological Models with inflation of Universe in the SBTG and derived the field equations. In the section of ‘Solution of the Field Equations and Inflationary Universe Model’, we have considered the inflationary phase of solution where the volume is the constant function of scalar field. In last two sections “Kinematical and Physical Parameters” have been discussed. The outcomes are summed up with ends in the last section.

II. MODEL AND THE FIELD EQUATIONS

Let us consider Five Dimensional Bianchi Type-V Space-Time in the form of

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2\alpha x} (dy^2 + dz^2) + C^2 dm^2, \tag{1}$$

Where, A, B & C are the potential functions of cosmic time t .

The field equation in SBTG is

$$G_{ij} - \tilde{\omega} \varphi^n \left(\varphi_{,i} \varphi_{,j} - \frac{1}{2} g_{ij} \varphi_{,a} \varphi^{,a} \right) = -T_{ij}, \tag{2}$$

where the scalar field ϕ satisfying the equation,

$$2\phi^m \phi_{,j}^j + m\phi^{m-1} \phi_{,b} \phi^{,b} = 0; \tag{3}$$

where ϕ is the Scalar field and m is constant.

The inflationary universe of scalar field energy momentum tensor can be defined as,

$$T_i^j = \psi_{,i} \psi^{,i} - \left\{ \frac{1}{2} \psi_{,k} \psi^{,k} + v(\psi) \right\} g_i^j, \tag{4}$$

$$T_j^i = \text{diag} \left[\frac{1}{2} \dot{\psi}^2 - v(\psi), \frac{1}{2} \dot{\phi}^2 - v(\psi), \frac{1}{2} \dot{\phi}^2 - v(\psi), \frac{1}{2} \dot{\phi}^2 - v(\psi), -\frac{1}{2} \dot{\psi}^2 - v(\psi) \right] \tag{5}$$

Hence the resulting Saez-Ballester field equations from utilizing (2), (3), (4) and (5) with space-time (1) are

$$\frac{2\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} - \frac{\tilde{\omega}}{2} \phi^n \dot{\phi}^2 + \frac{\dot{\psi}^2}{2} - v(\psi) = 0 \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} - \frac{\tilde{\omega}}{2} \phi^n \dot{\phi}^2 + \frac{\dot{\psi}^2}{2} - v(\psi) = 0 \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} - \frac{\tilde{\omega}}{2} \phi^n \dot{\phi}^2 + \frac{\dot{\psi}^2}{2} - v(\psi) = 0 \tag{8}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + 2\frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} - \frac{\tilde{\omega}}{2} \phi^n \dot{\phi}^2 + \frac{\dot{\psi}^2}{2} - v(\psi) = 0 \tag{9}$$

$$\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right] = 0 \tag{10}$$

$$\ddot{\psi} + \dot{\psi} \left[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C} \right] + \frac{dv}{d\psi} = 0 \tag{11}$$

$$\ddot{\phi} + \left[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C} \right] \dot{\phi} + \frac{n}{2} \frac{\dot{\phi}^2}{\phi} = 0 \tag{12}$$

Here the overhead dot denotes the ordinary differentiation with respect to the cosmic time t and $\tilde{\omega}$ is constant.

III. SOLUTION OF THE FIELD EQUATIONS AND INFLATIONARY UNIVERSE MODEL

The arrangement of Equation (6) to (12) is having with six directly autonomous conditions with ten unknowns A, B, C, ϕ, ψ & v . In order to obtain its solution, we consider power-law form of the scale factor which has a significant importance in cosmology since it elegantly illustrates different cosmic evolutionary phases. Here the flat region is considered where the potential is constant i.e.

$$v(\psi) = \text{constant}. \tag{13}$$

And from equation (10) we get,

$$B = k_1 A \tag{14}$$

Then using equation (13) and (14) the set of field equations (6) to (12) becomes

$$\frac{2\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{2\dot{A}\dot{C}}{AC} + \frac{\dot{A}^2}{A^2} - \frac{1}{A^2} - \frac{\tilde{\omega}}{2} \phi^n \dot{\phi}^2 + \frac{\dot{\psi}^2}{2} - v(\psi) = 0 \tag{15}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} - \frac{3}{A^2} - \frac{\tilde{\omega}}{2} \phi^n \dot{\phi}^2 + \frac{\dot{\psi}^2}{2} - v(\psi) = 0 \tag{16}$$

