International Journal of Mathematics Trends and Technology Volume 67 Issue 7, 26-37, August, 2021
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V67I8P503

Topological Indices and M-Polynomials of Wheel And Gear Graphs

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Abstract* — *Let G = (V, E) be a molecular graph with vertex set V and edge set E. The topological indices of molecular graphs are widely used for establishing relation between the structure of a molecular compound and its physicochemical properties or biological properties. The degree-based topological indices of wheel graph ($W_{4,m}$), antiwheel graph($AWW_{5,m}$), gear graph ($J_{8,m}$),and antiweb gear graph ($AWJ_{8,m}$), are studied from M-polynomials.

Keywords — *Antiweb gear graph, antiweb wheel graph, gear graph, m-level wheel, M-polynomial, topological index.*

I. INTRODUCTION

Let G be a simple graph with vertex set V(G) and edge set E(G).Let |V(G)| = n and |E(G)| = m. The topological indices of graphs are widely used for establishing correlations between the molecular compound and its physicochemical properties or biological properties. The topological index of a molecule structure can be considered as a non-empirical numerical quantity which quantifies the molecular structure and its branching pattern [1].The degree-based topological indices play a significant role in understanding various properties of a molecular structure. A topological index is a numerical parameter mathematically derived from the graph structure. The application of molecular descriptors is nowadays a standard procedure in the study of structure property relations, especially in the QSAR/QSPR study.

The Hosoya polynomial is the most well known polynomial which plays a vital role in determining distance-based topological indices such as Wiener index of graph [2].The Zagreb polynomials and forgotten polynomials are studied for vitamin D_3 by M.R.R.Kanna [3].The main advantage of M-polynomials is the information it contains about degree-based graph invariants. The sum of the degrees of neighbor vertices based fifth Zagreb index and polynomial are studied by P.Sarkar [4].The computation of degree-based topological indices from M-polynomials are studied by E.Deutsch and S.Klavzar [5].The metric dimension of wheel related graphs are studied by H.M.A.Siddique [6]. F-polynomial and fourth Zagreb polynomial of a molecular graph are studied by N.K.Raut et al. [7].In a wheel graph, the hub has degree n-1 and other nodes have degree 3.Generalized wheel graph, is also known as multilevel network $W_{m,n}$ [8].The multilevel wheel network $W_{m,n}$ is obtained from m-copies of cycles C_n and one copy of vertex, such that all vertices of every copy of C_n are adjacent to V. Thus $W_{m,n}$ has m+1 vertices and diameter 2, is shown in figure 1 [9,10].The web graph $W_{n,r}$ is a graph consisting of r concentric copies of cycle graph C_n with corresponding vertices connected by spokes. The $W_{m,n}$ graph has mn+1 vertices that is the centre and n-rim vertices and has diameter 2.Antiwheel graph is the complementary graph of a wheel. A graph is gear graph obtained from W_n by adding a vertex between rim vertices and is denoted by G_n, G_n has 2n+1 vertices and 3n edges. An antiweb gear graph obtained by replacing C_{2n} with $(C_{2n})^2$ in a gear graph J_{2n},it is denoted by AWJ_{2n}.

The first and second Zagreb indices are degree-based graph invariants. The first Zagreb polynomial is defined as $M_1(G,x) = \sum_{uv\in(G)} x^{d_u+d_v}$.The first and second Zagreb indices, modified second Zagreb index, Randic index, inverse Randic index, symmetric division index, harmonic index, inverse sum index and augmented Zagreb index are defined in [11-18].The M-polynomial of graph G is defined as [19-23],

M(G;x,y) = $\sum_{\delta \leq i \leq j \leq \Delta} m_{ij}(G)\, x^i y^j$,where δ = Min{d_v|v ∈ V(G)},Δ = Max{d_v|v ∈ V(G)},

and $m_{ij}(G)$ is the edge vu ∈ E(G) such that i≤ j, with $D_x = x\frac{\partial}{\partial x} f(x,y)$, $D_y = y\frac{\partial}{\partial y} f(x,y)$, $S_x = \int_0^x \frac{f(t,y)}{t} dt$,

$S_y = \int_0^y \frac{f(x,t)}{t} dt$, J(f(x,y)) = f(x,x), $Q_\alpha(f(x,y)) = x^\alpha f(x,y)$.

Our notation is standard and mainly taken from standard books of graph theory [24-29].In this paper we investigate the M-polynomial and topological indices of m-level wheel graph ($W_{4,m}$),m-level antiwheel graph ($AWW_{5,m}$), m-level gear graph ($J_{8,m}$) and m-level antiweb gear graph($AWJ_{8,m}$).The edge partitions of these graphs are represented in table numbers (1 to 4) and derivational formulas for studied topological indices are given in table number 5.

