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Secure Super Domination in Graphs

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Abstract *- Let $G = (V(G), E(G))$ be a connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $u \in V(G)\backslash S$, there exists $v \in S$ such that $uv \in E(G)$. A dominating set S is called a super dominating set if for very vertex $u \in V(G)\backslash S$, there exists $v \in S$ such that $N_G(v) \cap (V(G)\backslash S) = \{u\}$. A super dominating set S is called a secure super dominating set if for every vertex $u \in V(G)\backslash S$, there exists $v \in S$ such that $(S\backslash\{v\}) \cup \{u\}$ is a super dominating set of G. In this paper, we investigate the concept and give some important results.*

Keywords: *dominating set, super dominating set, secure dominating set, secure super dominating set*

I. INTRODUCTION

Suppose that $G = (V(G), E(G))$ is a simple graph with vertex set $V(G)$ and edge set $E(G)$. In simple graph, we mean, finite and undirected graph with neither loops nor multiple edges. For the general graph theoretic terminology, the readers may refer to [1].

A vertex v is said to dominate a vertex u if uv is an edge of G or $v = u$. A set of vertices $S \subseteq V(G)$ is called a dominating set of G if every vertex not in S is dominated by at least one member of S. The size of a set of least cardinality among all dominating sets for G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is called $\gamma - set$ of G. Domination in a graph has been a huge area of research in graph theory. It was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. Domination in graphs has been studied in [3, 4, 5, 6, 7, 8, 9, 10].

A dominating set S is called super dominating set if for every vertex $u \in V(G)\backslash S$, there exists $v \in S$ such that $N_G(v) \cap (V(G)\backslash S) = \{u\}$. The super domination number of G, is the minimum cardinality of a super dominating set of G and is dentoed by $\gamma_{sup}(G)$ [11]. A super dominating set cardinality $\gamma_{sup}(G)$ is called a $\gamma_{sup} - set$ of G. Super domination has been studied in [12, 13, 14, 15, 16, 17, 18, 19].

A dominating set S of $V(G)$ is a secure dominating set of G if for each $u \in V(G)\backslash S$, there exists $v \in S$ such that $uv \in E(G)$ and the set $(S\backslash\{v\}) \cup \{u\}$ is a dominating set of G. The secure domination number of G, is the minimum cardinality of a secure dominating set of G and is denoted by $\gamma_s(G)$. A secure dominating set of cardinality $\gamma_s(G)$ is called $\gamma_s - set$ of G. The secure domination has been studied in [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33].

Motivated by the idea of secure [20] and super [11] domination in graphs, we initiate the study of a secure super dominating set. A super dominating set S is called a secure super dominating set if for every vertex $u \in V(G)\backslash S$, there exists $v \in S$ such that $(S\backslash\{v\}) \cup \{u\}$ is a super dominating set of G. The secure super domination number of G, is the minimum cardinality of a secure super dominating set of G and is denoted by $\gamma_{ssup}(G)$. A secure super dominating set of cardinality $\gamma_{ssup}(G)$ is called $\gamma_{ssup} - set$ of G. In this paper, we investigate the concept and give some important results. We further give the characterization of a secure super dominating set in the join and corona of two graphs.

II. RESULTS

Definition 2.1 [12] *A set $S \in V(G)$ is called a secure dominating set of a graph G if for every vertex $u \in V(G)\backslash S$ there exists $v \in S \cap N_G(u)$ such that $(S\backslash\{v\}) \cup \{u\}$ is dominating. It is super secure dominating set if $N_G(v) \cap (V(G)\backslash S) = \{u\}$.*

Remark 2.2 *A secure super dominating set need not be a super secure dominating set of a graph G.*

Example 2.3 *In Figure 1, the subset $S = \{v_1, v_3, v_4\}$ is a secure dominating set and a super dominating set. Hence, S is a* *super secure dominating set. But S is not a secure super dominating set of G since $(S\backslash S_4) \cup \{v_5\} = \{v_1, v_3, v_5\}$ is not a super dominating set of G.*

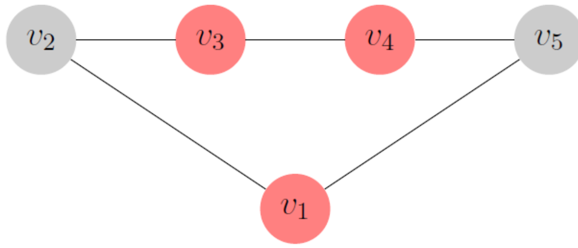

Figure 1: $S = \{v_1, v_3, v_4\}$ is a super secure dominating set.

