International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V67I7P513
Volume 67 Issue 7, 104-112, July, 2021
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Nano Generalized pre c-Irresolute and Nano Contra Generalized pre c-Irresolute Functions

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Abstract – *In this paper the concept of Nano Generalized pre c-Irresolute and Contra Generalized pre c-Irresolute functions are introduced. The Characterization and properties of these functions relating Ngpc-int,Ngpc-cl,Ngpc-ker,Ngpc-surf with Nint, Ncl and Nker are investigated. Also Nano Generalized pre c-closed and Nano Generalized pre c-open maps are introduced.*

Keywords — *Ngpc-closed set, Ngpc-continuous function, Ncgpc-continuous function, Ngpc-orresolute function,Ncgpc-irresolute function*

I. INTRODUCTION

The importance of irresolute functions are significant in various areas of Mathematics and related Sciences. The idea of irresoluteness was introduced in 1972 by Crossley and Hildebrand[1]. Various forms of nano irresolute functions have been investigated over the years.This paper gives the development of the theory of nano generalized pre c-irresolute and nano contra generalized pre c-irresolute functions.

II. PRELIMINARIES

We recall the following definitions.

Definition 2.1. Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be approximation space. Let X ⊆ U. Then

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = \cup_{x \in U} \{R(x): R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by $L_R(X)$.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = \cup_{x \in U} \{R(x): R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2. If (U, R) is an approximation space and $X, Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\emptyset) = U_R(\emptyset) = \emptyset$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever X ⊆ Y.
9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3. Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where X ⊆ U. Then $\tau_R(X)$ satisfies the following axioms.

(i) U and $\emptyset \in \tau_R(X)$.

(ii) The union of all the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

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Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call (U, $\tau_R(X)$) as a nano topological space. The elements of $\tau_R(X)$are called as nano open sets. The complement of the nano open sets are called nano closed sets.

Definition 2.4. [2] If (U, $\tau_R(X)$) is a nano topological space with respect to X where X ⊆ U and if A ⊆ U then

(i) The nano interior of A is defined as the union of all nano open subsets contained in A and is denoted by $Nint(A)$. That is $Nint(A)$) is the largest nano open subset of A.

(ii) The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$. That is $Ncl(A)$is the smallest nano closed set containing A.

Definition 2.5..[5] A subset A of a nano topological space $(U, \tau_R(X))$ is called a nano generalized pre c-closed set (briefly Ngpc-closed set) if $Npcl(A) \subseteq G$ whenever A ⊆ G and G is nano c-set.

Definition 2.6. [5] The Nano generalized pre c-interior of A is defined as the union of all Ngpc-open sets of U contained in A and it is denoted by Ngpc-int(A).

Definition 2.7. [5] The Nano generalized pre c-closure of A is defined as the intersection of all Ngpc-closed sets of U containing A and it is denoted by Ngpc-cl(A).

Definition 2.8. [5] The Nano generalized pre c-kernel of A is defined as the intersection of all Ngpc-open sets of U containing A and it is denoted by Ngpc-ker(A).

Definition 2.9. [6] The Nano generalized pre c-surface of A is defined as the union of all Ngpc-closed sets of U contained in A and it is denoted by Ngpc-surf (A).

Definition 2.10. Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$ be two nano topological spaces. The function $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is called

(i) [3]nano continuous on U if the inverse image of every nano open set in V is a nano open set in U.

(ii) [4]nano contra continuous on U if the inverse image of every nano open set in V is a nano closed set in U.

(iii) [6]Ngpc-continuous on U if the inverse image of every nano open set in V is a Ngpc-open set in U.

(iv) [6]Ncgpc-continuous on U if the inverse image of every nano open set in V is a Ngpc-closed set in U.

III. NANO GENERALIZED PRE C-IRRESOLUTE FUNCTIONS

In this section Nano generalized pre-c Irresolute function is defined and its characterizations and properties with respect to Ngpc-int, Ngpc-cl, Ngpc-ker and Ngpc-surf of sets are derived.

Definition 3.1. Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$ be two nano topological spaces. The function $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is said to be Nano generalized pre c-irresolute (briefly Ngpc- irresolute) on U if the inverse image of every Ngpc-open set in V is a Ngpc-open set in U.

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/_R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology with respect to X. Ngpc-closed sets are $\emptyset, U, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau'_R(Y) = \{\emptyset, V, \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology with respect to Y. Ngpc-closed sets are $\emptyset, V, \{y\}, \{z\}, \{w\}, \{x, z\}, \{y, z\}, \{z, w\}, \{x, y, z\}, \{x, z, w\}, \{y, z, w\}$. Define $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ as $f(a) = z, f(b) = x, f(c) = y, f(d) = w$. Then f is Ngpc-irresolute since the inverse image of every Ngpc-open set in V is a Ngpc-open set in U.

Theorem 3.3. A function $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ngpc-irresolute if and only if the inverse image of every Ngpc-closed set in V is Ngpc-closed in U.

Proof. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be Ngpc-irresolute. Let A be Ngpc-closed in V. Then A^C is Ngpc-open in V. Since f is Ngpc-irresolute $f^{-1}(A^C)$ is Ngpc-open in U. Therefore $f^{-1}(A)$ is Ngpc-closed in U. Thus the inverse image of every Ngpc-closed set in V is Ngpc-closed in U. Conversely let the inverse image of every Ngpc-closed set in V is Ngpc-closed in U. Let B be a Ngpc-open set in V. Then B^C is Ngpc-closed in V. By our assumption $f^{-1}(B^C) = (f^{-1}(B))^c$ is Ngpc-closed in U.Therefore $f^{-1}(B)$ is Ngpc-open in U. Thus the inverse image of every Ngpc-open set in V is Ngpc-open in U. Hence f is Ngpc-irresolute.

Theorem 3.4. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. Then the following statements are equivalent.

(i) f is Ngpc-irresolute.

(ii) For every subset A of U, Ngpc-cl $(f^{-1}($ Ngpc-cl$(f(A)))) = f^{-1}($ Ngpc-cl$(f(A)))$.

(iii) For every subset B of V, Ngpc-cl $(f^{-1}($Ngpc-cl $(B)))= f^{-1}($Ngpc-cl $(B))$.

Proof. (i) ⟺ (ii). Let f be Ngpc-irresolute and $A \subseteq U$. Then $f(A) \subseteq V$. Ngpc-cl$(f(A))$ is Ngpc- closed in V. Since f is Ngpc-irresolute, $f^{-1}($Ngpc-cl$(f(A)))$ is Ngpc-closed in U. Therefore Ngpc-cl$(f^{-1}($Ngpc-cl$(f(A))) = f^{-1}($Ngpc-cl$(f(A)))$.

Conversely let Ngpc-cl $(f^{-1}($ Ngpc-cl$(f(A)))) = f^{-1}($ Ngpc-cl$(f(A)))$ for every subset A of U. Let H be a Ngpc-closed set in V. Since $f^{-1}(H) \subseteq U$, Ngpc-cl $(f^{-1}($ Ngpc-cl$(f(f^{-1}(H))))) = f^{-1}($ Ngpc-cl$(f(f^{-1}(H))))$. That is Ngpc-cl $(f^{-1}(H)) = f^{-1}(H)$ implies that $f^{-1}(H)$ is Ngpc-closed in U. Hence f is Ngpc-irresolute.

