International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V67I7P518
Volume 67 Issue 7, 150-157, July, 2021

Estimation of Fourier coefficients of flux density for Surface Mounted Permanent Magnet (SMPM) Generators by Direct Search Optimization

Mamidi Ramakrishna Rao

DHI-QUEST Pvt. Ltd., India

Abstract — *For Surface Mounted Permanent Magnet (SMPM) generators, it is essential to predict its performance to analyze the magnet's air gap flux density wave shape. The flux density wave shape is neither a pure sine wave or square wave or combination. This is due to variation of air gap reluctance between stator and permanent magnets. The stator slot openings and the number of slots make the wave shape more complicated. In order to reduce the analysis complexity, approximations are done to the wave shape, and Fourier analysis is used.*

Instead of usual integration method, the Fourier coefficients, a_n and b_n, are obtained by direct search method optimization. The wave shape with optimized coefficients gave wave shape close to the desired wave shape. Harmonics amplitudes are worked out and compared with initial values.

It can be concluded that direct search method can be used for estimating Fourier coefficients for irregular wave shapes

Keywords — *Fourier analysis, Flux plot analysis, Direct search, SMPM generators, Optimization*

.

I. INTRODUCTION

IN offshore wind applications, direct drive turbines are used which are less complex than gear box turbines and give better performance including higher efficiency (>3%) [1].

These low-speed generators have a large rotor diameter and high number of magnetic poles. The performance parameters like torque, losses and efficiency largely depend upon the air gap flux density wave shape under the poles. Hence, detailed analysis of flux wave shape is necessary. This wave shape is neither a pure sinusoidal nor rectangular in shape and its complexity is due to non-uniform reluctance. The complexity increases with stator slot openings and the number of slots per pole. This complex wave shape is estimated by Finite Element Magnetic Methods (FEMM) simulations. For performance estimation, the wave shape is to be further analyzed into its fundamental and harmonic components.

The wave shape traditionally has been analyzed by 'Fourier analysis' which involves determining Fourier coefficients namely a_o, a_n and b_n through trigonometric integrations. Simplification of wave shape may reduce the accuracy of harmonics' prediction and performance. In this paper, a new method is suggested that does not call for complex integrations which sometimes can become tedious.

The flux density distribution under the pole is different from a simple rectangle wave shape. When FEMM analysis is carried out, it is observed that the error when calculated with rectangle wave shape is less accurate and the deviation is more than 10% [2]. Leakage flux between the adjacent poles and notches due to stator slot openings makes the wave shape different from a simple rectangle. Wave shapes of SMPM permanent magnets [3] in the slot less airgap and in the slotted air gap is shown below.

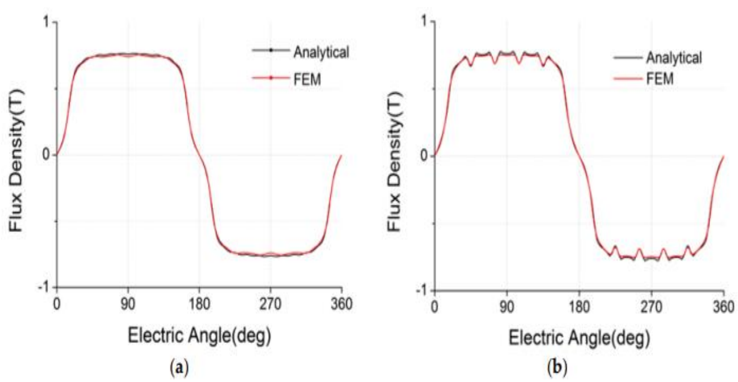

Fig. 1 Flux density wave shape in air gap in slot-less and in slotted air gap of permanent magnets in SMPM machine

Two typical cases namely single notch (case 1) and multi notch cases (case 2) are considered for analysis. Case 1 is shown in Fig 4. This occurs when the width of the magnet is less than pole pitch and stator slot opening is at the center of magnetic axis with leakage flux between pole to pole. Case 2 occurs generally in high-speed machines, when the magnet width is large and two or more slot openings are likely to come under a magnet.

