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General solution to Vaidya-Tikekar metric

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Abstract- Here we are considering the Vaidya-Tikekar metric which represents a 3- dimensional space with time time equals a constant, in a spheroidal super dense star. We report a general solution to Vaidya-Tikekar metric (in terms of hyper-geometric series), used in modelling Einstein's field equations. These models permits densities approximates to the order of $2 \times 10^{14} gmcm^{-3}$***, radii of the order of few kilometers and maximum mass up to four times the solar mass.***

Keywords: Space-time, Neutron star, Hypergeometric series

0.1 INTRODUCTION

Theory of general relativity consolidates gravity as a phenomenon that virtually relates with the geometry of space-time and succeeds in attaining at a broader understanding of the space times linked with distribution of physical fields of cosmological and astrophysical significance. The adequate mass of gravitational field as evident from the Einstein's field equations of general relativity accomplish constraints in attaining simple exact solutions which may serve as models of relativistic stars. Lack of dependable knowledge about the properties of central core area of comparatively compact stars is another hindrance which warrants of a general nature. Correspondingly, it is desiderate to have analytic solutions at hand, which may serve as significant models for these stars [Tolman (1939)]. If similar closed form solutions[Tolman (1939); Adler (1974); Leibovitz (1969); Buchdahl (1959)] adhere to certain general fundamental properties expected from fluids at ultra high masses and pressure; then it will be of astrophysical significance.

Tikekar and Vaidya (1982) have established that the space-times with t = constant having the geometry of a 3D-spheroid characterized by two parameters K measuring the oblateness and R showing the spherical nature of the spheroid are useful in developing easily surveyable relativistic design for super dense stars such as neutron stars. Perhaps these space-times can be used to establish static models portraying the strong gravitational field in the interior of superdense condensations of matter like white dwarfs and neutron stars. The physical soundness of the class of models by Tikekar and Vaidya was examined by Knutsen (1988) and concluded that these models are stable with respect to infinitesimal radial beats. Tikekar (1990) again reported another class of models with above geometry. References shows that only a limited number of analytic closed form solutions of Einstein's field equations for static spherical distributions of material can be useful as easily significant modes for superdense stars, it is necessary to investigate the suitability of other particular classes of models in this set up.

This paper deals with the study of spheroidal space time and its suitability to represent the interior of compact fluid spheres in equilibrium. The space times are characterised by two curvature parameters R and K. The requirement that the space time of a matter distribution in equilibrium be spheroidal determines the law of variation of density of matter in the configuration and the problem of solving a second order linear differential equation. Leach and Maharaj (1996) and Mukherjee et al. (1997) have discussed methods for solving this differential equation. We have discussed two methods for obtaining general solution of this differential equations one of which is similar to the one given by Maharaj and Leach (1996).

Our other method consists of converting this differential equation to hyper geometric function. Evaluation of the mass and size of a class of physically feasible static relativistic star models are an attractive feature of this model because boosting the limit of the maximum mass of a neutron star is of very high implication in deciding whether the undiscovered part is a black hole or a neutron star in a binary system of star.

0.2 MATTER DISTRIBUTION ON SPHEROIDAL SPACE TIME

Consider Tikekar and Vaidya's (1982) approach, we look at the static spherically symmetric space-time with the metric,

$$ds^2 = -\frac{1 - K\frac{r^2}{R^2}}{1 - \frac{r^2}{R^2}}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) + e^{\nu(r)}dt^2 \quad (1)$$

where $K = 1 - \frac{b^2}{R^2}$ represents the space-time in the interior of a spherical distribution of matter at rest.

Considering the physical content of the space time to an ideal fluid having associated energy momentum tensor as,

$$T_{ij} = (\rho + \frac{p}{c^2})u_i u_j - (\frac{p}{c^2})g_{ij} \quad (2)$$

where ρ represents matter density and p the fluid pressure.

Representing the unit four velocity field of matter mentioned as u^i= $(0, 0, 0, e^{-\nu/2})$, Einstein's field equations:

$$R_{ij} - \frac{1}{2}Rg_{ij} = \frac{8\pi G}{c^2}T_{ij} \quad (3)$$

reduce to the system of three equations given by

$$8\pi\rho = \frac{3(1-K)}{R}\frac{\left[1 - \frac{K}{3}\frac{r^2}{R^2}\right]}{\left[1 - K\frac{r^2}{R^2}\right]^2} \quad (4)$$

$$8\pi p = \left[\frac{\nu'}{r} + \frac{1}{r^2}\right]\frac{\left[1 - \frac{r^2}{R^2}\right]}{\left[1 - K\frac{r^2}{R^2}\right]} - \frac{1}{r^2} \quad (5)$$

and

$$c\left[1 - K\frac{r^2}{R^2}\right]\left[\nu'' + \frac{\nu'^2}{2} - \frac{\nu'}{r}\right] - \frac{1-K}{R^2}(r\nu' + 2) + \frac{2(1-K)}{R^2}\left[1 - K\frac{r^2}{R^2}\right] = 0 \quad (6)$$