Remark 2.4 *The set $S = V(G)$ is a super dominating set and a secure dominating set.*

Proof: If $S = V(G)$, then every vertex in $V(G)\backslash S = \emptyset$ vacuously satisfies the definitions of a super dominating set and a secure dominating set. ■

Remark 2.5 *Every graph G has a super dominating set and a secure dominating set.*

Proof: By Remark 2.4, $S = V(G)$ is a super dominating set and a secure dominating set. ■

From the definitions of secure super dominating set and Remark 2.5, the following is immediate.

Remark 2.6 *Let G be a nontrivial graph. Then $1 \leq \gamma(G) \leq \gamma_{ssup}(G) \leq n$.*

For a nontrivial connected graph G, the following result says that $\gamma_{ssup}(G)$ ranges over all integers from 1 to $n-1$.

Theorem 2.7 *Given positive integers $k, m,$ and n such that $1 \leq k \leq m \leq n-1$, where $n \geq 2$, there exists a connected graph G with $|V(G)| = n$, $\gamma(G) = k$, and $\gamma_{ssup}(G) = m$.*

Proof: Consider the following cases.
Case 1. Suppose that $1 = k \leq m = n-1$.

Consider $G = K_n = [v_1, v_2, \ldots, v_n]$ with $D = \{v_1\}$ a γ-*set* of G, and $S = V(G)\backslash D$ a γ_{ssup}-*set* of G (see Figure 2).

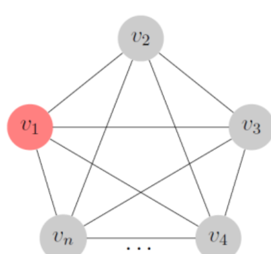

Figure 2: A graph G with $1 = k \leq m = n-1$.

Thus, $|V(G)| = |V(K_n)| = n, \gamma(G) = |D| = 1 = k$, and $\gamma_{ssup}(G) = |V(G)\backslash D| = |V(G)| - |D| = n - k = n - 1 = m$.
Case 2. Suppose that $1 \leq k \leq m \leq n-1$.

Then $G = P_k \circ P_p$ with $n = (k+1)p$ for all positive integers $k \geq 1$ and $p \geq 2$. (see Figure 3).

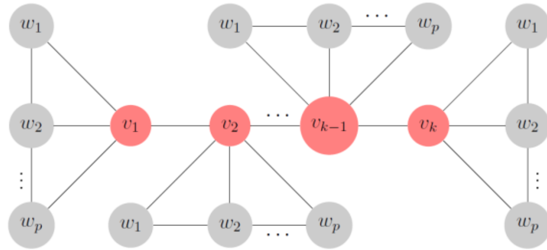

Figure 3: A graph G with $1 \leq k \leq m \leq n-1$.

Let $m = kp$. The set $D = \{v_1, v_2, \ldots, v_k\}$ is a γ-*set* of G and $S = V(G) \backslash D$ is a γ_{ssup}-*set* of G. Thus, $|V(G)| = |V(P_k \circ P_p)| = |V(P_k)||V(P_p)| + |V(P_k)| = kp + k = (p+1)k = n$, $\gamma(G) = |D| = k$, and $\gamma_{ssup}(G) = |S| = |V(G) \backslash D| = n - k = (p+1)k - k = pk = m$. ∎

Corollary 2.8 *The difference between $\gamma_{ssup}(G) - \gamma(G)$ can be arbitrarily large.*

Proof: By Theorem 2.7, there exists a connected graph G such that $\gamma(G) = 1$ and $\gamma_{ssup}(G) = n + 1$. Then $\gamma_{ssup}(G) - \gamma(G) = (n+1) - 1 = n$. Hence, the difference between $\gamma_{ssup}(G) - \gamma(G)$ can be made arbitrarily large. ∎

Let $P_n = [v_1, v_2, \ldots, v_n]$ such that $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(P_n) = \{v_1v_2, v_2\,v_3, \ldots, v_{n-1}v_n\}$. The next result shows the super inverse domination number of a path graph P_n.

