International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V67I8P505
Volume 67 Issue 8, 45-49, August, 2021

Effect of Discrete Time Delays On The Stability of A Dynamical System

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Abstract
In this study, the effect of discrete time delays on the stability of a dynamical system was considered. On the implementation of the computational techniques called ODE15s, it is shown that the dynamical system is dominantly unstable. It is also observed from the results that as the discrete time delays is increased then the yeast species 2 (Candida Parapsilosis) dominates yeast species 1 (Candida Albican) which implies thatyeast species 2will drive yeast species 1into extinction.

Keyword: *Time delays, stability, dynamical system and ODE15s.*

I. INTRODUCTION

Often times when real life situation are modeled and analyzed, it usually leads to ordinary differential equation or partial differential equation [1]. Dynamical system is a system that evolves in time through the iterated application of an underlying dynamical rule. It is a mathematical model that one usually constructs in order to investigate some physical phenomenon that evolves in time [2].[3], defined stability as the return to steady-state as determined by eigen values of the Jacobian matrix of a mathematical system. This definition is in total conformity with (4) whose emphasized that the type o stability for specific steady-state solutions should be tested for continuity and partial differentiability of the interacting functions that are imposed on the dynamical system. The stability of a dynamical system with continuous discrete time delays using Lambert W. functions and obtained a stable system was study by [5].[6], then investigated the stability analysis of a dynamical system using iterative algorithm method and obtained a stability in a delayed equation. Though [7] used numerical simulation technique to investigate the steady-state solution and its type of stability of the intrinsic growth rates of two interacting plant species. The result shows that irrespective of the variation of the intrinsic growth rates, the positive co-existence steady-state solution is dominantly stable. But [8] who studied the survival of two competing species in a polluted environment with the help of local stability analysis. The result revealed that the competitive outcomes may be affected in the presence of a toxicant. See also [9].[10] have extended the work of [11], by using a differential equation model to investigate whether the concept of constructing a feedback control with which to stabilize an unstable steady-state is applicable to stabilize a market population system. [10] then used feedback control to construct a controlled in which two unstable steady-states of two interacting stock market populations were stabilized. The relationship that existed between intraspecific and interspecific competition was investigated by [12]. But in this paper, we consider the effect of discrete time delays on the stability of a dynamical system.

II. MATHEMATICAL FORMULATION

We have considered multi-parameter continuous dynamical system of a nonlinear first order ODE:

$$\frac{dx}{dt} = \propto_1 x - \beta_1 x^2 - r_1 xy(t-h)$$
$$\frac{dy}{dt} = \propto_2 y - \beta_2 y^2 - r_2 xy(t-h)$$

Where all parameters are assumed to be positive constant which can be any real constant.

- $x(t)$ specifies the biomass of yeast species 1 (Candida Albican) at time t in the unit of weeks.
- $y(t)$ specifies the biomass of yeast species 2 (Candida Parapsilosis) at time t in the unit of weeks.
- $\propto_1$ and $\propto_2$ specifies the growth rate of yeast species 1 and yeast species 2 respectively.
- β_1 and β_2 specifies the intra-competition coefficient of yeast species 1 and yeast species 2 respectively.
- r_1 and r_2 specifies the inter-competition coefficient of yeast species 1 and yeast species 2 respectively in which r_1 is the contribution of the yeast species 1 to inhibit the growth of the yeast species 2 as r_2 is the contribution of the yeast species 2 to inhibit the growth of the yeast species 1.
- h is the discrete time delays with the following precise model parameters $\propto_1 = 0.1, \propto_2 = 0.08, \beta_1 = 0.0014$, $\beta_2 = 0.001, r_1 = 0.0012, r_2 = 0.0009$.

III. METHOD OF ANALYSIS

We have fully employ the ordinary differential equation of order 15s (ODE15s) as a computational technique to model and predict the effect of discrete time delays on the stability of the proposed dynamical system.

IV. RESULTS

On the application of the above mentioned computational techniques, we have obtained the following useful results which are presented and displayed as shown in table 1 – table 8.

Table 1: Quantifying the effect of time delays from h = 0 to h = 0.14 on the type of stability using ODE15s numerical method.

Example	Time delays h	x_e	y_e	λ_1	λ_2	TOS
1	0.00	0.5962	21.2827	0.0732	0.0365	Unstable
2	0.01	0.5980	21.2810	0.0732	0.0365	Unstable
3	0.02	0.5998	21.2793	0.0732	0.0365	Unstable
4	0.03	0.6016	21.2776	0.0732	0.0365	Unstable
5	0.04	0.6034	21.2759	0.0732	0.0365	Unstable
6	0.05	0.6052	21.2742	0.0732	0.0365	Unstable
7	0.06	0.607	21.2725	0.0732	0.0365	Unstable
8	0.07	0.6088	21.2708	0.0732	0.0365	Unstable
9	0.08	0.6106	21.2691	0.0732	0.0365	Unstable
10	0.09	0.6124	21.2674	0.0732	0.0365	Unstable
11	0.1	0.6142	21.2657	0.0732	0.0365	Unstable
12	0.11	0.616	21.264	0.0732	0.0365	Unstable
13	0.12	0.6178	21.2623	0.0732	0.0365	Unstable
14	0.13	0.6196	21.2606	0.0732	0.0365	Unstable
15	0.14	0.6214	21.2589	0.0732	0.0365	Unstable

Table 2: Quantifying the effect of time delays from h = 0.15 to h = 0.29 on the type of stability using ODE15s numerical method.