Here and in what follows an overhead prime mentions differentiation with respect to radial variable r. In our proposal the prevailing choice of state of matter is replaced with the option of the spheroidal geometry which demonstrate the rate of change with respect to r. Equation (4) displays that the density of the fluid is figured out by the curvature of the physical 3-space. The field equation (5) presents the variation of pressure with r when ν is chosen to satisfy Equation (6). It is shown by Tikekar (1990), that the relativistic condition for hydro static equilibrium is:

$$\frac{1}{c^2}\frac{dp}{dr} = -\frac{(\rho + \frac{p}{c^2})}{r^2}\left[\frac{m(r) + (\frac{4\pi Gp}{c^4})r^3}{1 - \frac{2m(r)}{r}}\right] \quad (7)$$

This usually replaces the field equation (6) with the explicit form:

$$\frac{1}{c^2}\frac{dp}{dr} = -\frac{\left[1-K\frac{r^2}{R^2}\right]}{\left[1-\frac{r^2}{R^2}\right]}\left[\frac{4\pi Gpr}{c^4}+\frac{(1-k)r}{2R^2\left[1-K\frac{r^2}{R^2}\right]}\right](\rho+\frac{p}{c^2}) \tag{8}$$

This law points out that the pressure gradient coupled with the repulsive force make up for the gravitational force of attraction of matter and thus establishes equilibrium. We shall examine how the law of variation of the density given by Equation (4) facilitates us to assess the mass and the radius of the configuration.

0.3 GENERAL SOLUTION OF FIELD EQUATIONS

Adopting new variable ψ and z^2 defined as,

$$\begin{aligned} \psi &= e^{\nu/2} \\ z^2 &= 1-\frac{r^2}{R^2} \end{aligned} \tag{9}$$

in to the second order, non linear ordinary differential equation (6), resulting in a second order linear differential equation of the form,

$$(1-K+Kz^2)\frac{d^2\psi}{dz^2}-Kz\frac{d\psi}{dz}+K(K-1)\psi=0 \tag{10}$$

Defining an independent variable,

$$u^2=\frac{K}{K-1}z^2, K<0$$

changes the differential equation (10) to the form

$$(1-u^2)\frac{d^2\psi}{du^2}+u\frac{d\psi}{du}+(1-K)\psi=0 \tag{11}$$

used by Tikekar and Vaidya (1982). Further considering the new independent variable $x = u^2$, the differential equation (11) can be written in the form of a hyper-geometric equation as

$$x(1-x)\frac{d^2\psi}{dx^2}+\frac{1}{2}\frac{d\psi}{du}+\frac{(1-K)}{4}\psi=0 \tag{12}$$

The function $\psi = e^{\nu/2}$ which satisfies the above equation can be equated to

$$\psi = AF\left[\frac{-1+\sqrt{2-K}}{2},\frac{-1-\sqrt{2-K}}{2},\frac{1}{2},x\right]+Bx^{1/2}F\left[\frac{\sqrt{2-K}}{2},\frac{-\sqrt{2-K}}{2},\frac{3}{2},x\right] \tag{13}$$

where $F[a,b,c,x]$ is the hyper-geometric function with its arguments A and B are arbitrary constants. The closed form solutions that can be obtained from (13) can be put into two classes based on the values of K.
The solution ψ for the equation (13) for both the cases of K can be determined.

Case 1: $K = 2-(2n^2-1), n = 2,3,4,\cdots,$

$$\psi = AF\left[\frac{-1+\sqrt{2-K}}{2},\frac{-1-\sqrt{2-K}}{2},\frac{1}{2},x\right]+Bx^{\frac{1}{2}}(1-x)^{\frac{3}{2}}F\left[\frac{3+\sqrt{2-K}}{2},\frac{3-\sqrt{2-K}}{2},\frac{3}{2},x\right]$$

Case 2: $K = 2-4n^2, n = 1,2,3,\cdots,$

$$\psi = A(1-x)^{\frac{3}{2}}F\left[\frac{2+\sqrt{2-K}}{2},\frac{2-\sqrt{2-K}}{2},\frac{1}{2},x\right]+Bx^{\frac{1}{2}}F\left[\frac{\sqrt{2-K}}{2},\frac{-\sqrt{2-K}}{2},\frac{3}{2},x\right]$$

The solution given by (13) can be used in general for all $K < 0$ as far as numerical calculations are concerned.
As we have reached the solution for the Einsteins field equation without making any assumption on the equation of state for its matter content, it is required to examine the physical plausibility of the solution thus obtained.