II. MATERIALS AND METHODS

A topological index is a numerical parameter mathematically derived from the graph structure. A wheel graph W_n is the join of K_1 and C_n. A wheel graph W_n has n+1 vertices and 2n edges. The m-level wheel graph ($W_{4,m}$) represented in figure 1. It is observed from this figure 1, there are two edges $E_{mn,3}$ and $E_{3,3}$with frequency mn and mn respectively. The graph of m-level antiwheel graph ($AWW_{5,m}$) is represented in figure 2. In this graph the degrees (d_u,d_v) are (mn,5) and (5,5) with number of edges mn and 2mn.The gear graph ($J_{8,m}$) is given in figure 3.This gear graph $J_{8,m}$ has edges $E_{mn,3}$ and $E_{2,3}$ with frequency mn and 2mn respectively. The m-level antiweb gear graph ($AWJ_{8,m}$) is represented in figure 4. It is observed from figure 4, there are four edges $E_{mn,5}$, $E_{5,5}$ $E_{4,4}$ and $E_{4,5}$ with frequency mn, mn, mn and 2mn respectively. The harmonic polynomial is defined as H(G,x) = 2 $\sum_{uv\in(G)} x^{d_u+d_v-1}$,where $\int_0^1 H(G,x)\,dx$ = H(G).The M-polynomial of G is defined as,

M(G;x,y) = $\sum_{\delta \leq i \leq j \leq \Delta} m_{ij}(G)\, x^i y^j$,where δ = Min{d_v| v∈V(G)} and Δ= Max{d_v|v∈V(G)}.

The harmonic index is studied from M-polynomial for which the function F = f(x,y) is f(x,y) =$\frac{2}{x+y}$. More on the f(x,y) functions defined in terms of topological indices is given in [12]. The first Zagreb index, second Zagreb index, modified second Zagreb index, Randic index, inverse Randic index, symmetric division index, harmonic index, inverse sum index and augmented Zagreb index are studied from M-polynomials for wheel graph, antiweb wheel graph, gear graph and antiweb gear graph. The edge partitions of wheel and gear graphs are represented in table number (1 to 4) and the formulas used in the computation of these topological indices are given in table 5. The computation of these topological indices from M(G;x,y) is given in the following section.

III. RESULTS AND DISCUSSION

Theorem 3.1 The M-polynomial of m-level wheel graph ($W_{4,m}$) is

$M(W_{4,m};x,y) = mnx^{mn} y^3 + mnx^3y^3$.

Proof. From figure 1 we notice that there two separate cases and the number of edges is different, namely $E_{\{mn,3\}}$,and $E_{\{3,3\}}$.

$E_{\{mn,3\}}$= {uv ∈ E($W_{4,m}$)|d_u= mn, d_v= 3},$E_{\{3,3\}}$ = {uv ∈ E($W_{4,m}$)|d_u=3,d_v=3}.

The number of edges $E_{\{mn,3\}}$,$E_{\{3,3\}}$ are mn and mn respectively.

$M(W_{4,m};x,y) = \sum_{\delta\leq i\leq j\leq\Delta} m_{ij}(G)\, x^i y^j = \sum_{mn\leq 3} m_{mn3}(W_{4,m})\, x^{mn} y^3 + \sum_{3\leq 3} m_{33}(W_{4,m})\, x^3 y^3$

$= |E_{\{mn,3\}}|x^{mn}y^3 + |E_{\{3,3\}}|x^3y^3.$

$= mnx^{mn} y^3 + mnx^3y^3.$

Proposition. Let $W_{4,m}$ be a graph of m-level wheel graph, we then have following topological indices.

1.$M_1(W_{4,m}) = (mn)^2+9mn$.

2.$M_2(W_{4,m}) =3(mn)^2+9mn$.

3.$^mM_2(W_{4,m}) == \frac{1}{3}(1+\frac{mn}{3})$.

4.$R_\alpha(W_{4,m}) = 3^\alpha(mn)^{\alpha+1}+3^{2\alpha} mn$.

5.$RR_\alpha(W_{4,m}) = \frac{mn}{3^\alpha(mn)^\alpha} +\frac{mn}{3^{2\alpha}}$.

6.$SSD(W_{4,m}) = \frac{1}{3}(mn)^2+2mn+3$.

7.$H(W_{4,m}) = \frac{2mn}{mn+3} + \frac{mn}{3}$.

8.$I(W_{4,m}) =\frac{3(mn)^2}{mn+3} + \frac{3mn}{2}$.

9.$A(W_{4,m}) = \frac{27(mn)^4}{(mn+3)^3} +12mn$.

Proof. The following are made in order to have the above topological indices.