(ii) ⟺ (iii). Assume (ii) holds. Let B be any subset of V. Then replacing A by $f^{-1}(B)$ in (ii) we have Ngpc-cl (f^{-1}(Ngpc-cl$\left(f\left(f^{-1}(B)\right)\right)$)) $= f^{-1}$(Ngpc-cl$\left(f\left(f^{-1}(B)\right)\right)$) . That is Ngpc-cl ($f^{-1}$(Ngpc-cl ($B$)))= f^{-1}(Ngpc-cl (B)).

Conversely suppose (iii) holds. Let A be any subset of U. Then $f(A) \subseteq V$. Let $B = f(A)$. Then we have Ngpc-cl (f^{-1}(Ngpc-cl $(f(A))$))= f^{-1}(Ngpc-cl $(f(A))$) for every subset A of U.

Theorem 3.5. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. Then the following statements are equivalent.

(i) f is Ngpc-irresolute.

(ii) For every subset A of U, Ngpc-int (f^{-1}(Ngpc-int$(f(A))$)) $= f^{-1}$(Ngpc-int$(f(A))$).

(iii) For every subset B of V, Ngpc-int (f^{-1}(Ngpc-int(B)))= f^{-1}(Ngpc-int(B)).

Proof. (i) ⟺ (ii). Let f be Ngpc-irresolute and $A \subseteq U$. Then $f(A) \subseteq V$. Ngpc-int($f(A)$) is Ngpc-open in V. Since f is Ngpc-irresolute, f^{-1}(Ngpc-int$(f(A))$) is Ngpc-open in U. Therefore Ngpc-int(f^{-1}(Ngpc-int$(f(A))$)) $= f^{-1}$(Ngpc-int$(f(A))$).
Conversely let Ngpc-int (f^{-1}(Ngpc-int$(f(A))$)) $= f^{-1}$(Ngpc-int$(f(A))$) for every subset A of U. Let G be a Ngpc-open set in V. Since $f^{-1}(G) \subseteq U$, Ngpc-int(f^{-1}(Ngpc-int $\left(f\left(f^{-1}(G)\right)\right)$)) $= f^{-1}$(Ngpc-int$\left(f\left(f^{-1}(G)\right)\right)$. That is Ngpc-int ($f^{-1}(G)$) $=$ $f^{-1}(G)$ implies that $f^{-1}(G)$ is Ngpc-open in U. Hence f is Ngpc-irresolute.

(ii) ⟺ (iii). Assume (ii) holds. Let B be any subset of V. Then replacing A by $f^{-1}(B)$ in (ii) we have Ngpc-int (f^{-1}(Ngpc-int$\left(f\left(f^{-1}(B)\right)\right)$)) $= f^{-1}$(Ngpc-int$\left(f\left(f^{-1}(B)\right)\right)$) . That is Ngpc-int ($f^{-1}$(Ngpc-int ($B$)))= f^{-1}(Ngpc-int (B)).

Conversely suppose (iii) holds. Let A be any subset of U. Then $f(A) \subseteq V$. Let $B = f(A)$. Then we have Ngpc-int (f^{-1}(Ngpc-int $(f(A))$)) $= f^{-1}$(Ngpc-int $(f(A))$) for every subset A of U.

Equality does not hold in theorems (3.4) and (3.5).

Example 3.6. In example (3.2)

Let $A = \{a, b, c\} \subseteq U$ and $B = \{x, y, w\} \subseteq V$. Then $f^{-1}(B) = \{b, c, d\}$.

(i) Now f^{-1}(Ngpc-cl($f(A)$)) $= f^{-1}$(Ngpc-cl($f(\{a, b, c\})$)) $= f^{-1}$(Ngpc-cl($\{x, y, z\}$) $= f^{-1}(\{x. y. z\}) = \{a, b, c\}$and Ngpc-cl($f^{-1}$(Ngpc-cl($f(A)$))) = Ngpc-cl ($\{a, b, c\}$) $= \{a, b, c\}$ and thus Ngpc-cl (f^{-1}(Ngpc-cl$(f(A))$)) $= f^{-1}$(Ngpc-cl$(f(A))$).
Also f^{-1}(Ngpc-cl(B)) $= f^{-1}$(Ngpc-cl($\{x, y, w\}$)) $= f^{-1}(V) = U$and Ngpc-cl(f^{-1}(Ngpc-cl(B))) = Ngpc-cl $(U) = U$. Thus Ngpc-cl (f^{-1}(Ngpc-cl (B)))= f^{-1}(Ngpc-cl (B)).

(ii) Now f^{-1}(Ngpc-int($f(A)$)) $= f^{-1}$(Ngpc-int($f(\{a, b, c\})$)) $= f^{-1}(\{x, y, z\}) = \{a, b, c\}$ and Ngpc-int(f^{-1}(Ngpc-int($f(A)$))) = Ngpc-int($\{a, b, c\}$) $= \{a, b, c\}$ and thus Ngpc-int (f^{-1}(Ngpc-int$(f(A))$)) $= f^{-1}$(Ngpc-int$(f(A))$).
Also f^{-1}(Ngpc-int(B)) $= f^{-1}$(Ngpc-int($\{x, y, w\}$)) $= f^{-1}(\{x, y, w\}) = \{b, c, d\}$ and Ngpc-int(f^{-1}(Ngpc-int(B))) = Ngpc-int ($\{b, c, d\}$) $= \{b, c, d\}$. Thus Ngpc-int (f^{-1}(Ngpc-int(B)))= f^{-1}(Ngpc-int(B)).

Theorem 3.7. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a Ngpc-irresolute function. Then we have

(i) Ngpc-int $(f(A)) \subseteq$ f (Ngpc-ker(A)) for every subset A of U.

(ii) f (Ngpc-surf(A)) ⊆ Ngpc-cl $(f(A))$ for every subset A of U.

(iii) f^{-1}(Ngpc-int (B)) ⊆ Ngpc-ker $(f^{-1}(B))$ for every subset B of V.

(iv) Ngpc-surf $(f^{-1}(B)) \subseteq$ f^{-1}(Ngpc-cl (B)) for every subset B of V.

Proof. Let f be Ngpc-irresolute and A be a subset of U.

(i) Then $f(A) \subseteq V$ and Ngpc-int $(f(A))$ is Ngpc-open in V. Since f is Ngpc-irresolute f^{-1}(Ngpc-int $(f(A))$) is Ngpc-open in U. Therefore Ngpc-ker (f^{-1}(Ngpc-int $(f(A))$)) $= f^{-1}$(Ngpc-int $(f(A))$) . But Ngpc-int$(f(A)) \subseteq$ $f(A)$ implies f^{-1}(Ngpc-int$(f(A))$) $\subseteq A$. This implies Ngpc-ker(f^{-1}(Ngpc-int$(f(A))$)) ⊆ Ngpc-ker (A). Hence f^{-1}(Ngpc-int $(f(A))$) ⊆ Ngpc-ker (A) shows that Ngpc-int $(f(A)) \subseteq$ f (Ngpc-ker(A)).

(ii) Then $f(A) \subseteq V$ and Ngpc-cl $(f(A))$ is Ngpc-closed in V. Since f is Ngpc-irresolute f^{-1}(Ngpc-cl$(f(A))$) is Ngpc-closed in U. Therefore Ngpc-surf (f^{-1}(Ngpc-cl $(f(A))$)) $= f^{-1}$(Ngpc-cl $(f(A))$) . Since $f(A) \subseteq$ Ngpc-cl$(f(A))$, $A \subseteq f^{-1}$(Ngpc-cl$(f(A))$) . This implies Ngpc-surf (A) ⊆ Ngpc-surf f^{-1}(Ngpc-cl$(f(A))$). Hence Ngpc-surf (A) ⊆ f^{-1}(Ngpc-cl $(f(A))$) shows that f (Ngpc-surf(A)) ⊆ Ngpc-cl $(f(A))$ for every subset A of U.