0.4 PHYSICAL PLAUSIBILITY AND BOUNDARY CONDITIONS

For the metric of field equation given by (1) with $e^{\nu/2}$ given by (13) to have a physically meaningful solution, it must satisfy the following requirements. (Knutsen 1988)

1. The metric should join continuously with the Schwarzschild's exterior metric which is given by,

$$ds_1^2 = -\left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right) + \left(1 - \frac{2m}{r}\right)dt^2 \tag{14}$$

2. Across the boundary $r = a$, $p(a)$ must be 0.
3. The matter density ρ and the fluid pressure p should be positive within the configuration.
4. The gradients $\frac{dp}{dr}$ and $\frac{d\rho}{dr}$ should be negative.
5. $\frac{dp}{d\rho} < 1$.

The first two boundary conditions leads to the relations,

$$\begin{aligned} \frac{1 - \frac{a^2}{R^2}}{1 - K\frac{a^2}{R^2}} &= 1 - \frac{2m}{a} \\ e^{\nu(a)} &= 1 - \frac{2m}{a} \\ and \qquad p(a) &= 0 \end{aligned}$$

These conditions helps to decide the constants A, B and m in terms of a, R and K.
The remaining conditions helps to derive,

$$\frac{d\rho}{dr} = \frac{(1-K)Kr}{8\pi R^4}\frac{\left(5 - K\frac{r^2}{R^2}\right)}{\left(1 - K\frac{r^2}{R^2}\right)^3}, \tag{15}$$

and that implies for $K < 0$, $\frac{d\rho}{dr} < 0$.
The expression for fluid pressure in general can be expressed as

$$8\pi p = \frac{1}{1 - K + Kz^2}\left[\frac{\nu'}{r}z^2 + \frac{K-1}{z^2}\right],$$

and the positive nature of matter pressure within the configuration gives,

$$\frac{\nu'}{r}z^2 \geq \frac{1-K}{z^2}\frac{d\rho}{dr} \tag{16}$$

Differentiating p with respect to r gives the relation,

$$\frac{dp}{dr} = -(p+\rho)\frac{1 - K\frac{r^2}{R^2}}{1 - \frac{r^2}{R^2}}\left[4\pi pr + \frac{(1-K)r}{2R^2\left(1 - K\frac{r^2}{R^2}\right)}\right] \tag{17}$$

Satisfying (16) by the above equation, implies $\frac{dp}{dr} < 0$ for $K < 0$
In the case of **isentropic fluids**, the speed of sound is given by $\sqrt{\frac{dp}{d\rho}}$ and all perfect fuid being isentropic implies that $\sqrt{\frac{dp}{d\rho}} < 1$.
Merging (15) and (17), gives

$$\frac{dp}{d\rho} = \frac{\dfrac{2\pi R^2(p+\rho)\left[(1-K)+8\pi pR^2\left(1-K\frac{r^2}{R^2}\right)\right]}{\left(1-K\frac{r^2}{R^2}\right)^3}}{K(K-1)\left(1-\frac{r^2}{R^2}\right)\left(5-K\frac{r^2}{R^2}\right)} \tag{18}$$

At the centre $r = 0$,

$$\left(\frac{dp}{d\rho}\right)_0 = \frac{2\pi R^2(p(0)+\rho(0))\left[(1-K)+ 8\pi p(0)R^2\right]}{5K(K-1)} \tag{19}$$

where $p(0)$ and $\rho(0)$ denotes the fluid pressure and density at the center. For the fluid distribution to complement with the strong energy condition at the centre, $\rho(0)-3p(0) \geq 0$, it follows,

$$\left(\frac{dp}{d\rho}\right)_0 \leq \frac{2(K-1)}{5K} \tag{20}$$

and hence for $\left(\frac{dp}{d\rho}\right)_0 < 1$, $K < \frac{-2}{3}$.