$M(W_{4,m};x,y) = mnx^{mn} y^3 + mnx^3y^3.$

$D_x(f(x,y)) = (mn)^2x^{mn} y^3 + 3mnx^3y^3,$

$D_y(f(x,y)) = 3mnx^{mn} y^3 + 3mn x^3y^3,$

$D_xD_y(f(x,y)) = 3(mn)^2x^{mn} y^3 + 9mnx^3y^3,$

$S_xD_y(f(x,y)) = 3x^{mn} y^3 +mn x^3y^3,$

$S_y(f(x,y)) = \frac{mn}{3}x^{mn} y^3 + \frac{mn}{3}x^3y^3,$

$D_xS_y(f(x,y)) = \frac{(mn)^2}{3} x^{mn} y^3 + mn x^3y^3,$

$S_xS_y(f(x,y)) =\frac{1}{3} x^{mn} y^3 + \frac{mn}{9}x^3y^3,$

$J(f(x,y)) = mnx^{mn+3} + mnx^6,$

$S_xJ(f(x,y)) = \frac{mn}{mn+3} x^{mn+3} + \frac{mn}{6} x^6,$

$S_xJD_xD_y(f(x,y)) = \frac{3(mn)^2}{mn+3} x^{mn+3} y^3 + \frac{3mn}{2}x^6,$

$S_x^3 Q_{-2} J D_x^3 (f(x,y)) = 27\frac{(mn)^4}{(mn+1)^3} x^{mn+1}+12mn x^4,$

$D_x^\alpha D_y^\alpha(f(x,y)) = 3^\alpha (mn)^{\alpha+1}x^{mn} y^3 +3^{2\alpha} mnx^3y^3,$

$S_x^{\alpha}\ S_x^{\alpha}(f(x,y)) = \frac{mn}{3^{\alpha}(mn)^{\alpha}} x^{mn} y^3 + \frac{mn}{3^{2\alpha}}\ x^3\ y^3.$

The topological indices described in table 5 are now obtained by using all the above mentioned values.

1.First Zagreb index

$M_1(W_{4,m}) = (D_x+D_y)(f(x,y))|_{x=y=1} = (mn)^2+9mn.$

2. Second Zagreb index

$M_2(W_{4,m}) = D_xD_y(f(x,y))|_{x=y=1} = 3(mn)^2+9mn.$

3. Modified second Zagreb index

${}^{m}M_2(W_{4,m}) = S_xS_y\,(f(x,y))|_{x=y=1} = \frac{1}{3}(1+\frac{mn}{3}).$

4. Randic index

$R_{\alpha}(W_{4,m}) = D_x^{\alpha}\,D_y^{\alpha}\,(f(x,y))|_{x=y=1} = 3^{\alpha}(mn)^{\alpha+1}+3^{2\alpha}\ mn.$

5. Inverse Randic index

$RR_{\alpha}(W_{4,m}) = S_x^{\alpha}\,S_y^{\alpha}\,(f(x,y))|_{x=y=1} = \frac{mn}{3^{\alpha}(mn)^{\alpha}} + \frac{mn}{3^{2\alpha}}.$

6. Symmetric division index

$SSD(W_{4,m}) = (D_xS_y\ +\ D_y\,S_x)(f(x,y))|_{x=y=1}\ = \frac{1}{3}\,(mn)^2+2mn+3.$

7. Harmonic index

$H(W_{4,m}) = 2S_xJ\ (f(x,y))|_{x=1} = \frac{2mn}{mn+3} + \frac{mn}{3}.$

8. Inverse sum index

$I(W_{4,m}) = S_xJD_xD_y(f(x,y))|_{x=1} = \frac{3(mn)^2}{mn+3} + \frac{3mn}{2}.$

9. Augmented Zagreb index

$A(W_{4,m}) = S_x^3\,Q_{-2}\,J\ D_x^3\,D_y^3\ (f(x,y))|_{x=1} = \frac{27\,(mn)^4}{(mn+3)^3} +12mn.$

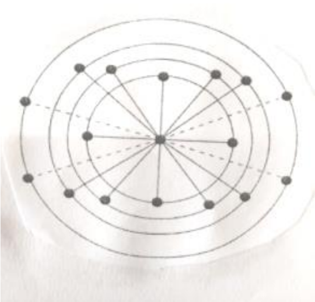


Figure 1: m-level wheel graph ($W_{4,m}$).

(d_u,d_v),where $u,v \in E(G)$	Number of edges E(G)
(mn,3)	mn
(3,3)	mn

Table 1: Edge partition for wheel graph ($W_{4,m}$).

Theorem.3.2. The M-polynomial of m-level antiwheel graph ($AWW_{5,m}$) is

$M(W_{5,m};x,y) = mnx^{mn} y^3 + 2mnx^5y^5$.

Proof. From figure 2 we notice that there two separate cases and the number of edges is different, namely $E_{\{mn,5\}}$ and $E_{\{5,5\}}$.

$E_{\{mn,5\}}= \{uv\in E(AWW_{5,m})|d_u= mn, d_v=5\}, E_{\{5,5\}} = \{uv\in E(AWW_{5,m})|d_u=5,d_v=5\}$.