Theorem 2.9 *Let $G = P_n$ of order $n \geq 2$. Then,*

$$\gamma_{ssup}(G) \begin{cases} \frac{2n}{3}, & if\ n = 3\ or\ n = 6 \\ \frac{2n-3}{3}, & if\ n \equiv 0(mod\ 3), n \neq 3, n \neq 6 \\ 3, & if\ n = 4 \\ \frac{2(n-1)}{3}, & if\ n \equiv 1(mod\ 3), n \neq 4 \\ \frac{2n-1}{3}, & if\ n \equiv 2(mod\ 3) \end{cases}$$

Proof: Let $G = P_n$ of order $n \geq 2$.

Case 1. $n \equiv 0(mod\ 3)$. Consider the graph G below (see Figure 4).

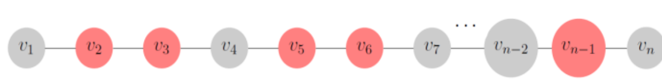

Figure 4: A graph G with $\gamma_{ssup}G = \frac{2n-3}{3}$, $n \neq 3$ and $n \neq 6$.

If $n \neq 3$ and $n \neq 6$, then the set $S = \left\{V_{3i} : i = 1, 2, \ldots, \frac{n-3}{3}\right\} \cup \left\{V_{3i-1} : i = 1, 2, \ldots, \frac{n}{3}\right\}$ is a $\gamma_{ssup} - set$ of G. Thus, $\gamma_{ssup}(G) = |S| = \frac{n-3}{3} + \frac{n}{3} = \frac{2n-3}{3}$. If $n = 3$ or $n = 6$, then $\gamma_{ssup}(G) = 2$ or $\gamma_{ssup}(G) = 4$ is clear (see Figure 4). Thus, $\gamma_{ssup}(G) = \frac{2n}{3}$.

Case 2. $n \equiv 1(mod\ 3)$. Consider the graph G below (see Figure 5).

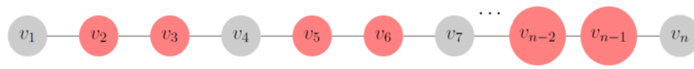

Figure 5: A graph G with $\gamma_{ssup}(G) = \frac{2(n-1)}{3}, n \neq 4$.

If $n = 4$, then the set $S = \left\{V_{3i}: i = 1, 2, \ldots, \frac{n-1}{3}\right\} \cup \left\{V_{3i-1}: i = 1, 2, \ldots, \frac{n-1}{3}\right\}$ is a $\gamma_{ssup} - set$ of G. Thus, $\gamma_{ssup}(G) = |S| = \frac{n-1}{3} + \frac{n-1}{3} = \frac{2(n-1)}{3}$. If $n = 4$, then $\gamma_{ssup}(G) = 3$ is clear.

Case 3. $n \equiv 2(mod\ 3)$. Consider the graph G below (see Figure 6).

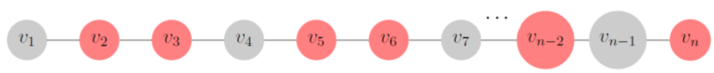

Figure 6: A graph G with $\gamma_{ssup}(G) = \frac{2n-1}{3}$.

The set $S = \left\{V_{3i}: i = 1, 2, \ldots, \frac{n-2}{3}\right\} \cup \left\{V_{3i-1}: i = 1, 2, \ldots, \frac{n+1}{3}\right\}$ is a $\gamma_{ssup} - set$ of G. Thus, $\gamma_{ssup}(G) = |S| = \frac{n-2}{3} + \frac{n+1}{3} = \frac{2n-1}{3}$. If $n = 4$ then $\gamma_{ssup}(G) = 3$ is clear. ■

Let $C_n = [v_1, v_2, \ldots, v_n]$ such that $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(C_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1\}$. The next result shows the super inverse domination number of a cycle graph C_n.