At the boundary, $r = a$,

$$\left(\frac{dp}{d\rho}\right)_a = \frac{-2\pi R^2\rho(a)\left[1-K\frac{a^2}{R^2}\right]^3}{K\left[1-\frac{a^2}{R^2}\right]\left[5-K\frac{a^2}{R^2}\right]} \tag{21}$$

Using the expression (4) for $\rho(a)$, the above equation becomes,

$$\left(\frac{dp}{d\rho}\right)_a = \frac{3(K-1)}{4K}\frac{\left[1-\frac{Ka^2}{3R^2}\right]\left[1-K\frac{a^2}{R^2}\right]}{\left[1-\frac{a^2}{R^2}\right]\left[5-K\frac{a^2}{R^2}\right]} \tag{22}$$

The last condition $\frac{dp}{d\rho} < 1$ will be satisfied if and only if,

$$\frac{a^2}{R^2} \leq \frac{-12+\sqrt{144+(K-5)(3+17K)}}{K(K-5)} \tag{23}$$

which gives the condition $K < \frac{-3}{17}$.

0.5 THE SOLUTION FOR $K = -1$

The line element describing the space-time metric for the value of $K = -1$ has the form

$$ds^2 = -\frac{1+\frac{r^2}{R^2}}{1-\frac{r^2}{R^2}}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) + e^{\nu(r)}dt^2 \tag{24}$$

If the matter content of the space time is a perfect fluid in equilibrium, ν is determined by solving the differential equation (10). The equation for $K = -1$ reduces to

$$(2-z^2)\frac{d^2\psi}{dz^2} + z\frac{d\psi}{dz} + 2\psi = 0 \tag{25}$$

The hyper-geometric solution of (10) for the choice of $K = -1$ gives

$$\psi = e^{\nu/2} = AF\left[\frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}, \frac{1}{2}, x\right] + Bx^{\frac{1}{2}}(1-x)^{\frac{3}{2}}F\left[\frac{\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}, \frac{3}{2}, x\right] \quad (26)$$

Here A and B are constants. The expressions for matter density and pressure is given by,

$$8\pi\rho = \frac{6}{R^2}\frac{[1+\frac{1}{3}\frac{r^2}{R^2}]}{[1+\frac{r^2}{R^2}]^2} \quad (27)$$

$$8\pi p = \frac{1}{2-z^2}\left[\frac{\nu'}{r}z^2 - \frac{2}{z^2}\right], \quad (28)$$

where

$$\frac{\nu'}{r} = -\frac{2}{R^2}\frac{\begin{array}{c} -AF\left[\frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}, \frac{3}{2}, x\right] - \frac{1}{2}Bx^{1/2} \\ F\left[\frac{2+\sqrt{3}}{2}, \frac{2-\sqrt{3}}{2}, \frac{5}{2}, x\right] - F\left[\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, \frac{3}{2}, x\right] \end{array}}{\begin{array}{c} AF\left[\frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{1}}{2}, \frac{3}{2}, x\right] + Bx^{1/2} \\ F\left[\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, \frac{3}{2}, x\right] \end{array}} \quad (29)$$

The density and the pressure attains the value ρ_0 and p_0 at the centre (r=0) and is given by,

$$8\pi\rho_0 = \frac{6}{R^2} \quad (30)$$

$$8\pi p_0 = \frac{-2}{R^2}\left[\frac{-0.23589A + 0.161366B}{0.23589A + 0.32274B} - 2\right] \quad (31)$$

The pressure at the center being positive is ensured by imposing the condition,

$$\frac{A}{B} > -1.6840 \quad (32)$$

The boundary condition that ensures continuity of metric (24) with e^{ν} implies

$$1 - \frac{2m}{a} = \frac{1-\frac{a^2}{R^2}}{1+\frac{a^2}{R^2}} \quad (33)$$

$$1 - \frac{2m}{a} = e^{\nu(a)} = \psi^2 = \left(AF\left[\frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}, \frac{1}{2}, x\right] + BzF\left[\frac{\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}, \frac{3}{2}, x\right]\right)^2 \quad (34)$$

Imposing the strong energy condition $\rho_0 - 3p_0 \geq 0$ at the centre gives further restriction on A and B such that

$$\frac{A}{B} \geq -1.3712 \quad (35)$$

A physically plausible fluid distribution is expected to comply with the requirement of causality condition that the speed of light $\frac{dp}{d\rho}$ will be the limiting speed of propagation of any signals in it. Accordingly, the causality requirement demand $\left(\frac{dp}{d\rho}\right)_0 < 1$.
Since $\rho_0 > 3p_0$ then from (20),

$$\left(\frac{dp}{d\rho}\right)_0 \leq 0.8 \quad (36)$$

At $r = a$,

$$\frac{a^2}{R^2} \leq \frac{-12+\sqrt{144+6\times 14}}{6} \quad (37)$$

then $\frac{dp}{d\rho} < 1$ at $r = a$.
Hence it is expected that speed of sound will not exceed speed of light throughout the configuration for fluid spheres with $a^2 < 0.5166R^2$.