Example	Time delays h	x_e	y_e	λ_1	λ_2	TOS
1	0.15	0.0457	27.4652	0.0669	0.025	Unstable
2	0.16	0.0459	27.4614	0.067	0.025	Unstable
3	0.17	0.0461	27.4576	0.0671	0.025	Unstable
4	0.18	0.0463	27.4538	0.0672	0.025	Unstable
5	0.19	0.0465	27.45	0.0673	0.025	Unstable
6	0.20	0.0467	27.4462	0.0674	0.025	Unstable
7	0.21	0.0469	27.4424	0.0675	0.025	Unstable
8	0.22	0.0471	27.4386	0.0676	0.025	Unstable
9	0.23	0.0473	27.4348	0.0677	0.025	Unstable
10	0.24	0.0475	27.431	0.0678	0.025	Unstable
11	0.25	0.0477	27.4272	0.0679	0.025	Unstable
12	0.26	0.0479	27.4234	0.068	0.025	Unstable
13	0.27	0.0481	27.4196	0.0681	0.025	Unstable
14	0.28	0.0483	27.4158	0.0682	0.025	Unstable
15	0.29	0.0485	27.412	0.0683	0.025	Unstable

Table 3: Quantifying the effect of time delays from h = 0.30 to h = 0.44 on the type of stability using ODE15s numerical method.

Example	Time delays h	x_e	y_e	λ_1	λ_2	TOS
1	0.30	0.0006	34.9485	0.0581	0.0101	Unstable
2	0.31	0.0006	34.9441	0.0581	0.0101	Unstable
3	0.32	0.0006	34.9397	0.0581	0.0101	Unstable
4	0.33	0.0006	34.9353	0.0581	0.0101	Unstable
5	0.34	0.0006	34.9309	0.0581	0.0101	Unstable
6	0.35	0.0006	34.9265	0.0581	0.0101	Unstable
7	0.36	0.0006	34.9221	0.0581	0.0101	Unstable
8	0.37	0.0006	34.9177	0.0581	0.0101	Unstable
9	0.38	0.0006	34.9133	0.0581	0.0101	Unstable
10	0.39	0.0006	34.9089	0.0581	0.0101	Unstable
11	0.40	0.0006	34.9045	0.0581	0.0101	Unstable
12	0.41	0.0006	34.9001	0.0581	0.0101	Unstable
13	0.42	0.0006	34.8957	0.0581	0.0101	Unstable
14	0.43	0.0006	34.8913	0.0581	0.0101	Unstable
15	0.44	0.0006	34.8869	0.0581	0.0101	Unstable

Table 4: Quantifying the effect of time delays from h = 0.45 to h = 0.59 on the type of stability using ODE15s numerical method.

Example	Time delays h	x_e	y_e	λ_1	λ_2	TOS
1	0.45	0	42.8582	0.0486	-0.0057	Unstable
2	0.46	0	42.8535	0.0486	-0.0057	Unstable
3	0.47	0	42.8488	0.0486	-0.0057	Unstable
4	0.48	0	42.8441	0.0486	-0.0057	Unstable
5	0.49	0	42.8394	0.0486	-0.0057	Unstable
6	0.50	0	42.8347	0.0486	-0.0057	Unstable
7	0.51	0	42.8300	0.0486	-0.0057	Unstable
8	0.52	0	42.8253	0.0486	-0.0057	Unstable
9	0.53	0	42.8206	0.0486	-0.0057	Unstable
10	0.54	0	42.8159	0.0486	-0.0057	Unstable
11	0.55	0	42.8112	0.0486	-0.0057	Unstable
12	0.56	0	42.8065	0.0486	-0.0057	Unstable
13	0.57	0	42.8018	0.0486	-0.0057	Unstable
14	0.58	0	42.7971	0.0486	-0.0057	Unstable
15	0.59	0	42.7924	0.0486	-0.0057	Unstable

Table5: Quantifying the effect of time delays from h = 0.60 to h = 0.74 on the type of stability using ODE15s numerical method.

Example	Time delays h	x_e	y_e	λ_1	λ_2	TOS
1	0.60	0	50.5388	0.0394	-0.0211	Stable
2	0.61	0	50.5344	0.0394	-0.0211	Stable
3	0.62	0	50.53	0.0394	-0.0211	Stable
4	0.63	0	50.5256	0.0394	-0.0211	Stable
5	0.64	0	50.5212	0.0394	-0.0211	Stable
6	0.65	0	50.5168	0.0394	-0.0211	Stable
7	0.66	0	50.5124	0.0394	-0.0211	Stable
8	0.67	0	50.508	0.0394	-0.0211	Stable
9	0.68	0	50.5036	0.0394	-0.0211	Stable
10	0.69	0	50.4992	0.0394	-0.0211	Stable
11	0.7	0	50.4948	0.0394	-0.0211	Stable
12	0.71	0	50.4904	0.0394	-0.0211	Stable
13	0.72	0	50.486	0.0394	-0.0211	Stable
14	0.73	0	50.4816	0.0394	-0.0211	Stable
15	0.74	0	50.4772	0.0394	-0.0211	Stable

Table6: Quantifying the effect of time delays from h = 0.75 to h = 0.89 on the type of stability using ODE15s numerical method.