II. FOURIER ANALYSIS

f(x) is a function of period 2π, it can be represented by [4]

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}) \qquad (1)$$

The a_0, a_n and b_n coefficients are called Fourier coefficients of f(x) and are given by

$$a_0 = (1/2\pi) \int_{-\pi}^{\pi} f(x)dx \qquad (2)$$

$$a_n = (1/\pi) \int_{-\pi}^{\pi} f(x)cos\,(nx)dx \qquad (3)$$

$$b_n = (1/\pi) \int_{-\pi}^{\pi} f(x)sin\,(nx)dx \qquad (4)$$

If the function is even, then f (-x) = f (x).If the function is an odd function, then f (-x) = -f (x), the Fourier series reduces to a Fourier sine series

$$f(x) = a_0 + \sum_{n=1}^{\infty} b_n \cos\,(n\frac{\pi}{L}\,x) \qquad (5)$$

with Fourier coefficients

$$b_n = (2/L) \int_0^L f(x)sin\,\left(n\frac{\pi}{L}x\right)dx \qquad (6)$$

III. OPTIMIZATION

Objective: To minimize the difference between flux density values (calculated and the desired values).
Variables: They are the coefficients of Fourier series 'a_n' or 'b_n'. The number of harmonics considered are 20. As the function is odd, the number of variables is only 20 coefficients (b_n).
Evaluation points: The flux densities are calculated at 10 degrees interval, over pole pitch pair.
Optimization method: Search method 'Direct Search' optimization is used.[5][6][7] is used.
Exploratory search and pattern search with Golden Section for identification are used. The flow chart and explanation are given below.

A. Fibonacci search and the method of Golden Sections

To find the maximum or minimum of an unimodal function, direct search method is used. In this method, the segment of line in which minimum lies is established.

Fig. 2 Explanation Fig. for "Golden Section"

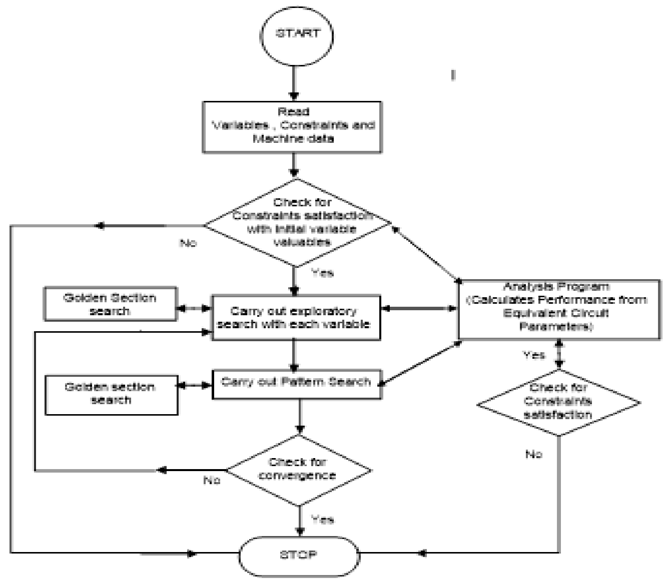

Fig. 3. Flow chart for the computer program

The diagram shows the line along which lies the interest of finding a minimum of function F. At points P , Q, and R the function values F_P, F_Q and F_R are evaluated. If $F_P < F_R < F_Q$ the minimum is to the left of P. If $F_P > F_R > F_Q$ the minimum is to the right of Q. If $F_P > F_R$ and $F_R < F_Q$ then the section PQ contains the minimum. If $F_Q < F_P$ the line is further explored towards the right of Q. After the segment PQ is established, the point R is decided such that PR/QR =1.618. Let the value of function F at R be F_R. If $F_P < F_R$ and $F_Q < F_R$ then the minimum lies in PR and once again PR can be divided in an identical way. In this manner, the interval of uncertainty can be reduced within few trials. This is the method of "Golden section."

IV. RESULTS AND ANALYSIS

A. Single Notch Wave (Case 1).

The desired wave shape with single notch is shown in Fig.4. The optimization search needs an initial value for the variables. So the initial values for all the Fourier coefficients (b_n) are taken 1 and the search continued. After exploration searches and 20 pattern searches as shown in flow chart 3, the Fourier coefficients values are shown in Fig 5.