Let f be Ncgpc-irresolute and B be a subset of V.

(iii) Then Ngpc-int (B) is Ngpc-open in V and f^{-1}(Ngpc-int(B) is Ngpc-open in U. Therefore Ngpc-ker (f^{-1}(Ngpc-int (B)) $= f^{-1}$(Ngpc-int (B). But Ngpc-int(B) ⊆ B implies f^{-1}(Ngpc-int(B)) ⊆ $f^{-1}(B)$. This implies Ngpc-ker(f^{-1}(Ngpc-int(B))) ⊆ Ngpc-ker($f^{-1}(B)$). Hence f^{-1}(Ngpc-int (B)) ⊆ Ngpc-ker ($f^{-1}(B)$).

(iv) Then Ngpc-cl (B) is Ngpc-closed in V and f^{-1}(Ngpc-cl(B) is Ngpc-closed in U. Therefore Ngpc-surf (f^{-1}(Ngpc-cl (B)) $= f^{-1}$(Ngpc-cl (B). Since $B \subseteq$ Ngpc-cl(B) , $f^{-1}(B) \subseteq f^{-1}$(Ngpc-cl($B$)) . This implies Ngpc-surf ($f^{-1}(B)$) ⊆ Ngpc-surf f^{-1}(Ngpc-cl(B)). Hence Ngpc-surf ($f^{-1}(B)$) ⊆ f^{-1}(Ngpc-cl (B) for every subset B of V.

Example 3.8. In Example (3.2)

(i) Let $A = \{a, c\} \subseteq U$.
Then Ngpc-int $f(A)) =$ Ngpc-int $(f\{a, c\}) =$ Ngpc-int $(\{y, z\}) = \{y\}$ and f(Ngpc-ker $(A)) = f$(Ngpc-ker $(\{a, c\})) = f(\{a, b, c\}) = \{x, y, z\}$. Thus Ngpc-int $(f(A)) \subseteq f$(Ngpc-ker (A)).
(ii) Let $A = \{b, c\} \subseteq U$.
Then f(Ngpc-surf $(A)) = f$(Ngpc-surf $(\{b, c\})) = f(\{c\}) = \{y\}$ and Ngpc-cl $f(A)) =$ Ngpc-cl $(f\{b, c\}) =$ Ngpc-cl $(\{x, y\}) = \{x, y, z\}$. Thus f(Ngpc-surf $(A)) \subseteq$ Ngpc-cl $f(A)$).
(iii) Let $B = \{z, w\} \subseteq V$.
Then f^{-1}(Ngpc-int($\{z, w\})) = f^{-1}(\{w\}) = \{d\}$ and Ngpc-ker($f^{-1}(\{z, w\})$) = Ngpc-ker $(\{a, d\}) = \{a, b, d\}$. Thus f^{-1}(Ngpc-int $(B)) \subseteq$ Ngpc-ker $\left(f^{-1}(B)\right)$.
(iv) Let $B = \{x, w\} \subseteq V$.
Then Ngpc-surf $(f^{-1}(\{x, w\}))$ = Ngpc-surf $(\{b, d\}) = \{d\}$ and f^{-1}(Ngpc-cl($\{x, w\})) = f^{-1}(\{x, z, w\}) = \{a, b, d\}$. Thus Ngpc-surf $\left(f^{-1}(B)\right) \subseteq f^{-1}$(Ngpc-cl (B)).

Theorem 3.9. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a Ngpc-irresolute function. Then we have
(i) Ngpc-ker $\left(f^{-1}(G)\right) = f^{-1}$(Ngpc-ker$(G)$) for every Ngpc-open subset G of V.
(ii) Ngpc-surf $\left(f^{-1}(H)\right) = f^{-1}$(Ngpc-surf (H)) for every Ngpc-closed subset H of V.

Proof. (i) Let f be Ngpc-irresolute and G be a Ngpc-open subset of V. Then Ngpc-ker $(G) = G$ and $f^{-1}(G)$ is Ngpc-open in U. Hence Ngpc-ker $\left(f^{-1}(G)\right) = f^{-1}(G) = f^{-1}$(Ngpc-ker (G)). This implies Ngpc-ker $\left(f^{-1}(G)\right) = f^{-1}$(Ngpc-ker$(G)$) for every Ngpc-open subset G of V.
(ii) Let f be Ngpc-irresolute and H be a Ngpc-closed subset of V. Then Ngpc-surf $(H) = H$ and $f^{-1}(H)$ is Ngpc-closed in U. Hence Ngpc-surf $\left(f^{-1}(H)\right) = f^{-1}(H) = f^{-1}$(Ngpc-surf (H)). This implies Ngpc-surf $\left(f^{-1}(H)\right) = f^{-1}$(Ngpc-surf (H)) for every Ngpc-closed subset H of V.

Example 3.10. In Example (3.2)
(i) Let G be Ngpc-open and $G = \{x, y\} \subseteq V$.
Then Ngpc-ker $\left(f^{-1}(G)\right) =$ Ngpc-ker$\left(f^{-1}(\{x, y\})\right)$ =Ngpc-ker $(\{b, c\}) = \{b, c\}$ and f^{-1}(Ngpc-ker$(G)) = f^{-1}$(Ngpc-ker($\{x, y\})) = f^{-1}(\{x, y\}) = \{b, c\}$. Thus Ngpc-ker $\left(f^{-1}(G)\right) = f^{-1}$(Ngpc-ker$(G)$) for every Ngpc-open subset G of V.
(ii) Let H be Ngpc-closed and $H = \{y, z\} \subseteq V$.
Then Ngpc-surf $\left(f^{-1}(H)\right) =$ Ngpc-surf$\left(f^{-1}(\{y, z\})\right)$ =Ngpc-surf $(\{a, c\}) = \{a, c\}$ and f^{-1}(Ngpc-surf$(H)) = f^{-1}$(Ngpc-surf($\{y, z\})) = f^{-1}(\{y, z\}) = \{a, c\}$. Thus Ngpc-surf $\left(f^{-1}(H)\right) = f^{-1}$(Ngpc-surf (H)) for every Ngpc-closed subset H of V.

Theorem 3.11. If a function $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ngpc-irresolute then f is Ngpc- continuous.

Proof. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be Ngpc-irresolute. Let A be any nano open set in V. Then A is Ngpc-open in V. Since f is Ngpc irresolute, $f^{-1}(A)$ is Ngpc-open in U. Thus the inverse image of every nano open set in V is Ngpc-open in U. Therefore any Ngpc-irresolute function is Ngpc-continuous.
The converse of the above theorem need not be true as shown in the following example.

Example 3.12. Let $U = \{a, b, c, d\}$ with $U/_R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology with respect to X. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau'_R(Y) = \{\emptyset, V, \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology with respect to Y. Define $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ as $f(a) = z, f(b) = y, f(c) = x, f(d) = w$. Then f is Ngpc-continuous but not Ngpc-irresolute since $f^{-1}(\{y\}) = \{b\}$ is not Ngpc-open in U for the Ngpc-open set $\{y\}$ in V.

Theorem 3.13. If $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ngpc-irresolute and $g: (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ is Ngpc-continuous then $g^\circ f: (U, \tau_R(X)) \to (W, \tau''_R(Z))$ is Ngpc-continuous.

