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Distance Polynomial, Distance Spectra And Distance Energy Of Some Edge Deleted Graphs

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Abstract - *The distance polynomial of G is defined as the determinant |μI-D|, where I is the unit matrix of the order same as that of D. The distance spectra of a connected graph G is the collection of distance eigenvalues of G and the distance energy of G is the absolute sum of the distance eigenvalues of G. In this paper, the distance spectra and the distance energy of the graph obtained by deleting the edges of complete subgraph K_r from the complete graph K_p are obtained.*

Keywords — *Complete graph, Distance eigenvalue of a graph, Distance spectrumof a graph, Distance energy of a graph.*

I. INTRODUCTION

Let V(G) be the vertex set of a simple connected graph G with vertices $v_1, v_2, \ldots, v_p$. The distance between the vertices v_i and v_j in G is the length of the shortest path between v_i and v_j in G. The distance matrix of G is denoted by D = D(G), is the square matrix with the diagonal entry $d_i i = 0$,for i = 1, 2, …,p and the non-diagonal entry d_{ij}, where dij is the distance between the vertices v_i and v_j in G, for i, j = 1, 2, …, p. The distance polynomial of G is defined as the determinant |μI-D|, where I is the unit matrix of the order same as that of D. The eigenvalues of D are said to be the distance eigenvalues (D-eigenvalues) of Gand the collection of distance eigenvalues of G forms the distance spectrum (D-spectrum) of G, denoted by specD(G) .

Since the distance matrix is symmetric, all its eigenvalues μ_i, $i = 1, 2, \ldots, p$,are real and can be labeled, so that $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_p$. If $\mu_1 > \mu_2 > \ldots > \mu_t$ are the distinct D-eigenvalues of G, then the D-spectrum can be written as,

$$\mathrm{Spec}_D(G) = \begin{pmatrix} \mu_1 & \mu_2 & \cdots & \mu_t \\ m_1 & m_2 & \cdots & m_g \end{pmatrix}$$

Where m_g indicates the algebraic multiplicity of the eigenvalue μ_t such that $m_1 + m_2 + \ldots + m_g = p$.

The distance energy or simply, the D-energy $E_D(G)$ of G is defined as,

$$E_D(G) = \sum_{i=1}^{p} |\mu_i| \qquad (1)$$

The concept of distance energy of a connected graph G is defined in [6]. This definition was motivated by the much older and nowadays extensively studied graph energy defined in a manner fully analogous to Eq. (1), but in terms of the ordinary graph spectrum (collection of eigenvalues of the adjacency matrix of a graph)[1 – 17].

Two graphs are said to be distance equienergetic graphs, if they have the same distance energy.

If M is a non – singular square matrix, then $\begin{vmatrix} M & N \\ P & Q \end{vmatrix} = |M||Q - PM^{-1}N|$ (2)

The spectral graph theoretic definitions in this paper follow the book[1]. All graphs considered in this paper are simple and connected.

II. Main Results

Let K_p be the complete graph on p vertices. An edge deleted graph G – *e* is the subgraph of G obtained by deleting the edge *e* from G without deleting the end vertices of G.

Two subgraphs G_1 and G_2 are said to be independent, if $V(G_1) \cap V(G_2)$ is an empty set.

Definition 2.1 [12]: Let K_{m_1} and K_{m_2} are the two independent copmlete subgraphs of a complete graph K_p. The graph obtained by deleting all the edges of $k_1 + k_2$ independent complete subgraphs, K_{m_1} (k_1 copies), K_{m_2} (k_2 copies) from K_p, is denoted by $K_{c_p}\left(p,(m_1,k_1),(m_2,k_2)\right)$ such that $\sum_{i=1}^{2} m_i k_i \le p$.

For the sake of brevity, we use $K_{c_p}(p,m,k)$ in place of $K_{c_p}\left(p,(m_1,k_1),(m_2,k_2)\right)$.