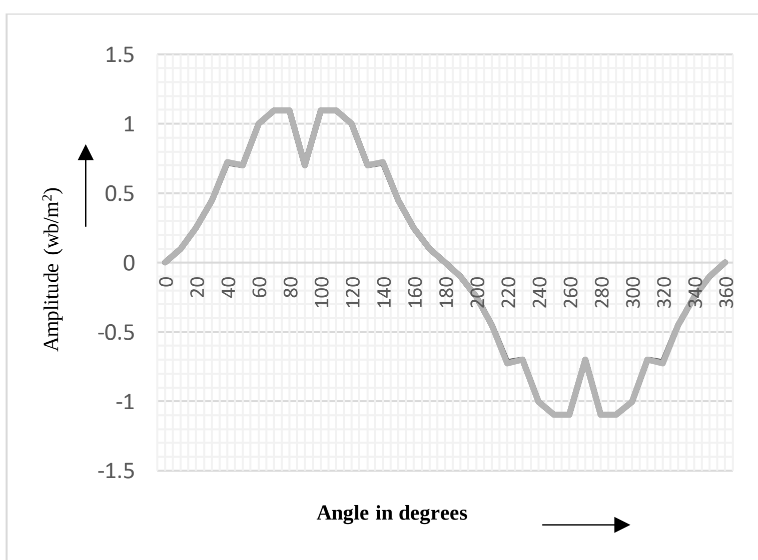

Fig 4. Flux density wave shape (with single notch) over pole pitch pair

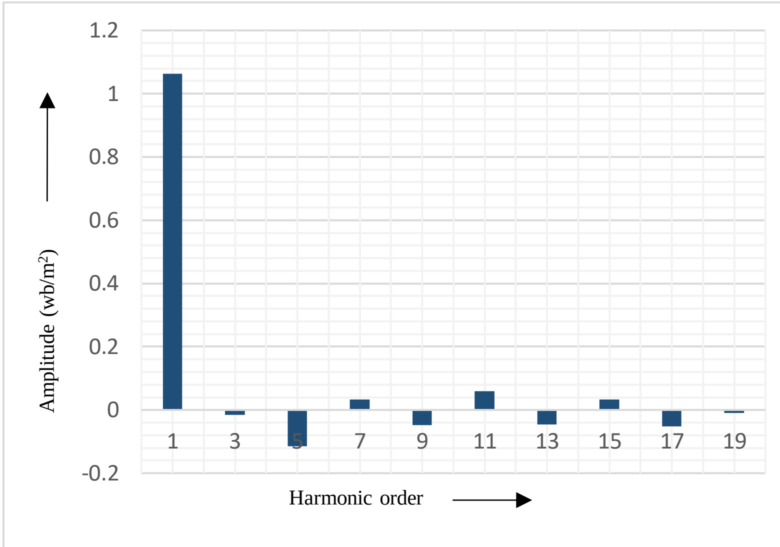

Fig 5. Fourier coefficient (b_n) after optimization (For single notch case)

With the search method-optimized-Fourier-coefficients, the y axis data points for a single notch wave shape are calculate and the comparison with desired values are is shown in Table I.

TABLE I
COMPARISON OF SINGLE NOTCH WAVE VALUES IN POLE-PITCH PAIR.

Angle	Desired Value	Actual Value	Difference
0	0	0	0
10	0.111	0.111	0
20	0.222	0.222	0
30	0.333	0.333	0
40	0.444	0.444	0
50	0.555	0.555	0
60	0.666	0.666	0
70	0.777	0.777	0
80	0.888	0.888	0
90	1	0.984	0.016
100	0.888	0.888	0
110	0.777	0.777	0
120	0.666	0.666	0
130	0.555	0.555	0
140	0.444	0.444	0
150	0.333	0.333	0
160	0.222	0.222	0
170	0.111	0.111	0
180	0	0	0
190	-0.111	-0.111	0
200	-0.222	-0.222	0
210	-0.333	-0.333	0
220	-0.444	-0.444	0
230	-0.555	-0.555	0

240	-0.666	-0.666	0
250	-0.777	-0.777	0
260	-0.888	-0.888	0
270	-1	-0.984	-0.016
280	-0.888	-0.888	0
290	-0.777	-0.777	0
300	-0.666	-0.666	0
310	-0.555	-0.555	0
320	-0.444	-0.444	0
330	-0.333	-0.333	0
340	-0.222	-0.222	0
350	-0.111	-0.111	0
360	0	0	0

From the above comparison, it can be observed, that the wave shape obtained by direct search values is in closed agreement with the desired values.