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{C}}{AC} - \frac{3}{A^2} + \frac{\tilde{\omega}}{2} \phi^n \dot{\phi}^2 - \frac{\dot{\psi}^2}{2} - v(\psi) = 0 \tag{17}$$

$$\ddot{\psi} + \dot{\psi} \left[\frac{3\dot{B}}{B} + \frac{\dot{C}}{C} \right] = 0 \tag{18}$$

$$\ddot{\phi} + \left[\frac{3\dot{B}}{B} + \frac{\dot{C}}{C} \right] \dot{\phi} + \frac{n}{2} \frac{\dot{\phi}^2}{\phi} = 0 \tag{19}$$

Adding equation (16) and (17)

$$\frac{\ddot{A}}{A} + 2 \frac{\dot{A}^2}{A^2} + \frac{\dot{A}\dot{C}}{AC} - \frac{2}{A^2} = \frac{2}{3} v_0 \tag{20}$$

Subtract (15) from (16) we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + 2 \frac{\dot{A}^2}{A^2} - \frac{2\dot{A}\dot{C}}{AC} - \frac{2}{A^2} = 0 \tag{21}$$

Again subtracting equation (21) from (20) we get

$$\left(\frac{\dot{C}}{C} \right)' + \frac{\dot{C}}{C} \left(\frac{\dot{C}}{C} + 3 \frac{\dot{A}}{A} \right) = \frac{2}{3} v_0 \tag{22}$$

Define $\frac{\dot{C}}{C} = \gamma$ and $\frac{\dot{A}}{A} = \alpha$ (23)

Equation (22) becomes

$$\left(\dot{\gamma} - \frac{2}{3} v_0 \right) + \gamma(\gamma + 3\alpha) = 0 \tag{24}$$

This can be solved with following two considerations

Case (i):

$$\dot{\gamma} = \frac{2}{3} v_0 \text{ and } \gamma = 0 \tag{25}$$

Case (ii):

$$\dot{\gamma} = \frac{2}{3} v_0 \text{ and } \gamma = -3\alpha \tag{26}$$

Hence from (25) we have $v_0 = 0$ and then (26) and (14) becomes

$$A = e^{\left(-\frac{v_0 t^2}{9} - \frac{m_1 t}{3} + m_3 \right)}, B = k_1 e^{\left(-\frac{v_0 t^2}{9} - \frac{m_1 t}{3} + m_3 \right)}, C = e^{\left(\frac{v_0 t^2}{3} + m_1 t \right)}. \tag{27}$$

The model of the universe (1) with metric potentials (27) is obtained as,

$$ds^2 = -dt^2 + e^{2\left(m_3 - \frac{v_0 t^2}{9} - \frac{m_1 t}{3} \right)} dx^2 + k_1^2 e^{2\left(qx + m_3 - \frac{v_0 t^2}{9} - \frac{m_1 t}{3} \right)} (dy^2 + dz^2) + e^{2\left(m_2 + \frac{v_0 t^2}{3} + m_1 t \right)} dm^2 \tag{28}$$

The above model (28) of the universe is the power law cosmological model and we got the potential functions of this model in terms of power law.

IV. KINEMATICAL PARAMETERS OF THE MODEL

The spatial volume with respect to the average scale factor which is nothing but the special form of deceleration parameter is found out to be,

$$V = R^3 \Rightarrow V = k_1 e^{\left(qx + 2m_3 + m_2 + \frac{v_0 t^2}{9} + \frac{m_1 t}{3} \right)} \tag{29}$$

The behavior of the spatial Volume is showing by the graph given below and it is seen that the nature of spatial volume is increases exponentially. Also it is observed from the above equation of spatial volume at cosmic time t=0 the spatial volume gives the constant nature.

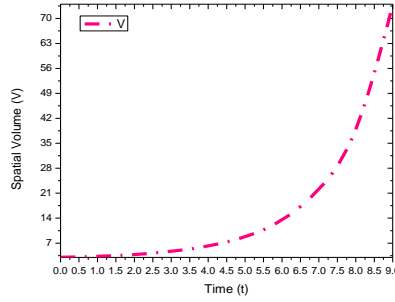


Figure (1): Plot of Spatial Volume Vs Time

The scale factor with respect to the special form of deceleration parameter is:

$$R = V^{1/3} = k_1^{1/3} e^{\left(\frac{qx+2m_3+m_2 + \frac{v_0 t^2}{9} + \frac{m_1 t}{3}}{3} \right)} \quad (30)$$