Theorem 2.1. Let $p,\ m_i, k_i,\ i = 1,2$ are positive integers with $\sum_{i=1}^{2} m_i k_i \le p$. Then for $p \ge 3$, the distance polynomial of $K_{c_p}\left(p,(m_1,k_1),(m_2,k_2)\right)$ is ,

$$\varphi[K_{c_p}(P,(m_1,k_1),(m_2,k_2)):\mu] =$$

$$(1+\mu)^{P-\sum_{i=1}^{2} m_i k_i - 1}(2+\mu)^{\sum_{i=1}^{2} m_i k_i - \sum_{i=1}^{2} k_i}(m_1-\mu-2)^{k_1-1}(m_2-\mu-2)^{k_2} \times$$

$$\{(P-m_1-\mu)(1+\mu) - \frac{m_2 k_2 (m_2-m_1)(1+\mu)}{m_2-\mu-2} - [(P-\sum_{i=1}^{2} m_i k_i - 2(1+\mu)](m_1-1)\}$$

Proof: Let 1, 2,…, m_1, m_1+1, m_1+2,…, $2m_1$, $2m_1+1$, $2m_1+2$,…, m_1k_1, m_1k_1+1, m_1k_1+2,…, $m_1k_1+m_2$, $m_1k_1+m_2+1$, $m_1k_1+m_2+2$,…, $m_1k_1+m_2k_2$, $m_1k_1+m_2k_2+1$, $m_1k_1+m_2k_2+2$,..., p are the vertices of K_p and hence the vertices of $K_{c_p}(p,m,k)$.

The distance polynomial of $K_{c_p}(p,m,k)$ is equal to the det|μI-D|. In the computations followed, we take det|D-μI| in place of det|μI-D| and at the end the result obtained will be multiplied by$(-1)^p$.

Now, det|D-μI| is equal to

1	2	…	m_1	m_1+1	m_1+2	…	$2m_1$	$2m_1+1$	$2m_1+2$	…	m_1k_1	m_1k_1+1	m_1k_1+2	…	$m_1k_1+m_2$	$m_1k_1+m_2+1$	$m_1k_1+m_2+2$	…	$m_1k_1+m_2k_2$	$m_1k_1+m_2k_2+1$	$m_1k_1+m_2k_2+2$	…	p		

$$\begin{vmatrix}
-\mu & 2 & \cdots & 2 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
2 & -\mu & \cdots & 2 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
. & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & & . \\
2 & 2 & \cdots & -\mu & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & -\mu & 2 & \cdots & 2 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 2 & -\mu & \cdots & 2 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
. & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & & . \\
1 & 1 & \cdots & 1 & 2 & 2 & \cdots & -\mu & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & -\mu & 2 & \cdots & 2 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 2 & -\mu & \cdots & 2 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
. & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & & . \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 2 & 2 & \cdots & -\mu & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & -\mu & 2 & \cdots & 2 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 2 & -\mu & \cdots & 2 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & & . \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 2 & 2 & \cdots & -\mu & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & -\mu & 2 & \cdots & 2 & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 2 & -\mu & \cdots & 2 & 1 & 1 & \cdots & & 1 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & & . \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 2 & 2 & \cdots & -\mu & 1 & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & -\mu & 1 & \cdots & & 1 \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & -\mu & \cdots & & 1 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & & . \\
 & & & & & & & & & & & & & & & & & & & & & & & -\mu & \\
1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & & -\mu
\end{vmatrix}$$

Subtracting first column from all other columns and setting $X = 1 + \mu$, we get,

$$\begin{vmatrix}
-\mu & 1+X & \cdots & 1+X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X \\
2 & -(1+X) & \cdots & 0 & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 \\
. & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
2 & 0 & \cdots & -(1+X) & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 & -1 & -1 & \cdots & -1 \\
1 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
. & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
1 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
. & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & . \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & . \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & . \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & -X & \cdots & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & . \\
 & & & & & & & & & & & & & & & & & & & & & & & \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -X
\end{vmatrix}$$