Proof. Let A be nano open in W. Since g is Ngpc-continuous $g^{-1}(A)$ is Ngpc-open in V. Since f is Ngpc-irresolute, $f^{-1}(g^{-1}(A)) = (g^\circ f)^{-1}(A)$ is Ngpc-open in U. Thus the inverse image of every nano open set in W is Ngpc-open in U.Therefore $g^\circ f$ is Ngpc-continuous.

Theorem 3.14. If $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ngpc-irresolute and $g: (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ is Ncgpc-continuous then $g^\circ f: (U, \tau_R(X)) \to (W, \tau''_R(Z))$ is Ncgpc-continuous.

Proof. Let A be nano open in W. Since g is Ncgpc-continuous $g^{-1}(A)$ is Ngpc-closed in V. Since f is Ngpc-irresolute, $f^{-1}(g^{-1}(A)) = (g^\circ f)^{-1}(A)$ is Ngpc-closed in U. Thus the inverse image of every nano open set in W is Ngpc-closed in U.Therefore $g^\circ f$ is Ncgpc-continuous.

Theorem 3.15. If $f: (U, \tau_R(X)) \to \left(V, \tau'_R(Y)\right)$ is Ngpc-irresolute and $g: (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ is nano continuous then $g^\circ f: (U, \tau_R(X)) \to (W, \tau''_R(Z))$ is Ngpc-continuous.

Proof. Proof is similar to theorem (3.13) since nano open set is a Ngpc-open set.

Theorem 3.16. If $f: (U, \tau_R(X)) \to \left(V, \tau'_R(Y)\right)$ is Ngpc-irresolute and $g: (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ is nano contra continuous

then $g \circ f: (U, \tau_R(X)) \to (W, \tau''_R(Z))$ is Ncgpc-continuous.
Proof. Proof is similar to theorem (3.14) since nano closed set is a Ngpc-closed set.
Theorem 3.17. If $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ and $g: (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ are Ngpc- irresolutes then $g \circ f$: $(U, \tau_R(X)) \to (W, \tau''_R(Z))$ is Ngpc-irresolute.
Proof. Let A be Ngpc-open in W. Since g is Ngpc-irresolute $g^{-1}(A)$ is Ngpc-open in V. Since f is Ngpc-irresolute, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is Ngpc-open in U. Thus the inverse image of every Ngpc-open set in W is Ngpc-open in U.Therefore $g \circ f$ is Ngpc- irresolute.
Example 3.18. Let $U = \{a, b, c, d\}$ with $U/_R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology on U. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau'_R(Y) = \{\emptyset, V, \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology on V. Let $W = \{p, q, r, s\}$ with $W/R'' = \{\{p\}, \{q, r\}, \{s\}\}$ and $Z = \{p, r\}$. Then $\tau''_R(Z) = \{\emptyset, W, \{p\}, \{p, q\}, \{p, q, r\}\}$ is a nano topology on W. Then $\tau^C_R(X) = \{\emptyset, U, \{a\}, \{a, b\}, \{a, c, d\}\}$, $\tau^C_{R'}(Y) = \{\emptyset, V, \{z\}, \{x, z\}, \{y, z, w\}\}$ and $\tau^C_{R''}(Z) = \{\emptyset, W, \{s\}, \{r, s\}, \{q, r, s\}\}$ are the complements of $\tau_R(X), \tau'_R(Y)$ and $\tau''_R(Z)$ respectively. Define $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ as $f(a) = z,\ f(b) = x,\ f(c) = y, f(d) = w$ and $g : (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ as $g(x) = p,\ g(y) = q,\ g(z) = s, g(w) = r$. The functions f and g are Ngpc-irresolutes. Then $g \circ f$ given by $(g \circ f)(\{a\}) = \{s\}, (g \circ f)(\{c\}) = \{q\}, (g \circ f)(\{d\}) = \{r\}, (g \circ f)(\{a, b\}) = \{p, s\}, (g \circ f)(\{a, c\}) = \{q, s\},\ (g \circ f)(\{a, d\}) = \{r, s\},\ (g \circ f)(\{a, b, c\}) = \{p, q, s\},\ (g \circ f)(\{a, b, d\}) = \{p, r, s\},\ (g \circ f)(\{a, c, d\}) = \{q, r, s\}$ is Ngpc-irresolute since the inverse image of every Ngpc-closed set in W is Ngpc-closed in U.

IV. NANO CONTRA GENERALIZED PRE C-IRRESOLUTE FUNCTIONS

In this section Nano contra generalized pre-c Irresolute function is defined and its characterizations and properties with respect to Ngpc-int, Ngpc-cl, Ngpc-ker and Ngpc-surf of sets are studied.
Definition 4.1. Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$ be two nano topological spaces. The function $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is said to be Nano contra generalized pre c-irresolute (briefly Ncgpc irresolute) on U if the inverse image of every Ngpc-open set in V is Ngpc-closed in U.
Example 4.2. Let $U = \{a, b, c, d\}$ with $U/_R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b, d\}\}$ is a nano topology with respect to X. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau'_R(Y) = \{\emptyset, V, \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology with respect to Y. Define $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ as $f(a) = y, f(b) = x$, $f(c) = w, f(d) = z$. Then f is Ncgpc-irresolute since the inverse image of every Ngpc-open set in V is Ngpc-closed in U.
Theorem 4.3. A function $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ncgpc-irresolute if and only if the inverse image of every Ngpc-closed set in V is Ngpc-open in U.
Proof. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be Ncgpc-irresolute. Let A be Ngpc-closed in V. Then A^C is Ngpc-open in V. Since f is Ncgpc irresolute $f^{-1}(A^C)$ is Ngpc-closed in U. Therefore $f^{-1}(A)$ is Ngpc-open in U. Thus the inverse image of every Ngpc-closed set in V is Ngpc-open in U. Conversely let the inverse image of every Ngpc-closed set in V is Ngpc-open in U. Let B be a Ngpc-open set in V. Then B^C is Ngpc-closed in V. By our assumption $f^{-1}(B^C) = (f^{-1}(B))^c$ is Ngpc-open in U.Therefore $f^{-1}(B)$ is Ngpc-closed in U. Thus the inverse image of every Ngpc-open set in V is Ngpc-closed in U. Hence f Ncgpc-irresolute.
Theorem 4.4. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. Then the following statements are equivalent.
(i) f is Ncgpc-irresolute.
(ii) $A \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl($f(A)$))) for every subset A of U.
(iii) $f^{-1}(B) \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl(B))) for every subset B of V.
Proof. (i) $\Leftrightarrow$ (ii). Let f be Ncgpc-irresolute and $A \subseteq U$. Then $f(A) \subseteq V$. Ngpc-cl($f(A)$) is Ngpc- closed in V. Since f is Ncgpc-irresolute, f^{-1}(Ngpc-cl$\big(f(A)\big)$) is Ngpc-open in U. Therefore Ngpc-int (f^{-1}(Ngpc-cl$\big(f(A)\big)$)) = f^{-1}(Ngpc-cl$\big(f(A)\big)$). But we know that $f(A) \subseteq$ Ngpc-cl$\big(f(A)\big)$ implies $A \subseteq f^{-1}$(Ngpc-cl$\big(f(A)\big)$). Hence $A \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl($f(A)$))) for every subset A of U.
Conversely let G be a Ngpc-closed set in V. Since $f^{-1}(G) \subseteq U$, we have $f^{-1}(G) \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl($f\big(f^{-1}(G)\big)$))) = Ngpc-int (f^{-1}(Ngpc-cl(G))). That is $f^{-1}(G) \subseteq$ Ngpc-int $\big(f^{-1}(G)\big)$ since G is Ngpc-closed. But we know that Ngpc-int$\big(f^{-1}(G)\big) \subseteq f^{-1}(G)$. Hence Ngpc-int$\big(f^{-1}(G)\big) = f^{-1}(G)$. This implies $f^{-1}(G)$ is Ngpc-open in $U$and hence f is Ncgpc irresolute.
(ii) $\Leftrightarrow$ (iii). Let $A \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl($f(A)$))) for every A in U. Let B be any subset of V. Then replacing A by $f^{-1}(B)$ we get $f^{-1}(B) \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl($f(f^{-1}(B)$)))). Hence $f^{-1}(B) \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl(B))) for every subset B of V.
Conversely let $f^{-1}(B) \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl(B))) for every subset B of V. Let A be a subset of U. Then $f(A) \subseteq V$ and we have $f^{-1}\big(f(A)\big) \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl($f(A)$))).