Theorem 2.10 *Let $G = C_n$ of order $n \geq 3$. Then,*

$$\begin{cases} \frac{2n}{3}, & if\ n \equiv 0(mod\ 3) \\ \frac{2n+1}{3}, & if\ n \equiv 1(mod\ 3) \\ \frac{2n-1}{3}, & if\ n \equiv 2(mod\ 3), n \neq 8 \\ 6, & if\ n = 8 \end{cases}$$

Proof: Let $G = C_n$ of order $n \geq 3$.

Case 1. $n \equiv 0(mod\ 3)$. Consider the graph G below (see Figure 7).

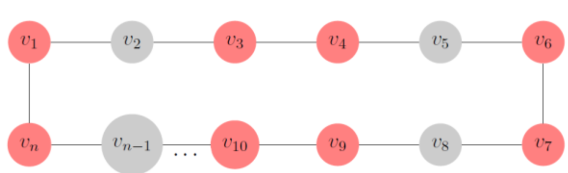

Figure 7: A graph G with $\gamma_{ssup}(G) = \frac{2n}{3}$.

The set $S = \left\{V_{3i}: i = 1, 2, \ldots, \frac{n}{3}\right\} \cup \left\{V_{3i-2}: i = 1, 2, \ldots, \frac{n}{3}\right\}$ is a $\gamma_{ssup} - set$ of G. Thus,

$$\begin{aligned}\gamma_{ssup}(G) &= |S| \\ &= \left|\left\{V_{3i}: i = 1, 2, \dots, \frac{n}{3}\right\} \cup \left\{V_{3i-2}: i = 1, 2, \dots, \frac{n}{3}\right\}\right| \\ &= \left|V_{3i}: i = 1, 2, \dots, \frac{n}{3}\right| + \left|V_{3i-2}: i = 1, 2, \dots, \frac{n}{3}\right| \\ &= \frac{n}{3} + \frac{n}{3} = \frac{2n}{3}\end{aligned}$$

Case 2. $n \equiv 1(mod\ 3)$. Consider the graph G below (see Figure 8).

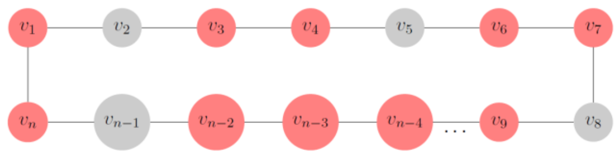

Figure 8: A graph G with $\gamma_{ssup}(G) = \frac{2n+1}{3}$.

The set $S = \left\{V_{3i}: i = 1, 2, \dots, \frac{n-4}{3}\right\} \cup \left\{V_{3i-2}: i = 1, 2, \dots, \frac{n+2}{3}\right\} \cup \{u_{n-2}\}$ is a $\gamma_{ssup} - set$ of G. Thus,

$$\begin{aligned}\gamma_{ssup}(G) &= |S| \\ &= \left|\left\{V_{3i}: i = 1, 2, \dots, \frac{n-4}{3}\right\} \cup \left\{V_{3i-2}: i = 1, 2, \dots, \frac{n+2}{3}\right\} \cup \{u_{n-2}\}\right| \\ &= \frac{n-4}{3} + \frac{n+2}{3} + 1 = \frac{2n+1}{3}\end{aligned}$$

Case 3. $n \equiv 2(mod\ 3)$. Consider the graph G below (see Figure 9).

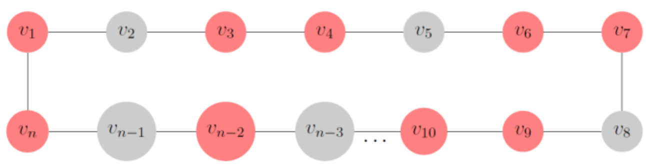

Figure 9: A graph G with $\gamma_{ssup}(G) = \frac{2n-1}{3}, n \neq 8$.