Example	Time delays h	x_e	y_e	λ_1	λ_2	TOS
1	0.75	0	57.4679	0.031	-0.0349	Unstable
2	0.76	0	57.4638	0.031	-0.0349	Unstable
3	0.77	0	57.4597	0.031	-0.0349	Unstable
4	0.78	0	57.4556	0.031	-0.0349	Unstable
5	0.79	0	57.4515	0.031	-0.0349	Unstable
6	0.8	0	57.4474	0.031	-0.0349	Unstable
7	0.81	0	57.4433	0.031	-0.0349	Unstable
8	0.82	0	57.4392	0.031	-0.0349	Unstable
9	0.83	0	57.4351	0.031	-0.0349	Unstable
10	0.84	0	57.431	0.031	-0.0349	Unstable
11	0.85	0	57.4269	0.031	-0.0349	Unstable
12	0.86	0	57.4228	0.031	-0.0349	Unstable
13	0.87	0	57.4187	0.031	-0.0349	Unstable
14	0.88	0	57.4146	0.031	-0.0349	Unstable
15	0.89	0	57.4105	0.031	-0.0349	Unstable

Table7: Quantifying the effect of time delays from h = 0.90 to h = 1.04 on the type of stability using ODE15s numerical method.

Example	Time delays h	x_e	y_e	λ_1	λ_2	TOS
1	0.90	0	63.3008	0.024	-0.0466	Unstable
2	0.91	0	63.2961	0.024	-0.0466	Unstable
3	0.92	0	63.2914	0.024	-0.0466	Unstable
4	0.93	0	63.2867	0.024	-0.0466	Unstable
5	0.94	0	63.282	0.024	-0.0466	Unstable
6	0.95	0	63.2773	0.024	-0.0466	Unstable
7	0.96	0	63.2726	0.024	-0.0466	Unstable
8	0.97	0	63.2679	0.024	-0.0466	Unstable
9	0.98	0	63.2632	0.024	-0.0466	Unstable
10	0.99	0	63.2585	0.024	-0.0466	Unstable
11	1	0	63.2538	0.024	-0.0466	Unstable
12	1.01	0	63.2491	0.024	-0.0466	Unstable
13	1.02	0	63.2444	0.024	-0.0466	Unstable
14	1.03	0	63.2397	0.024	-0.0466	Unstable
15	1.04	0	63.235	0.024	-0.0466	Unstable

Table 8: Quantifying the effect of time delays from h = 1.05 to h = 1.19 on the type of stability using ODE15s numerical method.

Example	Time delays h	x_e	y_e	λ_1	λ_2	TOS
1	1.05	0	67.9419	0.0185	-0.0559	Stable
2	1.06	0	67.9356	0.0185	-0.0559	Stable
3	1.07	0	67.9293	0.0185	-0.0559	Stable
4	1.08	0	67.923	0.0185	-0.0559	Stable
5	1.09	0	67.9167	0.0185	-0.0559	Stable
6	1.1	0	67.9104	0.0185	-0.0559	Stable
7	1.11	0	67.9041	0.0185	-0.0559	Stable
8	1.12	0	67.8978	0.0185	-0.0559	Stable
9	1.13	0	67.8915	0.0185	-0.0559	Stable
10	1.14	0	67.8852	0.0185	-0.0559	Stable
11	1.15	0	67.8789	0.0185	-0.0559	Stable
12	1.16	0	67.8726	0.0185	-0.0559	Stable
13	1.17	0	67.8663	0.0185	-0.0559	Stable
14	1.18	0	67.86	0.0185	-0.0559	Stable
15	1.19	0	67.8537	0.0185	-0.0559	Stable

V. DISCUSSION OF RESULTS

The results show clearly that when there is no discrete time delays, we found that the proposed dynamical system has one single positive unique steady-state solution which is unstable having two positive real eigen values. The positive eigen values contributes to the unbounded growth of the solution trajectory. It is clearly seen that the biomass of yeast species 2 is a better competitor compared to the biomass of yeast species 1. Though the biomass of yeast species 1 is increasing monotonically and the biomass of yeast species 2 is decreasing monotonically. It is observed that the dynamical system maintain instability despite the increase of the discrete time delays up to 1.19 with a common difference of 0.01.

VI. CONCLUSION

We applied computational numerical techniques of ordinary differential equation of order 15s (ODE15s) to ascertain the effect of discrete time delays on the stability of a dynamical system and observed that in the presence of a relatively small continuous discrete time delays that the dynamical system respond unstable dominantly.

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