Hence $A \subseteq$ Ngpc-int (f^{-1}(Ngpc-cl($f(A)$))) for every A in U.

Theorem 4.5. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. Then the following statements are equivalent.

(i) f is Ncgpc-irresolute.

(ii) Ngpc-cl (f^{-1}(Ngpc-int($f(A)$))) $\subseteq A$ for every subset A of U.

(iii) Ngpc-cl (f^{-1}(Ngpc-int(B))) $\subseteq f^{-1}(B)$for every subset B of V.

Proof. (i) $\Leftrightarrow$ (ii). Let f be Ncgpc-irresolute and $A \subseteq U$. Then $f(A) \subseteq V$. Ngpc-int($f(A)$) is Ngpc-open in V. Since f is Ncgpc-irresolute, f^{-1}(Ngpc-int$\big(f(A)\big)$) is Ngpc-closed in U. Therefore Ngpc-cl(f^{-1}(Ngpc-int$\big(f(A)\big)$)) = f^{-1}(Ngpc-int$\big(f(A)\big)$). But we know that Ngpc-int$\big(f(A)\big) \subseteq f(A)$ implies f^{-1}(Ngpc-int$\big(f(A)\big)$) $\subseteq A$. Hence Ngpc-cl(f^{-1}(Ngpc-int($f(A)$))) $\subseteq A$ for every subset A of U.

Conversely let G be a Ngpc-open set in V. Since $f^{-1}(G) \subseteq U$, we have Ngpc-cl (f^{-1}(Ngpc-int($f\big(f^{-1}(G)\big)$))) $\subseteq f^{-1}(G)$. That is Ngpc-cl (f^{-1}(Ngpc-int(G))) $\subseteq f^{-1}(G)$ implies Ngpc-cl $\big(f^{-1}(G)\big) \subseteq f^{-1}(G)$ since G is Ngpc-open. But we know that $f^{-1}(G) \subseteq$ Ngpc-cl$\big(f^{-1}(G)\big)$. Hence Ngpc-cl$\big(f^{-1}(G)\big) = f^{-1}(G)$. This implies $f^{-1}(G)$ is Ngpc-closed in U and hence f is Ncgpc irresolute.

(ii) $\Leftrightarrow$ (iii). Let Ngpc-cl (f^{-1}(Ngpc-int($f(A)$))) $\subseteq A$ for every A in U. Let B be any subset of V. Then replacing A by $f^{-1}(B)$ we get Ngpc-cl (f^{-1}(Ngpc-int($f\big(f^{-1}(B)\big)$))) $\subseteq f^{-1}(B)$. Hence Ngpc-cl (f^{-1}(Ngpc-int(B))) $\subseteq f^{-1}(B)$ for every subset B of V.

Conversely let Ngpc-cl (f^{-1}(Ngpc-int(B))) $\subseteq f^{-1}(B)$for every subset B of V. Let A be a subset of U. Then $f(A) \subseteq V$ and we have Ngpc-cl (f^{-1}(Ngpc-int($f(A)$))) $\subseteq f^{-1}\big(f(A)\big)$.

Hence Ngpc-cl (f^{-1}(Ngpc-int($f(A)$))) $\subseteq A$ for every A in U.

Example 4.6. In example (4.2)

(i) Let $A = \{a, b, c\} \subseteq U$.

Then Ngpc-int(f^{-1}(Ngpc-cl($f(A)$))) = Ngpc-int(f^{-1}(Ngpc-cl($\{x, y, w\}$))) = Ngpc-int $(U) = U$ and thus $A \subseteq$ Ngpc-int(f^{-1}(Ngpc-cl($f(A)$))).

Let $B = \{x, y\} \subseteq V$.

Then Ngpc-int(f^{-1}(Ngpc-cl(B))) = Ngpc-int(f^{-1}(Ngpc-cl($\{x, y\}$))) = Ngpc-int ($\{a, b, d\}$) = $\{a, b, d\}$. Thus $f^{-1}(B) \subseteq$ Ngpc-int(f^{-1}(Ncl(B)))

(ii) Let $A = \{b, d\} \subseteq U$.

Then Ngpc-cl(f^{-1}(Ngpc-int($f(A)$))) = Ngpc-cl(f^{-1}(Ngpc-int($\{x, z\}$))) = Ngpc-cl ($f^{-1}(\{x\})$) = Ngpc-cl($\{b\}$) = $\{b\}$ and thus Ngpc-int(f^{-1}(Ngpc-cl($f(A)$))) $\subseteq A$.

Let $B = \{y, z\} \subseteq V$ and $f^{-1}(B) = \{a, d\}$.

Then Ngpc-cl(f^{-1}(Ngpc-int(B))) = Ngpc-cl(f^{-1}(Ngpc-int($\{y, z\}$))) = Ngpc-cl ($f^{-1}(\{y\})$) = Ngpc-cl($\{a\}$) = $\{a\}$.Thus Ngpc-cl(f^{-1}(Ngpc-int(B))) $\subseteq f^{-1}(B)$

Theorem 4.7. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a Ncgpc-irresolute function. Then we have

(i) Ngpc-int $\big(f(A)\big) \subseteq f$ (Ngpc-surf(A)) for every subset A of U.

(ii) f (Ngpc-ker(A)) $\subseteq$ Ngpc-cl $\big(f(A)\big)$ for every subset A of U.

(iii) f^{-1}(Ngpc-int (B)) $\subseteq$ Ngpc-surf $\big(f^{-1}(B)\big)$ for every subset B of V.

(iv) Ngpc-ker $\big(f^{-1}(B)\big) \subseteq f^{-1}$(Ngpc-cl ($B$)) for every subset B of V.

Proof. (i) Let f be Ncgpc-irresolute and A be a subset of U. Then $f(A) \subseteq V$ and Ngpc-int $\big(f(A)\big)$ is Ngpc-open in V. Since f is Ngpc-irresolute f^{-1}(Ngpc-int $\big(f(A)\big)$) is Ngpc-closed in U. Therefore Ngpc-surf (f^{-1}(Ngpc-int $\big(f(A)\big)$)) = f^{-1}(Ngpc-int $\big(f(A)\big)$) . But Ngpc-int$\big(f(A)\big) \subseteq f(A)$ implies f^{-1}(Ngpc-int$\big(f(A)\big)$) $\subseteq A$. This implies Ngpc-surf(f^{-1}(Ngpc-int$\big(f(A)\big)$)) $\subseteq$ Ngpc-surf (A). Hence f^{-1}(Ngpc-int $\big(f(A)\big)$) $\subseteq$ Ngpc-surf (A) shows that Ngpc-int $\big(f(A)\big) \subseteq f$ (Ngpc-surf (A)) for every subset A of U.