The condition of pressure being 0 at the surface $r = a$ together with the continuity of metric coefficients gives

$$m = \frac{4\pi G}{c^2}\int_0^a \xi^2 p(\zeta)d\zeta = \frac{a^3}{a^2 + R^2} \tag{38}$$

and

$$\left(1 - \frac{2m}{a}\right)^{\frac{1}{2}} = AF\left[\frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}, \frac{1}{2}, x\right] + Bx^{\frac{1}{2}}F\left[\frac{\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}, \frac{3}{2}, x\right] \tag{39}$$

The pressure $p = 0$ at $r = a$ gives

$$-AF\left[\frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}, \frac{3}{2}, x\right] = \frac{1}{2}Bx^{\frac{1}{2}}F\left[\frac{2+\sqrt{3}}{2}, \frac{2-\sqrt{3}}{2}, \frac{5}{2}, x\right] \tag{40}$$

The conditions (39) and (40) determine the constants A and B while the condition (38) decides the total mass m of the distribution.

The density variation parameter λ is defined as,

$$\lambda = \frac{\rho_a}{\rho_0} = \frac{1 + \frac{a^2}{3R^2}}{(1 - \frac{a^2}{R^2})^2} \tag{41}$$

Following the strong energy condition, we reached out a condition that $\frac{a^2}{R^2} < 0.5166$ and this requirement is fulfilled for models with λ. The above equations are used for modelling, and finding the values of R, r M, $\frac{M}{M^a_\Theta}$, A and B and the same is tabulated below.

Table 1: Masses and equilibrium radii matching to ρ_a = 2×10^{14} gm/cm^3 for the class of relativistic star models.

λ	$R(km)$	$a(km)$	a^2/R^2	m	m/M_Θ	A	B
0.9	38.07	9.75	0.07	0.5995	0.4064	5574.942879	-4071.812627
0.85	36.99	11.88	0.10	0.1108	0.7531	5974.428971	-4363.904013
0.8	35.89	13.65	0.14	1.7253	1.1697	6516.443218	-4760.176525
0.75	34.75	15.18	0.19	2.43	1.65	7273.947259	-5313.959013
0.7	33.57	16.54	0.24	3.23	2.19	8375.914451	-6119.519593
0.65	32.35	17.75	0.30	4.11	2.79	10069.48439	-7357.494467
0.6	31.08	18.85	0.37	5.0695	3.4370	12881.25442	-9412.7759
0.55	29.76	19.85	0.44	6.1121	4.1438	18109.67936	-13234.40686
0.5	28.37	20.76	0.54	7.2359	4.9057	29721.51038	-21721.68567
0.45	26.92	21.58	0.64	8.4416	5.7231	64894.43843	-47429.76592
0.4	25.38	22.32	0.77	9.5094	6.44717	272783.69	-199375.9281
0.35	23.74	22.97	0.94	11.1053	7.5290	19297280.38	-14104341.85
0.3	21.98	23.53	1.15	12.5683	8.5209	-	-
0.25	20.06	24.00	1.43	14.1228	9.5747	-	-

Note: The mass m recorded in table is measured in km. 1 M_Θ= 1.475 km.

0.6 STABILITY ANALYSIS OF THE MODEL FOR $K = -1$

Chandrasekhar (1964) developed the method to investigate the stability of the star models with respect to infinitesimal radial pulsations. A normal mode of radial oscillation for an equilibrium configuration is defined as,

$$\delta r = \xi(r)_{trial}e^{i\sigma t} \tag{42}$$

and is said to be stable if and only if σ is real [Knutsen (1988)]. A trial function u is written as [Bardeen, Thorne and Meltzer]

$$u = \xi_{trial} r^2 e^{-\frac{\nu}{2}} \tag{43}$$

Chandrasekhars pulsation equation for the line element ,

$$ds^2 = -e^{\lambda} dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2)e^{\nu} dt^2 \tag{44}$$

is given as

$$\sigma^2 \int_0^a e^{\frac{1}{2}(3\lambda+\nu)(p+\rho)\frac{u^2}{rv^2}dr} = \int_0^a e^{\frac{1}{2}(\lambda+3\nu)\frac{p+\rho}{r^2}} \left[\left(\frac{-2}{r}\frac{d\nu}{dr} - \frac{1}{4}\left(\frac{d\nu}{dr}\right)^2 + 8\pi p e^{\lambda} \right) u^2 + \frac{dp}{d\rho}\left(\frac{du}{dr}\right)^2 \right] dr \tag{45}$$