The number of edges $E_{\{mn,5\}}, E_{\{5,5\}}$ are mn and 2mn respectively.

$M(AWW_{5,m};x,y) = \sum_{\delta\le i\le j\le\Delta} m_{ij}(G)\, x^i y^j$.

$= \sum_{mn\le 5} m_{mn5}(AWW_{5,m})\, x^{mn} y^5 + \sum_{5\le 5} m_{55}(AWW_{5,m})\, x^5 y^5$.

$= |E_{\{mn,5\}}|x^{mn}y^5 + |E_{\{5,5\}}|x^5y^5$.

$= mnx^{mn} y^5 + 2mnx^5y^5$.

Proposition. Let $AWW_{5,m}$ be a graph of m-level antiwheel graph, we then have following topological indices.

1. $M_1(AWW_{5,m}) = (mn)^2+25mn$.

2. $M_2(AWW_{5,m}) = 5(mn)^2+50mn$.

3. $^mM_2(AWW_{5,m}) = \frac{1}{5}+\frac{2mn}{25}$.

4. $R_\alpha(AWW_{5,m}) = 5^\alpha(mn)^{\alpha+1}+5^{2\alpha}\, 2\, mn$.

5. $RR_\alpha(AWW_{5,m}) = \frac{mn}{5^\alpha (mn)^\alpha} + \frac{2mn}{5^{2\alpha}}$.

6. $SSD(AWW_{5,m}) = \frac{1}{5}(mn)^2+4mn+5$.

7. $H(AWW_{5,m}) = \frac{2mn}{mn+5} + \frac{4mn}{10}$.

8. $I(AWW_{5,m}) = \frac{5(mn)^2}{mn+5}+ 5mn$.

9. $A(W_{5,m}) = \frac{125\,(mn)^4}{(mn+3)^3} + 61mn$.

Proof. The following are made in order to have the above topological indices.

$M(AWW_{5,m};x,y) = \sum_{\delta\le i\le j\le\Delta} m_{ij}(AWW_{5,m})\, x^i y^j$.

$M(AWW_{5,m};x,y) = mnx^{mn} y^5 + 2mnx^5y^5$.

$D_x(f(x,y)) = (mn)^2x^{mn} y^5+ 10mnx^5y^5$,

$D_y(f(x,y)) = 5mnx^{mn} y^5+ 10mn\, x^5y^5$,

$D_xD_y(f(x,y)) = 5(mn)^2x^{mn} y^5 + 50mnx^5y^5$,

$S_xD_y(f(x,y)) = 5x^{mn} y^5 +2mn\, x^5y^5$,

$S_y(f(x,y)) = \frac{mn}{5}x^{mn} y^5 +2\frac{mn}{5}x^5y^5$,

$D_xS_y(f(x,y)) = \frac{(mn)^2}{5}x^{mn} y^5 +2mnx^5y^5$,

$S_xS_y(f(x,y)) = \frac{mn}{5}x^{mn}\ y^5 + \frac{2mn}{25}x^5y^5$,

$J(f(x,y)) = mnx^{mn+5} + 2mn\ x^{10}$,

$S_xJ(f(x,y)) = \frac{mn}{mn+5}x^{mn+5} + \frac{mn}{5}x^{10}$,

$S_xJD_xD_y(f(x,y)) = 5\frac{(mn)^2}{mn+5}x^{mn+5} + 5mnx^{10}$,

$S_x^3\,Q_{-2}\,J\,D_x^3\,D_y^3(f(x,y)) = \frac{125\,(mn)^4}{(mn+3)^3} + 61mnx^8$,

$D_x^\alpha\,D_y^\alpha(f(x,y)) = 5^\alpha\,(mn)^{\alpha+1}x^{mn}\,y^5+5^{2\alpha}2\ mnx^5y^5$,

$S_x^\alpha\,S_x^\alpha(f(x,y)) = \frac{mn}{5^\alpha(mn)^\alpha}x^{mn}\ \ y^5+ 2\,\frac{mn}{5^{2\alpha}}x^5\ y^5$.

The topological indices described in table 5 are now obtained by using the above mentioned values.

1. First Zagreb index

$M_1(AWW_{5,m}) = (D_x+D_y)(f(x,y))|_{x=y=1} = (mn)^2+25mn$.

2. Second Zagreb index

$M_2(AWW_{5,m}) = D_xD_y(f(x,y))|_{x=y=1} = 5(mn)^2+50mn$.

3. Modified second Zagreb index

${}^mM_2(AWW_{5,m}) = S_xS_y\,(f(x,y))|_{x=y=1}= \frac{1}{5}+\frac{2mn}{25}$.

4. Randic index

$R_\alpha(AWW_{5,m}) = D_x^\alpha\,D_y^\alpha\,(f(x,y))|_{x=y=1} = 5^\alpha(mn)^{\alpha+1}+5^{2\alpha}\,2\ mn$.