(ii) Let f be Ncgpc-irresolute and A be a subset of U. Then $f(A) \subseteq V$ and Ngpc-cl $\big(f(A)\big)$ is Ngpc-closed in V. Since f is Ncgpc-irresolute f^{-1}(Ngpc-cl$\big(f(A)\big)$) is Ngpc-open in U. Therefore Ngpc-ker (f^{-1}(Ngpc-cl $\big(f(A)\big)$)) = f^{-1}(Ngpc-cl $\big(f(A)\big)$) . Since $f(A) \subseteq$ Ngpc-cl$\big(f(A)\big)$, $A \subseteq f^{-1}$(Ngpc-cl$\big(f(A)\big)$) . This implies Ngpc-ker (A) $\subseteq$ Ngpc-ker f^{-1}(Ngpc-cl$\big(f(A)\big)$). Hence Ngpc-ker (A) $\subseteq f^{-1}$(Ngpc-cl $\big(f(A)\big)$) shows that f (Ngpc-ker(A)) $\subseteq$ Ngpc-cl $\big(f(A)\big)$ for every subset A of U.

(iii) Let f be Ncgpc-irresolute and B be a subset of V. Then Ngpc-int (B) is Ngpc-open in V and f^{-1}(Ngpc-int(B) is Ngpc-closed in U. Therefore Ngpc-surf (f^{-1}(Ngpc-int (B)) = f^{-1}(Ngpc-int (B). But Ngpc-int(B) $\subseteq B$ implies f^{-1}(Ngpc-int(B)) $\subseteq f^{-1}(B)$. This implies Ngpc-surf(f^{-1}(Ngpc-int(B))) $\subseteq$ Ngpc-surf ($f^{-1}(B)$). Hence f^{-1}(Ngpc-int (B)) $\subseteq$ Ngpc-surf ($f^{-1}(B)$) for every subset B of V.

(iv) Let f be Ncgpc-irresolute and B be a subset of V. Then Ngpc-cl (B) is Ngpc-closed in V and f^{-1}(Ngpc-cl(B) is Ngpc-open in U. Therefore Ngpc-ker (f^{-1}(Ngpc-cl (B)) = f^{-1}(Ngpc-cl (B). Since $B \subseteq$ Ngpc-cl(B) , $f^{-1}(B) \subseteq f^{-1}$(Ngpc-cl($B$)) . This implies Ngpc-ker ($f^{-1}(B)$) $\subseteq$ Ngpc-ker f^{-1}(Ngpc-cl(B)). Hence Ngpc-ker ($f^{-1}(B)$) $\subseteq f^{-1}$(Ngpc-cl (B) for every subset B of V.

Example 4.8. In Example (4.2)

(i) Let $A = \{a, d\} \subseteq U$.

Then Ngpc-int $f(A)$) = Ngpc-int ($f\{a, d\}$) = Ngpc-int ($\{y, z\}$) = $\{y\}$ and f(Ngpc-surf (A)) = f(Ngpc-surf ($\{a, d\}$)) = $f(\{a, d\}) = \{y, z\}$. Thus Ngpc-int ($f(A)$) $\subseteq f$(Ngpc-surf (A)).

(ii) Let $A = \{a, b, c\} \subseteq U$.

Then f(Ngpc-ker (A)) = f(Ngpc-ker ($\{a, b, c\}$)) = $f(\{a, b, c\}) = \{x, y, w\}$ and Ngpc-cl $f(A)$) = Ngpc-cl ($f\{a, b, c\}$) = Ngpc-cl ($\{x, y, w\}$) = V. Thus f(Ngpc-ker (A)) $\subseteq$ Ngpc-cl $f(A)$).

(iii) Let $B = \{y, z, w\} \subseteq V$.

Then f^{-1}(Ngpc-int($\{y, z, w\}$)) = $f^{-1}(\{y, w\}) = \{a, c\}$ and Ngpc-surf($f^{-1}(\{y, z, w\})$) = Ngpc-surf ($\{a, c, d\}$) = $\{a, c, d\}$. Thus f^{-1}(Ngpc-int (B)) $\subseteq$ Ngpc-surf $\left(f^{-1}(B)\right)$.

(iv) Let $B = \{x, y\} \subseteq V$.

Then Ngpc-ker ($f^{-1}(\{x, y\})$) = Ngpc-ker ($\{a, b\}$) = $\{a, b\}$ and f^{-1}(Ngpc-cl($\{x, y\}$)) = $f^{-1}(\{x, y, z\}) = \{a, b, d\}$. Thus Ngpc-ker $\left(f^{-1}(B)\right) \subseteq f^{-1}$(Ngpc-cl ($B$)).

Theorem 4.9. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a Ncgpc-irresolute function. Then we have

(i) Ngpc-surf $\left(f^{-1}(G)\right) = f^{-1}$(Ngpc-ker($G$)) for every Ngpc-open subset G of V.

(ii) Ngpc-ker $\left(f^{-1}(H)\right) = f^{-1}$(Ngpc-surf ($H$)) for every Ngpc-closed subset H of V.

Proof. (i) Let f be Ncgpc-irresolute and G be a Ngpc-open subset of V. Then $f^{-1}(G)$ is Ngpc-closed in U. Therefore Ngpc-surf ($f^{-1}(G)$) = $f^{-1}(G)$. Since G is Ngpc-open, Ngpc-ker (G) = G implies f^{-1}(Ngpc-ker(G)) = $f^{-1}(G)$. Hence Ngpc-surf ($f^{-1}(G)$) = f^{-1}(Ngpc-ker(G)) for every Ngpc-open subset G of V.

(ii) Let f be Ncgpc-irresolute and H be a Ngpc-closed subset of V. Then $f^{-1}(H)$ is Ngpc-open in U. Therefore Ngpc-ker $\left(f^{-1}(H)\right) = f^{-1}(H)$. Since H is Ngpc-closed Ngpc-surf (H) = H implies f^{-1}(Ngpc-surf (H)) = $f^{-1}(H)$. Hence Ngpc-ker $\left(f^{-1}(H)\right) = f^{-1}$(Ngpc-surf ($H$)) for every Ngpc-closed subset H of V.

Example 4.10. In Example (4.2)

(i) Let G be Ngpc-open and $G = \{x, y\} \subseteq V$.

Then Ngpc-surf $\left(f^{-1}(G)\right)$ = Ngpc-surf$\left(f^{-1}(\{x, y\})\right)$ =Ngpc-surf ($\{a, b\}$) = $\{a, b\}$ and f^{-1}(Ngpc-ker(G)) = f^{-1}(Ngpc-ker($\{x, y\}$)) = $f^{-1}(\{x, y\}) = \{a, b\}$. Thus Ngpc-surf $\left(f^{-1}(G)\right) = f^{-1}$(Ngpc-ker($G$)) for every Ngpc-open subset G of V.

(ii) Let H be Ngpc-closed and $H = \{y, z\} \subseteq V$.