The condition that the Lagrangian change in pressure vanishes at the surface of the star $r = a$,

$$\Delta p = e^{\frac{-nu}{2}} \frac{1}{r^2} \nu p \frac{du}{dr} = 0 \tag{46}$$

is satisfied if

$$\left[\frac{du}{dr} \right]_{r=a} = 0 \tag{47}$$

Consider a trial function of the form,

$$u = R^3(1-z^2)^{\frac{3}{2}} \left[1 + a_1(1-z^2) + b_1(1-z^2)^2 + \cdots \right] \tag{48}$$

The condition (47) demands

$$3 + 5a_1 y + 7b_1 y^2 + \cdots = 0 \tag{49}$$

where $y = \frac{a^2}{R^2}$ and $a_1, b_1 \cdots$ are arbitrary constants.
The integral I_R on the right hand side of the pulsation equation is written as,

$$I_R = \frac{R}{8\pi\sqrt{2}} \int_0^a e^{(\lambda+3\nu)\frac{p+\rho}{2R^2}} y \left[y \left(\frac{2}{z}\frac{d\nu}{dz} - \frac{y}{4z^2}\left(\frac{d\nu}{dr}\right)^2 + 8\pi p e^{\lambda} \right) (1 + a_1 y + b_1 y^2)^2 + \frac{dp}{d\rho}(3 + 5a_1 y + 7b_1 y^2) \right] \sqrt{\frac{1}{y}} dy$$

where p, ρ, ν and λ are given by the equation (28), (27), (26) and (1). By assuming different combinations of values for a_1 and b_1, we found that the I_R remains positive for particular models. It gives a strong indication that the model presented here is stable with respect to infinitesimal radial pulsations.

0.7 DISCUSSIONS

When the thermonuclear sources of energy inside the star gets exhausted, it starts to contract under the control of gravitational interplay of its substance content until it ends up in its final fate as a black hole, white dwarf, or neutron star depending upon the mass. The models of Tikekar and Vaidya (1982) describe a super dense star having densities of their matter content in the range of 10^{14} - 10^{16} $gmcm^{-3}$ which is formed during these last stages of stellar development. We assume that at the boundary $r = a$, the density of the star is ρ_a = 2 × 10^{14} $gmcm^{-3}$ which corresponds to that of neutron star. Then, adopting the scheme explained by giving different values of $\lambda = \frac{\rho_a}{\rho_0}$ and for each chosen value of λ and the assumed value, ρ_a we calculate ρ_0. Equation (30) is then used to calculate R^2. Equation (27) then gives us an estimate of a, the radius of the star and finally Equation (33) gives the mass of the star. The value of m as given by Equation (33) will be in km. The mass of the

star in gm is obtained by $\frac{MG}{c^2} = m$ or $M = \frac{mc^2}{G}$. It is easier to express the mass of the star as a multiple of one solar mass M_θ. The results of the calculations for various values of λ are given in Table 1.
From Table 1 it is very well clear that $\frac{a^2}{R^2}$ is a decreasing function of λ. Now the physical requirement that ρ, p and $\rho - 3p/c^2$ be all 0 restricts to the condition (37) viz. $\frac{a^2}{R^2} \leq 0{\cdot}5166$. Therefore the corresponding restriction on λ is $\lambda \leq 0{\cdot}55$. Thus the first eleven values in the table gives a series of physically viable solutions to the above star-models. The most interesting fact is that each of these models possess an equilibrium radius which is equal to that of the radius of a neutron star. The maximum mass for the configuration is 4.14 M_θ and is obtained at the radius of 19.85 km. Both m and a are decreasing functions of λ .However if we relax the physical requirement to $\rho > 0$, $p \geq 0$, $\rho - 3\frac{p}{c^2} \geq$0, the restriction on $\frac{A}{B}$ is $\geq$ -1.3712 and considering this condition along with the other restrictions we end up with the highlighted value in the value as the most appropriate value for $\frac{m}{M_\Theta}$.
Graphs showing the variation of pressure (p), weak energy condition $(\rho - p)$, strong energy condition $(\rho - 3p)$ and rate of change of pressure with respect to density are plotted against different values of radius.