B. Multi Notch Wave (Case 2).

The number of wave shape notches are increased and the desired wave shape is shown in Fig 6. The search method of optimization is carried out, similar to case1.The comparison results are shown in Table II. The corresponding Fourier coefficients are shown in Fig 7.

In both single and multi-notches cases, values are in close agreement with the desired values. The deviation observed at some points in single notch case is less than 1.6% and in multi notch case it is less than 0.6%.

TABLE II
COMPARISON OF MULTI NOTCH WAVE VALUES IN POLE-PITCH PAIR

Angle	Desired Value	Actual Value	Difference
0	0	0	0
10	0.1	0.1	0
20	0.25	0.25	0
30	0.45	0.45	0
40	1	1.001	-0.001
50	0.7	0.7	0
60	1	0.994	0.006
70	1	1.001	-0.001
80	1	1	0
90	0.7	0.701	-0.001
100	1	1	0
110	1	1.001	-0.001
120	1	0.994	0.006
130	0.7	0.7	0
140	1	1.001	-0.001
150	0.45	0.45	0
160	0.25	0.25	0
170	0.1	0.1	0
180	0	0	0
190	-0.1	-0.1	0
200	-0.25	-0.25	0
210	-0.45	-0.45	0
220	-1	-1.001	0.001
230	-0.7	-0.7	0
240	-1	-0.994	-0.006
250	-1	-1.001	0.001

260	-1	-1	0
270	-0.7	-0.701	0.001
280	-1	-1	0
290	-1	-1.001	0.001
300	-1	-0.994	-0.006
310	-0.7	-0.7	0
320	-1	-1.001	0.001
330	-0.45	-0.45	0
340	-0.25	-0.25	0
350	-0.1	-0.1	0
360	0	0	0