Then Ngpc-ker $\left(f^{-1}(H)\right)$ = Ngpc-ker$\left(f^{-1}(\{y, z\})\right)$ =Ngpc-ker ($\{a, d\}$) = $\{a, d\}$ and f^{-1}(Ngpc-surf(H)) = f^{-1}(Ngpc-surf($\{y, z\}$)) = $f^{-1}(\{y, z\}) = \{a, d\}$.Thus Ngpc-ker $\left(f^{-1}(H)\right) = f^{-1}$(Ngpc-surf ($H$)) for every Ngpc-closed subset H of V.

Theorem 4.11. If a function $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ncgpc-irresolute then f is Ncgpc- continuous.

Proof. Let $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be Ncgpc-irresolute. Let A be any nano open set in V. Then A is Ngpc-open in V. Since f is Ncgpc-irresolute, $f^{-1}(A)$ is Ngpc-closed in U. Thus the inverse image of every nano open set in V is Ngpc-closed in U. Therefore any Ncgpc-irresolute function is Ncgpc-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 4.12. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology with respect to X. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau'_R(Y) = \{\emptyset, V, \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology with respect to Y. Define $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ as $f(a) = y, f(b) = z, f(c) = x, f(d) = w$. Then f is Ncgpc-continuous but not Ncgpc-irresolute since $f^{-1}(\{y\}) = \{a\}$ and $f^{-1}(\{y, z\}) = \{a, b\}$ are not Ngpc-open in U for the Ngpc-closed sets $\{y\}$ and $\{y, z\}$ in V.

Theorem 4.13. If $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ncgpc-irresolute and $g: (V, \tau'_R(Y)) \to \left(W, \tau''_R(Z)\right)$ is Ngpc-continuous then $g^\circ f: (U, \tau_R(X)) \to (W, \tau''_R(Z))$ is Ncgpc-continuous.

Proof. Let A be nano open in W. Since g is Ngpc-continuous $g^{-1}(A)$ is Ngpc-open in V. Since f is Ncgpc-irresolute, $f^{-1}(g^{-1}(A)) = (g^\circ f)^{-1}(A)$ is Ngpc-closed in U. Thus the inverse image of every nano open set in W is Ngpc-closed in U. Therefore $g^\circ f$ is Ncgpc-continuous.

Theorem 4.14. If $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ncgpc-irresolute and $g: (V, \tau'_R(Y)) \to (W, \tau''_R(Z))$ is Ncgpc-continuous then $g^\circ f: (U, \tau_R(X)) \to (W, \tau''_R(Z))$ is Ngpc-continuous.

Proof. Let A be nano open in W. Since g is Ncgpc-continuous, $g^{-1}(A)$ is Ngpc-closed in V. Since f is Ncgpc-irresolute, $f^{-1}(g^{-1}(A)) = (g^\circ f)^{-1}(A)$ is Ngpc-open in U. Thus the inverse image of every nano open set in W is Ngpc-open in U. Therefore $g^\circ f$ is Ngpc-continuous.

Theorem 4.15. If $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Ncgpc-irresolute and $g: (V, \tau_R'(Y)) \to (W, \tau_R''(Z))$ is nano continuous then $g^\circ f: (U, \tau_R(X)) \to (W, \tau_R''(Z))$ is Ncgpc-continuous.

Proof. The proof is similar to theorem (4.13) since every nano open set is Ngpc-open.

Theorem 4.16. If $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Ncgpc-irresolute and $g: (V, \tau_R'(Y)) \to (W, \tau_R''(Z))$ is nano contra continuous then $g^\circ f: (U, \tau_R(X)) \to (W, \tau_R''(Z))$ is Ngpc-continuous.

Proof. The proof is similar to theorem (4.14) since every nano closed set is Ngpc-closed.

Theorem 4.17. If $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ and $g: (V, \tau_R'(Y)) \to (W, \tau_R''(Z))$ are Ncgpc-irresolutes then $g^\circ f$: $(U, \tau_R(X)) \to (W, \tau_R''(Z))$ is Ngpc-irresolute.

Proof. Let A be Ngpc-open in W. Since g is Ncgpc-irresolute, $g^{-1}(A)$ is Ngpc closed in V. Since f is Ncgpc-irresolute, $f^{-1}(g^{-1}(A)) = (g^\circ f)^{-1}(A)$ is Ngpc-open in U. Thus the inverse image of every Ngpc-open set in W is Ngpc-open in U. Therefore $g^\circ f$ is Ngpc-irresolute.

Theorem 4.18. If $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Ncgpc-irresolute and $g: (V, \tau_R'(Y)) \to (W, \tau_R''(Z))$ is Ngpc-irresolute then $g^\circ f: (U, \tau_R(X)) \to (W, \tau_R''(Z))$ is Ncgpc-irresolute.

Proof. Let A be a Ngpc-closed set in W. Since $g: V \to W$ is Ngpc-irresolute, $g^{-1}(A)$ is Ngpc- closed in V. Since $f: U \to V$ is Ncgpc-irresolute, $f^{-1}(g^{-1}(A)) = (g^\circ f)^{-1}(A)$ is Ngpc-open in U. Thus the inverse image of every Ngpc-closed set in W is Ngpc-open in U. Therefore $g^\circ f$ is Ncgpc-irresolute.

Theorem 4.19. If $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Ngpc-irresolute and $g: (V, \tau_R'(Y)) \to (W, \tau_R''(Z))$ is Ncgpc-irresolute then $g^\circ f: (U, \tau_R(X)) \to (W, \tau_R''(Z))$ is Ncgpc-irresolute.

Proof. Proof is similar as theorem (4.18).

V. NANO GENERALIZED PRE C-CLOSED AND NANO GENERALIZED PRE C-OPEN MAPS

In this section Nano generalized pre c-closed and Nano Generalized pre c-open maps and are defined and some of their characterizations are presented.

Definition 5.1. The function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is said to be a Nano generalized pre c-closed map (briefly Ngpc-closed map) on U if the image of every nano closed set in $(U, \tau_R(X))$ is a Ngpc-closed set in $(V, \tau_R'(Y))$.

Definition 5.2. The function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is said to be a Nano generalized pre c-open map (briefly Ngpc- open map) on U if the image of every nano open set in $(U, \tau_R(X))$ is a Ngpc-open set in $(V, \tau_R'(Y))$.

Example 5.3. Let $U = \{a, b, c, d\}$ with $U/_R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology on U. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau_R'(Y) = \{\emptyset, V, \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology on V. Then $\tau_R^C(X) = \{\emptyset, U, \{a\}, \{a, b\}, \{a, c, d\}\}$ and $\tau_{R'}^C(Y) = \{\emptyset, V, \{z\}, \{x, z\}, \{y, z, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_R'(Y)$ respectively. Define $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ as $f(a) = z,\ f(b) = y,\ f(c) = x, f(d) = w$. Then the image of every nano closed (nano open) set in U is Ngpc-closed (Ngpc-open) in V. Hence f is both Ngpc-closed and Ngpc-open map.

Definition 5.4. A map $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is said to be a Strongly Nano generalized pre c-closed map (briefly Sngpc-closed map) on U if the image of every Ngpc-closed set in $(U, \tau_R(X))$ is a Ngpc-closed set in $(V, \tau_R'(Y))$.

Definition 5.5. A map $f: (U, \tau_R(X)) \to \left(V, \tau_R'(Y)\right)$ is said to be a Strongly Nano generalized pre c-open map (briefly Sngpc-open map) on U if the image of every Ngpc-open set in $(U, \tau_R(X))$ is a Ngpc-open set in $\left(V, \tau_R'(Y)\right)$.