Multiplying the rows 2,3,…,m_1 by $-X$ and hence dividing the determinant by $(-X)^{m_1-1}$

$$=(-X)^{1-m_1}\begin{vmatrix}
-\mu & 1+X & \cdots & 1+X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X \\
-2X & X(1+X) & \cdots & 0 & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X \\
. & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
-2X & 0 & \cdots & X(1+X) & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X \\
1 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
. & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
1 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
. & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & . & . & . & . & . \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & . & . & . & . & . \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & . & . & . & . & . \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & -X & \cdots & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \ddots & . \\
 & & & & & & & & & & & & & & & & & & & & & & & \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -X
\end{vmatrix}$$

Adding the rows $\sum_{i=1}^{2} m_i k_i + 1, \ldots, p$ to the rows 1, 2,…,m_1 and set $b = p - \sum_{i=1}^{2} m_i k_i - \mu$, $c = p - \sum_{i=1}^{2} m_i k_i - 2X$

$$
= (-X)^{1-m_1} \left| \begin{array}{cccccccccccccccccccccccc}
b & 1+X & \cdots & 1+X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & 0 & 0 & \cdots & 0 \\
c & X(1+X) & \cdots & 0 & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
c & 0 & \cdots & X(1+X) & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & X & X & \cdots & X & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & -X & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -X
\end{array} \right|
$$

Above determinant is in the form $\begin{vmatrix} M & N \\ P & Q \end{vmatrix}$, where N is a ZERO block. Therefore, Using Eq.(2), we get,

$$
|D - \mu I| = (-X)^{p - \sum_{i=1}^{2} m_i k_i - m_1 + 1} |M| \qquad (3)
$$

Where,

$$
|M| = \left| \begin{array}{ccccccccccccc}
b & 1+X & 1+X & \cdots & 1+X & X & X & \cdots & X & X & X & \cdots & X \\
c & X(1+X) & 0 & \cdots & 0 & X & X & \cdots & X & X & X & \cdots & X \\
c & 0 & X(1+X) & \cdots & 0 & X & X & \cdots & X & X & X & \cdots & X \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
c & 0 & 0 & \cdots & X(1+X) & X & X & \cdots & X & X & X & \cdots & X \\
1 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X
\end{array} \right|
$$

Extracting $1 + X$ from each of the columns 2, 3,…, m_1

$$
=(1+X)^{m_1-1}\begin{vmatrix}
b & 1 & 1 & \cdots & 1 & X & X & \cdots & X & X & X & \cdots & X \\
c & X & 0 & \cdots & 0 & X & X & \cdots & X & X & X & \cdots & X \\
c & 0 & X & \cdots & 0 & X & X & \cdots & X & X & X & \cdots & X \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
c & 0 & 0 & \cdots & X & X & X & \cdots & X & X & X & \cdots & X \\
1 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X
\end{vmatrix}
$$

Subtracting the columns 2,3,…,m_1 from $m_1+1, m_1+2, \ldots, m_1k_1+1, m_1k_1+2, \ldots m_2k_2$

i.e. Applying $C_i-\sum_{j=2}^{m_1} C_j, i=m_1+1, m_1+2, \ldots, m_2k_2$ and setting $Z=X-m_1-1$, we get,

$$
=(1+X)^{m_1-1}\begin{vmatrix}
b & 1 & 1 & \cdots & 1 & Z & Z & \cdots & Z & Z & Z & \cdots & Z \\
c & X & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
c & 0 & X & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
c & 0 & 0 & \cdots & X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X
\end{vmatrix}
$$

Adding the rows $m_1+1, m_1+2, \ldots, \sum_{i=1}^{2} m_i k_i$ to first row and setting $t=p-m_1-\mu$, we get,

$$
=(1+X)^{m_1-1}\begin{vmatrix}
t & 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & m_2-m_1 & m_2-m_1 & \cdots & m_2-m_1 \\
c & X & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
c & 0 & X & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
c & 0 & 0 & \cdots & X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & -X & \cdots & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & -X
\end{vmatrix}
$$