5. Inverse Randic index

$RR_\alpha(AWW_{5,m}) = S_x^\alpha\,S_y^\alpha(f(x,y))|_{x=y=1} = \frac{mn}{5^\alpha(mn)^\alpha} + \frac{2mn}{5^{2\alpha}}$.

6. Symmetric division index

$SSD(AWW_{5,m}) = (D_xS_y + D_y\,S_x)(f(x,y))|_{x=y=1} = \frac{1}{5}\,(mn)^2+4mn+5$.

7. Harmonic index

$H(AWW_{5,m}) = 2S_xJ\,(f(x,y))|_{x=1}= \frac{2mn}{mn+5} + \frac{2mn}{5}$.

8. Inverse sum index

$I(AWW_{5,m}) = S_xJ\ D_xD_y\ (f(x,y))|_{x=1} = \frac{5(mn)^2}{mn+5} + 5mn$.

9. Augmented Zagreb index

$A(AWW_{5,m}) = S_x^3\,Q_{-2}\,J\,D_x^3\,D_y^3\,(f(x,y))|_{x=1} = \frac{125\,(mn)^4}{(mn+3)^3} + 61mn$.

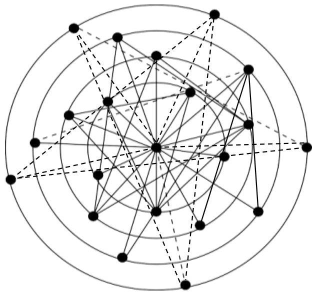


Figure 2: m-level antiwheel graph ($AWW_{5,m}$).

(d_u, d_v),where u,v ∈ E(G)	Number of edges E(G)
(mn,5)	mn
(5,5)	2mn

Table 2: Edge partition for antiweb wheel graph ($AWW_{5,m}$).

Theorem 3.3. The M-polynomial of m-level gear graph ($J_{8,m}$) is

$M(J_{8,m};x,y) = mnx^{mn} y^3 + 2mnx^3y^3$.

Proof. From figure 3 we notice that there two separate cases and the number of edges is different, namely $E_{\{mn,3\}}, E_{\{2,3\}}$.

$E_{\{mn,3\}} = \{uv \in E(J_{8,m}) | d_u = mn, d_v=3\}, E_{\{2,3\}} = \{uv \in E(J_{8,m}) | d_u=2, d_v=3\}$.

The number of edges $E_{\{mn,3\}}, E_{\{2,3\}}$ are mn and 2mn respectively.

$M(J_{8,m};x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(G)\, x^i y^j$.

$= \sum_{mn \le 3} m_{mn3}(J_{8,m})\, x^{mn} y^3 + \sum_{2 \le 3} m_{23}(j_{8,m})\, x^2 y^3$.

$= |E_{\{mn,3\}}| x^{mn} y^3 + |E_{\{2,3\}}| x^2 y^3$.

$= mnx^{mn} y^3 + 2mnx^2y^3$.

Proposition. Let $J_{8,m}$ be a graph of m-level antiwheel graph, we then have following topological indices.

1. $M_1(J_{8,m}) = (mn)^2+13mn$.

2. $M_2(J_{8,m}) = 3(mn)^2+12mn$.

3. $^mM_2(J_{8,m}) = \frac{1}{3}+\frac{mn}{3}$.

4. $R_\alpha(J_{8,m}) = 3^\alpha (mn)^{\alpha+1}+2^{\alpha+1}\ 3^\alpha mn$.

5. $RR_\alpha(J_{8,m}) = \frac{mn}{3^\alpha (mn)^\alpha} + \frac{2mn}{3^\alpha 2^\alpha}$.

6. $SSD(J_{8,m}) = \frac{1}{3}(mn)^2+4mn+3$.

7. $H(J_{8,m}) = \frac{2mn}{mn+3} + \frac{2mn}{5}$

8. $I(J_{8,m}) = \frac{3(mn)^2}{mn+3} + \frac{12mn}{5}$.

9. $A(J_{8,m}) = \frac{27(mn)^4}{(mn+1)^3} + 4mn$.

Proof. The following are made in order to have the above topological indices.

$M(J_{8,m};x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(J_{8,m})\, x^i y^j$.

$M(J_{8,m}; x,y) = mn\, x^{mn} y^3 + 2mnx^2y^3$.