If $n \neq 8$, then the set $S = \left\{V_{3i}: i = 1, 2, \dots, \frac{n-2}{3}\right\} \cup \left\{V_{3i-2}: i = 1, 2, \dots, \frac{n-2}{3}\right\} \cup \{v_n\}$ is a $\gamma_{ssup} - set$ of G. Thus,

$$\begin{aligned}\gamma_{ssup}(G) &= |S| \\ &= \left|\left\{V_{3i}: i = 1, 2, \dots, \frac{n-2}{3}\right\} \cup \left\{V_{3i-2}: i = 1, 2, \dots, \frac{n-2}{3}\right\} \cup \{u_{n-2}\}\right| \\ &= \frac{n-2}{3} + \frac{n-2}{3} + 1 = \frac{2n-1}{3}\end{aligned}$$

If $n = 8$, then the set $S = \{v_1, v_3, v_4, v_5, v_6, v_8\}$ is a $\gamma_{ssup} - set$ of G. Hence, $\gamma_{ssup}(G) = |S| = |\{v_1, v_3, v_4, v_5, v_6, v_8\}| = 6$. ■

A complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices. The next following result shows the domination number of $K_1 + H$ where H is a nontrivial graph.

Theorem 2.11 *Let $G = K_1 + H$ be a connected graph of order $n \geq 2$. Then, $\gamma_{ssup}(G) = n - 1$.*

Proof: Let $G = K_1 + H$ be a connected graph of order $n \geq 2$. Clearly, $S = V(G)\backslash V(K_1) = V(H)$ is a secure super dominating set of G. Hence, $\gamma_{ssup}(G) \leq |S|$. Let $V(K_1) = \{v\}$ and $x \in S$. Then $x \in V(H)$. Since H is nontrivial, $V(H)\backslash\{x\} \neq \emptyset$. Let $y \in V(H)\backslash\{x\} = S_1$.

Case 1. If $xy \in E(G)$, then there exists $x \in V(G)\backslash S_1$ and $y \in S_1$ such that $N_G(y) \cap (V(G)\backslash S_1) = \{v, x\}$. Hence, S_1 is not a super dominating set of G.

Case 2. If $xy \notin E(G)$, then there exists $x \in V(G)\backslash S_1$ that is not dominated by any elements of S_1. Hence, $S_1$is not a dominating set of G.

In any case, S_1 is not a secure super dominating set of G. Similarly, any proper subset of S' of $V(H)$ is not a secure super dominating set of $V(G)$. Therefore, $S = V(H)$ must be a $\gamma_{ssup} - set$ of G, that is,

$$\begin{aligned}\gamma_{ssup}(G) &= |S| \\ &= |V(G)\backslash V(K_1)| \\ &= |V(G)| - |V(K_1)| = n - 1 \blacksquare\end{aligned}$$

Corollary 2.12 *If a graph G is a wheel $W_n = K_1 + C_{n-1}$, or a star $S_n = K_1 + \overline{K}_{n-1}$, or a fan $F_n = K_1 + P_{n-1}$, or a complete graph K_n, then $\gamma_{ssup}(G) = n - 1$.*

Proof: Clearly, $K_n = K_1 + K_{n-1}$. Thus, if G is W_n, S_n, F_n, or K_n, then $\gamma_{ssup}(G) = n - 1$ by Theorem 2.11. ■

A complete bipartite graph $K_{m,n}$ is a graph that has its vertex set partitioned into two subsets of m and n vertices, respectively with an edge between pair of vertices if and only if one vertex in the pair is in the first subset and the other vertex is in the second subset.

Remark 2.13 *If a graph G is a complete bipartite $K_{m,n}$ with $m \geq 2$ and $n \geq 2$, then $\gamma_{ssup}(G) = n - 1$.*

III. CONCLUSIONS

In this paper, we introduced the concept of secure super domination in graphs and prove that given positive integers k, m and n such that $1 \leq k \leq m \leq n - 1$, where $n \geq 2$, there exists a connected graph G with $|V(G)| = n$, $\gamma(G) = k$, and $\gamma_{\text{sup}}(G)^{-1} = m$. Further, we prove the domination number of a path graph P_n, a cycle C_n, a wheel graph W_n, a fan graph F_n, a star graph S_n, a complete graph K_n, and a complete bipartite $K_{m,n}$. Some related problems on secure super domination in graphs are still open for research.

1. Characterize the secure super dominating sets of the join, corona, Cartesian product, and lexicographic product of two graphs.
2. Find the secure super dominating sets of the join, corona, Cartesian product, and lexicographic product of two graphs.

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