Example 5.6. Let $U = \{a, b, c, d\}$ with $U/_R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology on U. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau_R'(Y) = \{\emptyset, V, \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology on V. Then $\tau_R^C(X) = \{\emptyset, U, \{a\}, \{a, b\}, \{a, c, d\}\}$ and $\tau_{R'}^C(Y) = \{\emptyset, V, \{z\}, \{x, z\}, \{y, z, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_R'(Y)$ respectively. Define $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ as $f(a) = z,\ f(b) = x,\ f(c) = y, f(d) = w$. Then f is Sngpc-closed (Sngpc-open) since the image of every Ngpc-closed (Ngpc-open) set in U is Ngpc-closed (Ngpc-open) in V.

Theorem 5.7. A function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Ngpc-closed if and only if for each subset A of V and for each nano open set G of $(U, \tau_R(X))$ containing $f^{-1}(A)$, there is a Ngpc-open set B of $(V, \tau_R'(Y))$ such that $A \subseteq B$ and $f^{-1}(B) \subseteq G$.

Proof. Let A be a subset of $(V, \tau_R'(Y))$ and G be a nano open set of $(U, \tau_R(X))$ such that $f^{-1}(A) \subseteq G$. Then $U - G$ is a nano closed set of U. Since f is Ngpc-closed, $f(U - G)$ is Ngpc-closed in $(V, \tau_R'(Y))$. Now $B = V - f(U - G)$ is a Ngpc-open set containing A in V such that $f^{-1}(B) \subseteq G$.

Conversely let H be a nano closed set of $(U, \tau_R(X))$, then $f^{-1}(V - f(H)) \subseteq U - H$ and $U - H$ is nano open. By our assumption there is a Ngpc-open set B of $(V, \tau_R'(Y))$ such that $V - f(H) \subseteq B$ and $f^{-1}(B) \subseteq U - H$. Hence $V - B \subseteq f(H)$and $H \subseteq U - f^{-1}(B)$.Thus $V - B \subseteq f(H) \subseteq f(U - f^{-1}(B) \subseteq V - B$ which implies $f(H) = V - B$. Since $V - B$ is Ngpc-closed, $f(H)$ is a Ngpc-closed set in $(V, \tau_R'(Y))$. That is $f(H)$ is Ngpc-closed in V for every nano closed set H of U. Hence f is a Ngpc-closed map.

Theorem 5.8. A function $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ is Ngpc-open if and only if for each subset A of V and for each nano closed set H of $(U,\tau_R(X))$ containing $f^{-1}(A)$, there is a Ngpc-closed set B of $(V,\tau'_R(Y))$ such that $A\subseteq B$ and $f^{-1}(B)\subseteq H$.

Proof. Let A be a subset of $(V,\tau'_R(Y))$ and H be a nano closed set of $(U,\tau_R(X))$ such that $f^{-1}(A)\subseteq H$. Then $U-H$ is a nano open set of $U.$ Since f is Ngpc-open, $f(U-H)$ is Ngpc-open in $(V,\tau'_R(Y))$. Now $B=V-f(U-H)$ is a Ngpc-closed set containing A in V such that $f^{-1}(B)\subseteq H$.

Conversely let G be a nano open set of $(U,\tau_R(X))$, then $f^{-1}(V-f(G))\subseteq U-G$ and $U-G$ is nano closed. By our assumption there is a Ngpc-closed set B of $(V,\tau'_R(Y))$ such that $V-f(G)\subseteq B$ and $f^{-1}(B)\subseteq U-G$. Hence $V-B\subseteq f(G)$and $G\subseteq U-f^{-1}(B)$. Thus $V-B\subseteq f(G)\subseteq f(U-f^{-1}(B)\subseteq V-B$ which implies $f(G)=V-B$. Since $V-B$ is Ngpc-open, $f(G)$ is a Ngpc-open set in $(V,\tau'_R(Y))$. That is $f(G)$ is Ngpc-open in V for every nano open set G of U. Hence f is a Ngpc-open map.

Theorem 5.9. Let $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ be a function. Then the following statements are equivalent

(i) f is Sngpc-closed.

(ii) For every subset A of V and every Ngpc-open set G of $(U,\tau_R(X))$ containing $f^{-1}(A)$, there is a Ngpc-open set B of $(V,\tau'_R(Y))$ with $A\subseteq B$ and $f^{-1}(B)\subseteq G$.

Proof. Let A be a subset of $(V,\tau'_R(Y))$ and G be a Ngpc-open set of $(U,\tau_R(X))$ such that $f^{-1}(A)\subseteq G$. Then $U-G$ is a Ngpc-closed set of $U.$ Since f is Sngpc-closed, $f(U-G)$ is Ngpc-closed in $(V,\tau'_R(Y))$. Now $B=V-f(U-G)$ is a Ngpc-open set containing A in V such that $f^{-1}(B)\subseteq G$.

Conversely let H be a Ngpc-closed set of $(U,\tau_R(X))$, then $f^{-1}(V-f(H))\subseteq U-H$ and $U-H$ is Ngpc-open. By our assumption there is a Ngpc-open set B of $(V,\tau'_R(Y))$ such that $V-f(H)\subseteq B$ and $f^{-1}(B)\subseteq U-H$. Hence $V-B\subseteq f(H)$and $H\subseteq U-f^{-1}(B)$.Thus $V-B\subseteq f(H)\subseteq f(U-f^{-1}(B)\subseteq V-B$ which implies $f(H)=V-B$. Since $V-B$ is Ngpc-closed, $f(H)$ is a Ngpc-closed set in $(V,\tau'_R(Y))$. That is $f(H)$ is Ngpc-closed in V for every Ngpc-closed set H of U. Hence f is a Sngpc-closed map.

Theorem 5.10. Let $f:(U,\tau_R(X))\to(V,\tau'_R(Y))$ be a function. Then the following statements are equivalent

(i) f is Sngpc-open,

(ii) For every subset A of V and every Ngpc-closed set H of $(U,\tau_R(X))$ containing $f^{-1}(A)$, there is a Ngpc-closed set B of $(V,\tau'_R(Y))$ with $A\subseteq B$ and $f^{-1}(B)\subseteq H$.

Proof. Let A be a subset of $(V,\tau'_R(Y))$ and H be a Ngpc-closed set of $(U,\tau_R(X))$ such that $f^{-1}(A)\subseteq H$. Then $U-H$ is a Ngpc-open set of $U.$ Since f is Sngpc-open, $f(U-H)$ is Ngpc-open in $(V,\tau'_R(Y))$. Now $B=V-f(U-H)$ is a Ngpc-closed set containing A in V such that $f^{-1}(B)\subseteq H$.

Conversely let G be a Ngpc-open set of $(U,\tau_R(X))$, then $f^{-1}(V-f(G))\subseteq U-G$ and $U-G$ is Ngpc-closed. By our assumption there is a Ngpc-closed set B of $(V,\tau'_R(Y))$ such that $V-f(G)\subseteq B$ and $f^{-1}(B)\subseteq U-G$. Hence $V-B\subseteq f(G)$and $G\subseteq U-f^{-1}(B)$. Thus $V-B\subseteq f(G)\subseteq f(U-f^{-1}(B)\subseteq V-B$ which implies $f(G)=V-B$. Since $V-B$ is Ngpc-open, $f(G)$ is a Ngpc-open set in $(V,\tau'_R(Y))$. That is $f(G)$ is Ngpc-open in V for every Ngpc- open set G of U. Hence f is a Sngpc-open map.

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