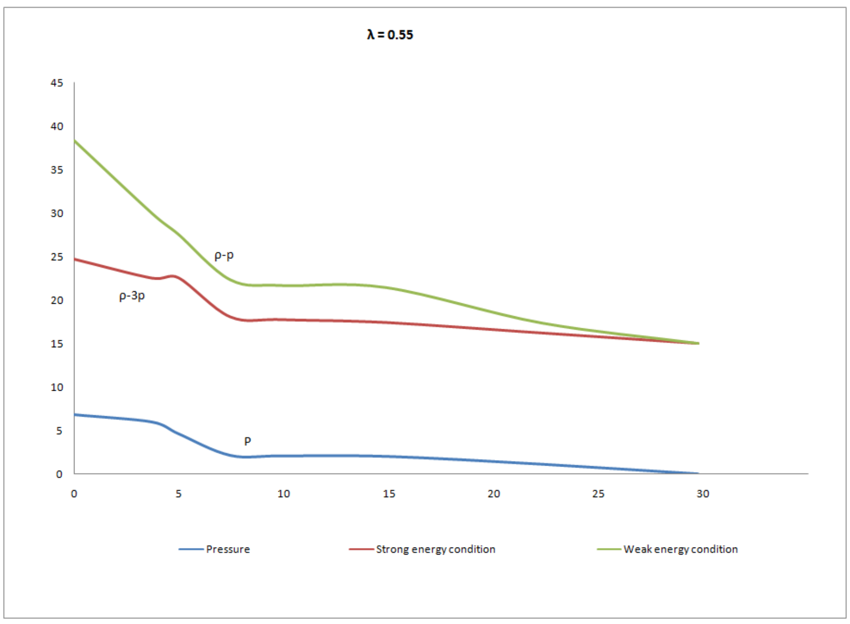

Fig. 1. Graph of p, $\rho - 3p$ and $\rho - p$ corresponding to $\lambda = 0.55$ is plotted against radius.

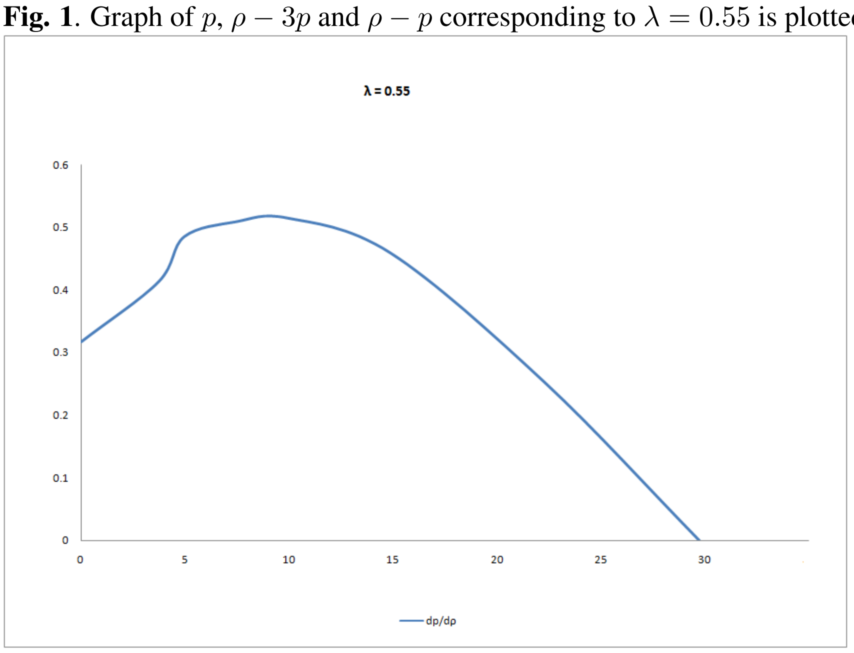

Fig. 2. Graph of $\frac{dp}{d\rho}$ at $\lambda = 0.55$ is plotted against radius.