Subtracting the rows $m_1+2, m_1+3, \ldots, 2m_1$ from m_1+1

$2m_1+2, 2m_1+3, ..., 2m_1$ from $2m_1$

……………………………………

$\sum_{i=1}^{2} m_i k_i - m_2 + 2, ..., \sum_{i=1}^{2} m_i k_i$ from $\sum_{i=1}^{2} m_i k_i + 1$

$$= (1+X)^{m_1-1}\begin{vmatrix} t & 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & m_2-m_1 & m_2-m_1 & \cdots & m_2-m_1 \\ c & X & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ c & 0 & X & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c & 0 & 0 & \cdots & X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1+X & -(1+X) & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1+X & 1 & \cdots & -(1+X) & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X & 1 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1+X & -(1+X) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1+X & 0 & \cdots & -(1+X) \end{vmatrix}$$

From second column blocks onwards, in each of the column blocks, adding all the columns to first column, we get,

$$= (1+X)^{m_1-1}\begin{vmatrix} t & 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & m_2^2-m_1m_2 & m_2-m_1 & \cdots & m_2-m_1 \\ c & X & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ c & 0 & X & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c & 0 & 0 & \cdots & X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 & -X+m_1-1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -(1+X) & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -(1+X) & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X+m_2-1 & 1 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & -(1+X) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -(1+X) \end{vmatrix}$$

Dividing first column of each column blocks by $-X+m_i-1$ and subtract all these from C$_1$,

i.e. $C_1 + \dfrac{\sum_{j=1}^{\sum_{i=1}^{2} m_i k_i} C_{j+1}}{-X+m_2+1}$ and setting $\beta = t - \dfrac{k_2(m_2^2 - m_1 m_2)}{-X+m_2-1}$, we get,

$$= (1+X)^{m_1-1}\begin{vmatrix} \beta & 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & m_2^2-m_1m_2 & m_2-m_1 & \cdots & m_2-m_1 \\ c & X & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ c & 0 & X & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c & 0 & 0 & \cdots & X & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & -X+m_1-1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -(1+X) & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -(1+X) & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -X+m_2-1 & 1 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & -(1+X) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -(1+X) \end{vmatrix}$$

On simplification, we get,

$$|M| = (-1)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i - m_1 + 1}(1+X)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i}(m_1 - X - 1)^{k_1-1}(m_2 - X - 1)^{k_2}\begin{vmatrix} \beta & 1 & 1 & \cdots & 1 \\ c & X & 0 & 0 & 0 \\ c & 0 & X & 0 & 0 \\ c & 0 & 0 & \ddots & 0 \\ c & 0 & 0 & 0 & X \end{vmatrix}$$

Applying $R_i - R_2$ for i=3,4,…,m₁, we get,

$$|M| = (-1)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i - m_1 + 1}(1+X)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i}(m_1 - X - 1)^{k_1-1}(m_2 - X - 1)^{k_2}\begin{vmatrix} \beta & 1 & 1 & \cdots & 1 \\ c & X & 0 & 0 & 0 \\ 0 & -X & X & 0 & 0 \\ 0 & -X & 0 & \ddots & 0 \\ 0 & -X & 0 & 0 & X \end{vmatrix}$$

Operating $C_2 + \sum_{i=3}^{m_1} C_i$, gives

$$= (-1)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i - m_1 + 1}(1+X)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i}(m_1 - X - 1)^{k_1-1}(m_2 - X - 1)^{k_2}\begin{vmatrix} \beta & m_1-1 & 1 & \cdots & 1 \\ c & X & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & X \end{vmatrix}$$

$$= (-1)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i - m_1 + 1}(1+X)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i}(m_1 - X - 1)^{k_1-1}(m_2 - X - 1)^{k_2} X^{m_1-2}\begin{vmatrix} \beta & m_1-1 \\ c & X \end{vmatrix} \quad (4)$$

Simplification of (5) gives,

$$|M| = X^{m_1-2}(1+X)^{\sum_{i=1}^{2} m_ik_i - \sum_{i=1}^{2} k_i}(m_1 - X - 1)^{k_1-1}(m_2 - X - 1)^{k_2} \times \left[(p - m_1 - \mu)X - \frac{m_2k_2(m_2 - m_1)X}{m_2 - X - 1} - (p - \sum_{i=1}^{2} m_ik_i - 2X)(m_1 - 1)\right] \quad (5)$$

Using (5) in (3), and multiplying the obtained result by $(-1)^{p}$, we get the result of Theorem 2.1.