$D_x(f(x,y)) = (mn)^2 x^{mn} y^3 + 4mnx^2y^3$,

$D_y(f(x,y)) = 3mnx^{mn} y^3 + 6mn\, x^2y^3$,

$D_xD_y(f(x,y)) = 3(mn)^2x^{mn} y^3 + 12mnx^2y^3$,

$S_xD_y(f(x,y)) = 3x^{mn} y^3 + 3mn\; x^2y^3$,

$S_y(f(x,y)) = \frac{mn}{3}x^{mn} y^3 + 2\frac{mn}{3}x^2y^3$,

$D_xS_y(f(x,y)) = \frac{(mn)^2}{3}x^{mn} y^3 + 4\frac{mn}{3}x^2y^3$,

$S_xS_yM(J_{8,m}, x,y) = \frac{1}{3}x^{mn} y^3 + \frac{mn}{3}x^2y^3$,

$J(f(x,y)) = mnx^{mn+3} + 2mn\, x^5$,

$S_xJ(f(x,y)) = \frac{mn}{mn+3}x^{mn+3} + \frac{2mn}{5}x^5$,

$S_xJD_xD_y(f(x,y)) = 3\frac{(mn)^2}{mn+3}x^{mn+3} + \frac{12mn}{5}x^5$,

$S_x^3 Q_{-2} J D_x^3 D_y^3 (f(x,y)) = \frac{27}{(mn+1)^2}(mn)^4 x^{mn+1} + 4mnx^3$,

$D_x^\alpha D_y^\alpha(f(x,y)) = 3^\alpha (mn)^{\alpha+1}x^{mn} y^3 + 2^{\alpha+1}3^\alpha mnx^2y^3$,

$S_x^\alpha S_x^\alpha(f(x,y)) = \frac{mn}{3^\alpha (mn)^\alpha} x^{mn} y^3 + \frac{2mn}{3^\alpha 2^\alpha} x^2y^3$.

The topological indices described in table 5 are now obtained by using the above mentioned values.

1. First Zagreb index

$M_1(J_{8,m}) = (D_x+D_y)(f(x,y))|_{x=y=1} = (mn)^2+13mn$.

2. Second Zagreb index

$M_2(J_{8,m}) = D_xD_y\,(f(x,y))|_{x=y=1} = 3(mn)^2+12mn$.

3. Modified second Zagreb index

${}^mM_2(J_{8,m}) = S_xS_y\,(f(x,y))|_{x=y=1} = \frac{1}{3}+\frac{mn}{3}$.

4. Randic index

$R_\alpha(J_{8,m}) = D_x^\alpha D_y^\alpha(f(x,y))|_{x=y=1} = 3^\alpha(mn)^{\alpha+1}+2^{\alpha+1}\,3^\alpha\, mn$.

5. Inverse Randic index

$RR_\alpha(J_{8,m}) = S_x^\alpha S_y^\alpha(f(x,y))|_{x=y=1} = \frac{mn}{3^\alpha (mn)^\alpha} + \frac{2mn}{3^\alpha 2^\alpha}$.

6. Symmetric division index

$SSD(J_{8,m}) = (D_xS_y + S_xD_y)(f(x,y))|_{x=y=1} = \frac{1}{3}(mn)^2+4mn+3.$

7. Harmonic index

$H(J_{8,m}) = 2S_xJ\ (f(x,y))|_{x=1} = \frac{2mn}{mn+3} + \frac{2mn}{5}.$

8. Inverse sum index

$I(J_{8,m}) = S_xJD_xD_y\ (f(x,y))|_{x=1} = \frac{3(mn)^2}{mn+3} + \frac{12mn}{5}.$

9. Augmented Zagreb index

$A(J_{8,m}) = S_x^3\, Q_{-2}\, J\, D_x^3\, D_y^3\ (f(x,y))|_{x=1} = \frac{27(mn)^4}{(mn+1)^3} + 4mn.$

(d_u, d_v),where u,v ∈E(G)	Number of edges E(G)
(mn,3)	mn
(2,3)	2mn

Figure 3: m-level gear graph ($J_{8,m}$). **Table 3: Edge partition for gear graph ($J_{8,m}$).**

Theorem 3.4. The M-polynomial of m-level antiweb gear graph ($AWJ_{8,m}$) is

$M(AW_{8,m};x,y) = mn\, x^{mn}\, y^5 + mnx^5y^5+mnx^4\, y^4 +2\, mnx^4y^5.$

Proof. From figure 4 we notice that there four separate cases and the number of edges is different, namely $E_{\{mn,5\}}, E_{\{5,5\}}, E_{\{4,4\}}$and $E_{\{4,5\}}$,

$E_{\{mn,5\}} = \{uv \in E(J_{8,m})|d_u= mn,d_v=5\}, E_{\{5,5\}} = \{uv \in E(J_{8,m})|d_u=5,d_v=5\},$

$E_{\{4,4\}} = \{uv \in E(J_{8,m})|d_u=4,d_v=4\}$ and $E_{\{4,5\}} = \{uv \in E(J_{8,m})|d_u=4,d_v=5\}.$

The number of edges $E_{\{mn,5\}}, E_{\{5,5\}}, E_{\{4,4\}}, E_{\{4,5\}}$ are mn, mn, mn and 2mn respectively.