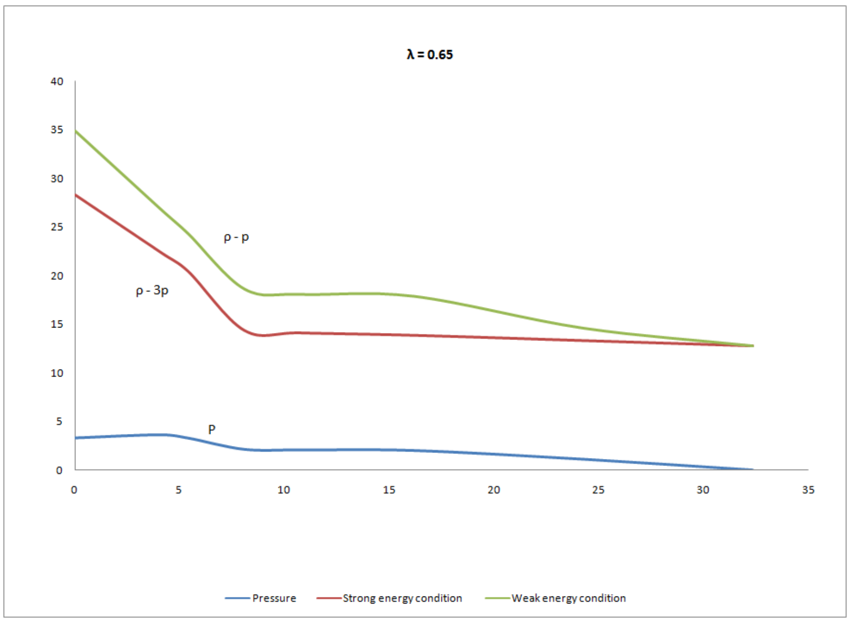

Fig. 3. Graph of p, $\rho - 3p$ and $\rho - p$ corresponding to $\lambda = 0.65$ is plotted against radius.

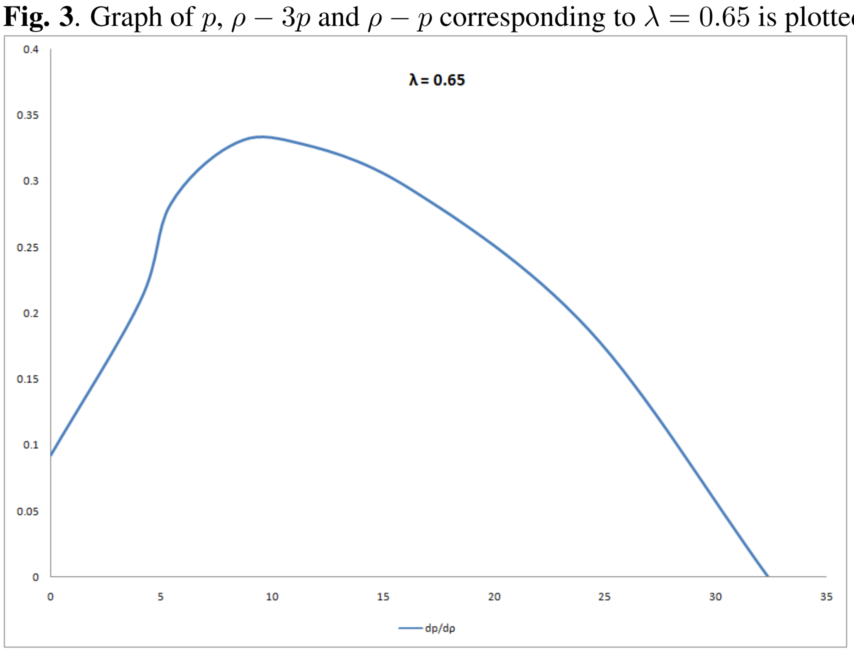

Fig. 4. Graph of $\frac{dp}{d\rho}$ at $\lambda = 0.65$ is plotted against radius

One good feature of Vaidya-Tikekar model is that they permit comparatively higher values of maximum mass of a neutron star than the values permitted by a similar non-nuclear analysis by Rhoades and Ruffini (1974). According to Tikekar and Vaidya the solution to the differential equations involves solving a recurrence relation and hence that solution can be applicable only to certain values of K (few integral values like $K = -2, -7, -14, -23 \cdots$). There arises the importance of our solution that we have mentioned in this paper. We had derived a hypergeometric solution to Vaidya-Tikekar metric, that it could be used for any values of K with $K < \frac{-3}{17}$ and henace the solution can be considered as a general solution to V-T metric. The same general solution can be used even for pseudospheroidal geometry in which many researchers are working on. In order to check the validity of the solution, a particular value of $K(K = -1)$ is considered and the physical plausibility of the system was studied. All the five boundary conditions are evaluated for $K = 1$ and its noted that all the boundary conditions go well in hand with the general conditions. The most interesting and significant result is that with $K = -1$, our general solution to V-T metric is awarding a better limit to the maximum mass a neutron star can hold (4.14 M_θ). Thus the series of equilibrium configurations given by these space-time metric, each having mass, radii and surface density of the same order as in a neutron star.

0.8 ACKNOWLEDGEMENTS

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