The resulting Scalar field with respect to the scale factor is found out to be,

$$\psi = \frac{k_1^3 e^{-3m_3}}{m_4} t \quad (31)$$

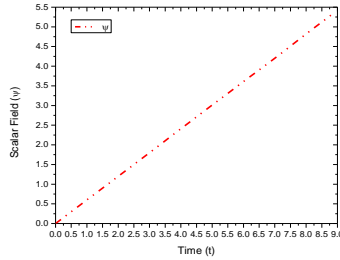


Figure (2): Plot of Scalar Field Vs Time

By observing above graph of scalar field it can be seen that the nature is totally linear and graph is straight line passing through origin and it is moving towards the infinite in nature with infinite time interval.

Utilizing the metric potentials (28) the resulting Saez- Ballester scalar potential of the Universe is found out to be,

$$\phi = \left[\frac{2m_5}{(n+1)3k_1^3 m_3} \right]^{2/n+2} (t + m_6)^{2/n+2} \quad (32)$$

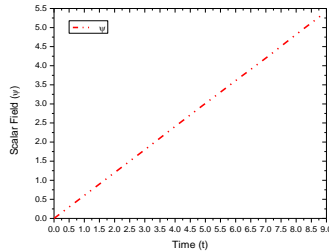


Figure (3): Plot of Saez-Ballester Scalar Field Vs Time

The nature of Saez-Ballester Scalar field of the inflationary universe is also a straight line and varying with infinite time interval.

IV. PHYSICAL PARAMETERS

The Hubble parameter for model (1) is found out to be,

$$H = \frac{\dot{R}}{R} = \frac{2v_0t + 3m_1}{27} \tag{33}$$

The graphical behavior of the Hubble parameter is shown by the graph mention below and the scale is linear and is straight line passing through the origin and varies with infinite time interval.

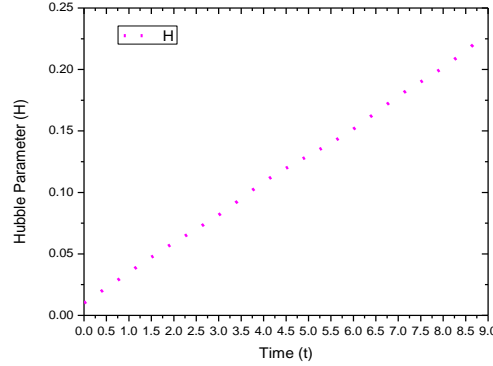


Figure (4): Plot of Hubble Parameter Vs Time

The obtained expansion scalar θ is

$$\theta = 3H = \frac{2v_0t + 3m_1}{9} \tag{34}$$

The overall graphical nature of the Expansion Scalar of the given inflationary universe is as shown in the following graph:

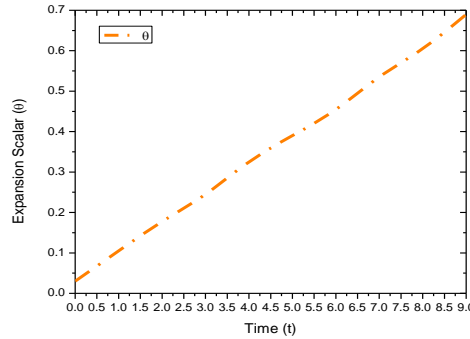


Figure (5): Plot of Expansion Scalar Vs Time

The mean anisotropy parameter for the given expansion of the model (1) is found out to be,

$$A_m = \frac{1}{3} \sum_{j=1}^4 \left\{ \frac{\Delta H_j}{H} \right\}^2 = 12(2v_0t + 3m_1)^2 - 4/3 \tag{34}$$

The resulting shear scalar of the given model is

$$\sigma^2 = \frac{3}{2} H^2 A_m = \frac{3}{2} \left[\frac{2v_0t + 3m_1}{27} \right]^2 \left\{ 12(2v_0t + 3m_1)^2 - 4/3 \right\} \tag{35}$$

V. CONCLUSIONS

In this present article, we have explored the inflationary universe models with five dimensional Bianchi Type-V metric in the frame work of SBTG. We have thought of Special form of deceleration parameter in this situation. We got the equations for pressure and energy density of inflationary model of the universe. The Scalar field and the Saez-Ballester Scalar Field of the given inflationary model is having a straight line in nature and also incenses infinitely with infinitetime interval. Some physical and kinematical parameters of the models have been graphically described.

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