$M(AWJ_{8,m};x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(G)\, x^i y^j.$

$= \sum_{mn \le 5} m_{mn5}(AWJ_{8,m})\, x^{mn} y^5 + \sum_{5 \le 5} m_{55}(AWJ_{8,m})\, x^5 y^5 + \sum_{4 \le 4} m_{44}(AWJ_{8,m})\, x^4 y^4 + \sum_{4 \le 5} m_{45}(AWJ_{8,m})\, x^4 y^5.$

$= |E_{\{mn,5\}}|x^{mn}y^5 + |E_{\{5,5\}}|x^5y^5 + |E_{\{4,4\}}|x^4y^4 + |E_{\{4,5\}}|x^4y^5.$

$= mnx^{mn}\, y^5 + mnx^5y^5 + mnx^4\, y^4 + 2mnx^4y^5.$

Proposition. Let $AWJ_{8,m}$ be a graph of m-level antiweb gear graph, we then have following topological indices.

1. $M_1(AWJ_{8,m}) = (mn)^2+41mn.$

2. $M_2(AWJ_{8,m}) = 5(mn)^2+81mn.$

3. $^mM_2(AWJ_{8,m}) == 0.2+0.56mn.$

4.$R_\alpha(AWJ_{8,m}) = 5^\alpha (mn)^{\alpha+1} + 5^{2\alpha}$ mn + $4^{2\alpha}$mn + $4^\alpha 5^\alpha$ 2mn.

5.$RR_\alpha(AWJ_{8,m}) = \frac{mn}{5^\alpha (mn)^\alpha} + \frac{mn}{5^{2\alpha}} + \frac{mn}{4^{2\alpha}} + \frac{2mn}{5^\alpha 4^\alpha}$.

6. $SSD(AWJ_{8,m}) = \frac{1}{5}(mn)^2 + 8mn + 5$.

7. $H(AWJ_{8,m}) = \frac{2mn}{mn+5} + 0.89mn$.

8. $I(AWJ_{8,m}) = \frac{5(mn)^2}{mn+5} + \frac{25mn}{10} + 2mn + \frac{40mn}{9}$.

9. $A(AWJ_{8,m}) = 125\frac{(mn)^4}{(mn+3)^3} + 97mn$.

Proof. The following are made in order to have the above topological indices.

$M(AWJ_{8,m};x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(AWJ_{8,m})\, x^i y^j$.

$M(AWJ_{8,m};x,y) = mn\, x^{mn} y^5 + mnx^5y^5 + mnx^4 y^4 + 2mnx^4y^5$.

$D_x(f(x,y)) = (mn)^2x^{mn} y^5 + 5\, mnx^5y^5 + 4mnx^4 y^4 + 8mnx^4y^5$,

$D_y(f(x,y)) = 5mnx^{mn} y^5 + 5mnx^5y^5 + 4mnx^4 y^4 + 10mnx^4y^5$,

$D_xD_y(f(x,y)) = 5(mn)^2x^{mn} y^5 + 25mnx^5y^5 + 16mnx^4 y^4 + 40mnx^4y^5$,

$S_xD_y(f(x,y)) = 5x^{mn} y^5 + mn\, x^5y^5 + mn\, x^4 y^4 + \frac{10}{4}mn\, x^4y^5$,

$S_y(f(x,y)) = \frac{mn}{5}x^{mn} y^5 + \frac{mn}{5}x^5y^5 + \frac{mn}{4}x^4 y^4 + 2\frac{mn}{5}x^4y^5$,

$D_xS_y(f(x,y)) = \frac{(mn)^2}{5}x^{mn} y^5 + mnx^5y^5 + mnx^4 y^4 + 8\frac{mn}{5}x^4y^5$,

$S_xS_y(f(x,y)) = \frac{1}{5}x^{mn} y^5 + \frac{mn}{25}x^5y^5 + \frac{mn}{16}x^4y^4 + \frac{mn}{10}x^4y^5$,

$J(f(x,y)) = mnx^{mn+5} + mn\, x^{10} + mnx^8 + 2mn\, x^9$,

$S_xJ(f(x,y)) = \frac{mn}{mn+5}x^{mn+5} + \frac{mn}{10}x^{10} + \frac{mn}{8}x^8 + \frac{2mn}{9}x^9$,

$S_xJD_xD_y(f(x,y)) = 5\frac{(mn)^2}{mn+5}x^{mn+5} + 25\frac{mn}{10}x^{10} + 2mnx^8 + \frac{40mn}{9}x^9$,

$S_x^3 Q_{-2} J D_x^3 D_y^3 (f(x,y)) = 125\frac{(mn)^4}{(mn+5)^3}x^{mn+3} + 31mnx^8 + 19mnx^6 + 47mnx^7$,

$D_x^\alpha D_y^\alpha(f(x,y)) = 5^\alpha (mn)^{\alpha+1}x^{mn} y^5 + mn5^{2\alpha}x^5y^5 + 4^{2\alpha}\, mn\, x^4 y^4 + 2mn\, 5^\alpha 4^\alpha x^4 y^5$,

$S_x^\alpha S_x^\alpha(f(x,y)) = \frac{mn}{5^\alpha (mn)^\alpha}x^{mn} y^5 + \frac{mn}{5^{2\alpha}}mn\, x^5 y^5 + \frac{mn}{4^{2\alpha}}x^4 y^4 + 2\frac{mn}{5^\alpha 4^\alpha}x^4 y^5$.