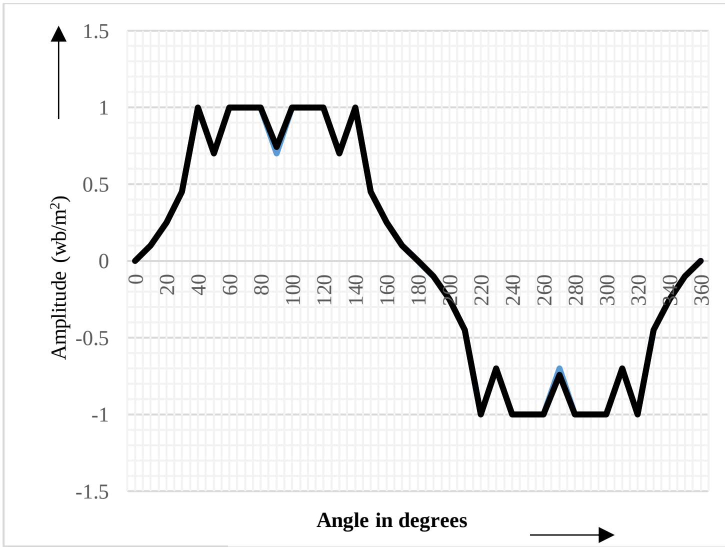

Fig 6. Flux density wave shape (with multi notch) over pole pitch pair

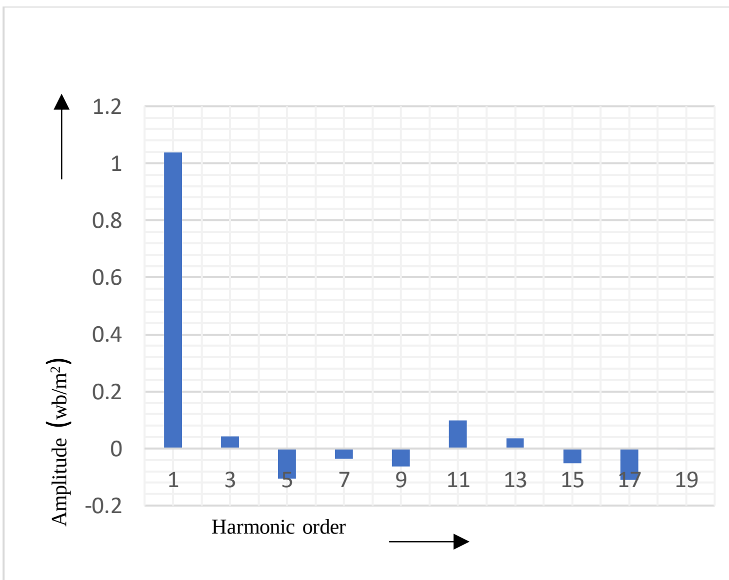

Fig .7 Fourier coefficient (b_n) after optimization (For multi notch case)

V. VALIDATION

Validation is done by comparing the Fourier coefficients obtained by

i) Computing coefficients by for a 'Even Triangle Wave' wave shown in Fig 8. by standard integration method [8]. The Fourier coefficient

$$a_n = \begin{cases} 4A\frac{1-(-1)^n}{\pi^2 n^2}, & n\ odd \\ 0, & n\ even \end{cases} \qquad (7)$$

With A =1, the values for a_n are given in the Table III.

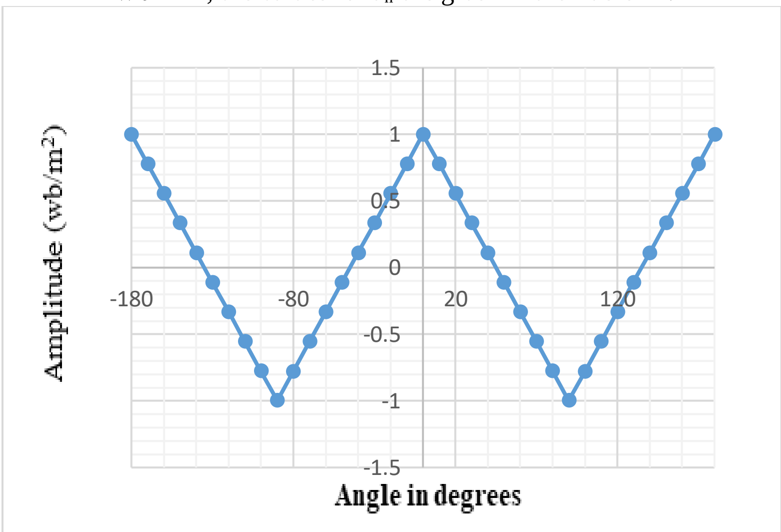

Fig .8 Even Triangle Wave

ii) Computing the coefficients by Direct search method as explained in both case 1 and case 2. The Table IV shows a detailed harmonic analysis

.TABLE: IV

HARMONIC AMPLITUDES BY DIRECT SEARCH

Angle	1st	3rd	5th	7th
-180	**-0.812**	**-0.0921**	**-0.0345**	**-0.0189**
-170	-0.7997	-0.0797	-0.0222	-0.0065
-160	-0.7631	-0.046	0.006	0.0145
-150	-0.7032	0	0.0298	0.0163
-140	-0.6221	0.046	0.0324	-0.0033
-130	-0.522	0.0797	0.0118	-0.0186
-120	-0.406	0.0921	-0.0172	-0.0094
-110	-0.2777	0.0797	-0.0339	0.0121
-100	-0.141	0.046	-0.0264	0.0177
-90	0	0	0	0
-80	0.141	-0.046	0.0264	-0.0177
-70	0.2777	-0.0797	0.0339	-0.0121
-60	0.406	-0.0921	0.0172	0.0094
-50	0.522	-0.0797	-0.0118	0.0186
-40	0.6221	-0.046	-0.0324	0.0033
-30	0.7032	0	-0.0298	-0.0163
-20	0.7631	0.046	-0.006	-0.0145
-10	0.7997	0.0797	0.0222	0.0065
0	**0.812**	**0.0921**	**0.0345**	**0.0189**
10	0.7997	0.0797	0.0222	0.0065
20	0.7631	0.046	-0.006	-0.0145