III. DISTANCE SPECTRA AND DISTANCE ENERGY OF $K_{c_p}\left(p,(m_1,k_1),(m_2,k_2)\right)$

Theorem 3.1. For $p \geq 3, \sum_{i=1}^{2} m_i k_i < p$, the distance spectra of $K_{c_p}(p,m,k)$ contains the D-eigenvalues

-1	$(p-\sum_{i=1}^{2} m_i k_i - 1$ times),
-2	$(\sum_{i=1}^{2} m_i k_i - \sum_{i=1}^{2} k_i$ times),
m_1-2	(k_1-1 times),
m_2-2	(k_2-1 times),
α_1	(1 time),
α_2	(1 time),
α_3	(1 time)

Where α_1, α_2 and α_3 are the roots of the equation

$$\{(p-m_1-\mu)(m_2-\mu-2)-m_2k_2(m_2-m_1)\}(1+\mu)-\{p-\sum_{i=1}^{2} m_i k_i - 2(1+\mu)\}(m_2-\mu-2)(m_1-1)=0$$

Proof: Proof follows easily by Theorem 2.1.

Theorem 3.2. For $p \geq 3, \sum_{i=1}^{2} m_i k_i < p$, the distance energy of $K_{c_p}(p,m,k)$ is

$$E_D[K_{c_p}(p,m,k)] = p + 2\sum_{i=1}^{2} m_i k_i - 4\sum_{i=1}^{2} k_i - \sum_{i=1}^{2} m_i + 3 + |\alpha_1| + |\alpha_2| + |\alpha_3|$$

Proof. D-energy of G is given by $E_D(G) = \sum_{i=1}^{p} |\mu_i|$, where μ_i is D-eigenvalue of G, for i = 1, 2, …, p. Hence the result of Theorem 3.2. follows by using the D-spectra obtained in Theorem 3.1.

Definition 3.1 [12]: Let $K_{m_i}; i = 1,2,...,l$,are the *l* independent complete subgraphs of K_p. The graph obtained by deleting all the edges of $\sum_{i=1}^{l} k_i$, independent complete subgraphs K_{m_1} (k_1 copies), K_{m_2} (k_2 copies),…, K_{m_l} (k_l copies) from a complete graph K_p is denoted by $K_{c_p}\left(p,(m_1,k_1),(m_2,k_2),...,(m_l,k_l)\right)$, such that $\sum_{i=1}^{l} m_i k_i \leq p$.

Theorem 3.4. Let $p,\ m_i, k_i,\ i = 1,2,...,l$ are positive integers with $\sum_{i=1}^{l} m_i k_i \leq p$. Then for $p \geq 3$, the distance polynomial of $K_{c_p}\left(p,(m_1,k_1),(m_2,k_2),...,(m_l,k_l)\right)$ is ,

$$\phi(K_{c_p}\left(p,(m_1,k_1),(m_2,k_2),...,(m_l,k_l)\right):\mu) =$$

$$(1+\mu)^{p-\sum_{i=1}^{l} m_i k_i - 1}(2+\mu)^{\sum_{i=1}^{l} m_i k_i - \sum_{i=1}^{l} k_i}(m_1-\mu-2)^{k_1-1} \times$$

$$\prod_{i=2}^{l}(m_i-\mu-2)^{k_i} \times \{[(p-m_1-\mu)(1+\mu) - \sum_{i=2}^{l} \frac{k_i(1+\mu)(m_i^2-m_1m_i)}{m_i-\mu-2} - (p-\sum_{i=1}^{l} m_i k_i - 2\mu-2)(m_1-1)\}$$

Proof. Proof of this theorem is omitted as it follows on the same lines as that of Theorem 2.1.

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