The topological indices described in table 5 are now obtained by using the above mentioned values.

1. First Zagreb index

$M_1(AWJ_{8,m}) = (D_x+D_y)(f(x,y))|_{x=y=1} = (mn)^2 + 41mn$.

2. Second Zagreb index

$M_2(AWJ_{8,m}) = D_xD_y\,(f(x,y))|_{x=y=1} = 5(mn)^2+81mn.$

3. Modified second Zagreb index

${}^{m}M_2(AWJ_{8,m}) = S_xS_y\,(f(x,y))|_{x=y=1} = 0.2+0.56mn.$

4. Randic index

$R_\alpha(AWJ_{8,m}) = D_x^{\alpha}D_y^{\alpha}\,(f(x,y))|_{x=y=1} = 5^{\alpha}(mn)^{\alpha+1}+5^{2\alpha}\,mn+4^{2\alpha}\,mn+5^{\alpha}4^{\alpha}\;2mn.$

5. Inverse Randic index

$RR_\alpha(AWJ_{8,m}) = S_x^{\alpha}\,S_x^{\alpha}\,(f(x,y))|_{x=y=1} = \frac{mn}{5^{\alpha}(mn)^{\alpha}} + \frac{mn}{5^{2\alpha}} + \frac{mn}{4^{2\alpha}} + \frac{2mn}{5^{\alpha}4^{\alpha}}.$

6. Symmetric division index

$SSD(AWJ_{8,m}) = (S_yD_x + S_xD_y)\,(f(x,y))|_{x=y=1} = \frac{1}{5}(mn)^2+8mn+5.$

7. Harmonic index

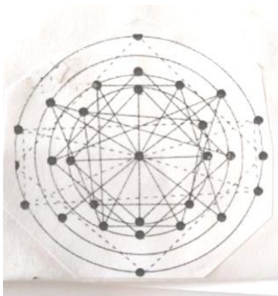
$H(AWJ_{8,m}) = 2S_xJ\,(f(x,y))|_{x=1} = \frac{2mn}{mn+5} + 0.89mn.$

8. Inverse sum index

$I(AWJ_{8,m}) = S_xJD_xD_y\,(f(x,y))|_{x=1} = \frac{5(mn)^2}{mn+5} + \frac{25mn}{10} + 2mn + \frac{40mn}{9}.$

9. Augmented Zagreb index

$A(WJ_{8,m}) = S_x^3\,Q_{-2}\,J\,D_x^3\,D_y^3\,(f(x,y))|_{x=1} = 125\,\frac{(mn)^4}{(mn+3)^3} + 97mn.$



(d_u, d_v),where u,v ∈E(G)	Number of edges E(G)
(mn,5)	mn
(5,5)	mn
(4,4)	mn
(4,5)	2mn

Figure 4: m-level antiweb gear graph ($AWJ_{8,m}$).

Table 4: Edge partition for antiweb gear graph ($AWJ_{8,m}$).

Topological index	Derivation from M(G;x,y)
First Zagreb index	$(D_x+D_y)(M(G;x,y))|_{x=y=1}$
Second Zagreb index	$(D_xD_y)(M(G;x,y))|_{x=y=1}$
Modified Second Zagreb index	$(S_xS_y)(M(G;x,y))|_{x=y=1}$

Randic index	$(D_xD_y)(M(G;x,y))\|_{x=y=1}$
Inverse Randic index	$(S_xS_y)(M(G;x,y))\|_{x=y=1}$
Symmetric division index	$(D_xS_y+ D_yS_x)(M(G;x,y))\|_{x=y=1}$
Harmonic index	$(2S_xJ)(M(G;x,y))\|_{x=1}$
Inverse sum index	$(S_xJ\ D_xD_y)(M(G;x,y))\|_{x=1}$
Augmented Zagreb index	$(Sx^3Q_{-2}J\ Dx^3\ D_y^3)(M(G;x,y))\|_{x=1}$

Table 5: Derivation of topological indices from M-polynomial.

IV. CONCLUSIONS

In this paper first Zagreb index, second Zagreb index, modified second Zagreb index, Randic index, inverse Randic index, symmetric division index, harmonic index, inverse sum index and augmented Zagreb index are studied from M-polynomials for wheel, antiweb wheel graph, gear graph and antiweb gear graph.

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