30	0.7032	0	-0.0298	-0.0163
40	0.6221	-0.046	-0.0324	0.0033
50	0.522	-0.0797	-0.0118	0.0186
60	0.406	-0.0921	0.0172	0.0094
70	0.2777	-0.0797	0.0339	-0.0121
80	0.141	-0.046	0.0264	-0.0177
90	0	0	0	0
100	-0.141	0.046	-0.0264	0.0177
110	-0.2777	0.0797	-0.0339	0.0121
120	-0.406	0.0921	-0.0172	-0.0094
130	-0.522	0.0797	0.0118	-0.0186
140	-0.6221	0.046	0.0324	-0.0033
150	-0.7032	0	0.0298	0.0163
160	-0.7631	-0.046	0.006	0.0145
170	-0.7997	-0.0797	-0.0222	-0.0065
180	**-0.812**	**-0.0921**	**-0.0345**	**-0.0189**

TABLE III
FOURIER COEFFICIENTS _(TRIANGLE WAVE UP TO 7TH HARMONIC)

n	**an**
0	0
1	0.8106
2	0
3	0.0901
4	0
5	0.0324
6	0
7	0.0165

The harmonics considered are up to 20. But for convenience, in the above Table the amplitudes of harmonics up to 7th are shown. The difference is 0.17%, 2.2%, 6.48% and 14% for 1st, 3rd,5th and 7th harmonics respectively.

VI. CONCLUSION

It is observed from above, that the wave shape obtained from Fourier coefficients from direct search method is in close agreement with desired wave shape plot (obtained from FEMM analysis). This direct search method does not call any trigonometric integrations. The direct search method of determining Fourier coefficients can be used for irregular periodic wave shapes of poles which occur generally in SMPM machines.

VII. REFERENCES

[1] Alexandre Marx, An in-depth comparative study of direct drive versus gear box wind turbines, KTH School of Industrial Engineering and Management. Master of Science Thesis, Stockholm, Sweden. FULLTEXT01.pdf (diva-portal.org) 1-67
[2] F.Libert and J. Soulard, DESIGN STUDY OF A DIRECT-DRIVEN SURFACE MOUNTED PERMANENT MAGNET MOTOR FOR LOW SPEED APPLICATION, Department of Electrical Machines and Power Electronics, Royal Institute of Technology (KTH), Stockholm, Sweden.pages, 1-6. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.532.197&rep=rep1&type=pdf
[3] Amuliu Bogdan Proca, Ali Keyhani, Ahmed EL, Analytical Model for Permanent Magnet Motors with Surface Mounted Magnets, IEEE Transactions of energy conversion, 18(3) (2003) 386-391
[4] Dmitriv Sergeevich Nikitin Fouriers Analysis Lecture 8:, Tomsk Polytechnic University NikitinDmSr@yandex.ru
[5] Mamidi Ramakrishna Rao, Estimation of parameters for Induction motor's analytical model by Direct search method, 2008 International Conference on Electrical Machines and Systems Wuhan, China, Conference date, IEEE Xplore (2nd Feb 2009), (2008) 17-20.
[6] R Ramarathnam, Optimization of Polyphase Induction Motor Design, Dissertation, Indian Institute of Technolgy, (1969). Appendix-II
[7] Wei-Ta Chu,One -Dimensional Search Methods, https://web2.qatar.cmu.edu/~gdicaro/15382/additional/one-dimensional-search-methods.pdf Spring (2014).
[8] Professor Erik Cheever,The Fourier Series, Department of Engineering, SwarthmoreCollege, https://lpsa.swarthmore.edu/Fourier/Series/WhyFS.html

VIII. ACKNOWLEDGEMENT

Mamidi Ramakrishna Rao thanks Ms. Mamidi Sri Rekha for her sincere help in preparing the paper and presentation. He thanks "DHI-QUEST Pvt. Ltd, India" for sponsoring the technical paper for presentation and allowing it for publication.