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On Some Results And Application of The Inverse Exponential Transform

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Abstract

In this paper I have introduced the concept of the inverse exponential transform, the inverse exponential transform of some basic functions and some basic properties like linearity, shifting, convolution theorem and some important results with application to initial value problem are discussed.

I. Introduction

Integral transforms have been successfully used for almost two centuries in solving many problems in applied mathematics, mathematical physics, and engineering science.

The origin of the integral transforms including the Laplace and Fourier transforms can be traced back to back to celebrated work of P. S. Laplace (1729-1827) on probability theory in the 1780s and to monumental Treatise of Joseph Fourier (1768-1830) on 'La Theorie Analytique de la Chaleur' published in 1822, Laplace classic book on 'La Theorie Analytique des Probabilities' includes some basic results of the Laplace transforms which is one of the oldest and most commonly used integral transform available in the mathematical literature. This has effectively been used in finding the solution of linear differential equations and integral equations

Several authors [2-15] discussed the applications of different integral transformations along with its properties. Recently N. S. Ambarkhane, H. A. Dhirbasi, K.L. Bondar [1] introduced an integral transform 'Exponential Transform' and proved its existence, some properties like linearity, shifting, second shifting, change of scale. Moreover, exponential transform of some basic functions are derived. Many finite transforms like 'finite Laplace Transform', finite Fourier's sine and cosine transforms, finite Hankel transforms have many more applications in applied mathematics, physics and in different branches of engineering also. Main aim of this paper is to prove the further results of the exponential transform for the development and application point of view.

II. Preliminaries

EXPONENTIAL TRANSFORM [1]

Definition 2.1: Let $f(t)$ be function defined for all positive values of t, then

$$\overline{f}(s) = \int_{o}^{\infty} a^{-st} f(t)dt, \quad a > 1,$$

provided the integral exists is called exponential transform of $f(t)$. It is denoted as

$$A[f(t)] = \overline{f}(s) = \int_{o}^{\infty} a^{-st} f(t)dt, \quad a > 1,$$

here A is called exponential transformation operator, the parameter *s* is real or complex number.

In general, the parameter *s* is taken to be a real positive number.

Theorem 2.2 : *[Existence of Exponential Transform]*[1] *If $f(t)$ is a function of class A, then exponential transform of $f(t)$ exists or Suppose $f(t)$ is piece-wise continuous in every finite interval and is of exponential order k as $t \to \infty$ then $\overline{f}(s)$ exists for all $(s \log a) > k$, that is exponential transform exists.*

Exponential transform of some functions **[1]**

I) $A[1] = \dfrac{1}{(s \log a)}, a > 1$

II) $A[t^n] = \dfrac{n!}{[s \log a]^{n+1}}, a > 1$

III) $A[e^{kt}] = \dfrac{1}{(s \log a - k)}, a > 1$

IV) $A[\cosh kt] = \dfrac{(s \log a)}{\left[(s \log a)^2 - k^2\right]}, a > 1, (s \log a)^2 > k^2$

V) $A[\sinh kt] = \dfrac{k}{(s \log a)^2 - k^2}, a > 1, (s \log a)^2 > k^2$

VI) $A[\sin kt] = \dfrac{k}{(s \log a)^2 + k^2}$

VII) $A[\cos kt] = \dfrac{(s \log a)}{(s \log a)^2 + k^2}$

Properties of Exponential Transform **[1]**

I) Linearity Property

$$A[k_1 f_1(t) + k_2 f_2(t)] = k_1 A[f_1(t)] + k_2 A[f_2(t)]$$

II) Shifting Property

$$\text{If } \ A[f(t)] = \overline{f}(s), \ \text{then } \ A[e^{kt} f(t)] = \overline{f}(s - \frac{k}{\log a})$$

III) Change of Scale Property

$$\text{If } A[f(t)] = \overline{f}(s), \ \text{then } \ A[f(kt)] = \frac{1}{k} \overline{f}\left(\frac{s}{k}\right)$$

IV) Second Shifting theorem

$$If \quad A[f(t)] = \overline{f}(s) \text{ and } G(t) = \begin{cases} F(t-k), & t > k \\ 0, & t < k \end{cases},$$

$$then \quad A[G(\text{t})] = a^{-ks}\overline{f}(s)$$

Exponential Transform of the derivative of a function [2]

Theorem 3.1: $If \ A[f(t)] = \overline{f}(s), \ then \ A[f'(t)] = (s\log a).A[f(t)] - f(0)$

Theorem 3.2: $If \ A[f(t)] = \overline{f}(s), \ then$

$A[f''(t)] = (s\log a)^2 A[f(t)] - (s\log a).f(0) - f'(0)$

Theorem 3.3: $If \ A[f(t)] = \overline{f}(s), \ then$

$A[f'''(t)] = (s\log a)^3 A[f(t)] - (s\log a)^2 f(0) - (s\log a) f'(0) - f''(0)$

Theorem3.4: $If \ A[f(t)] = \overline{f}(s), \ then$

$A[f^n(t)] = (s\log a)^n A[f(t)] - (s\log a)^{n-1} f(0) - (s\log a)^{n-2} f'(0)$
$-(s\log a)^{n-3} f''(0) - \cdots\cdots\cdots\cdots - f^{n-1}(0)$

Exponential Transform of Integral of a function[2]

Theorem 3.5: $If \ A[f(t)] = \overline{f}(s), \ then \ \ A\left[\int_0^t f(t)dt\right] = \frac{1}{(s\log a)}\overline{f}(s)$

Theorem 3.6: $If \ A[f(t)] = \overline{f}(s), \ then \ \ A[t^n.f(t)] = \left[\frac{(-1)^n}{(\log a)^n}\right]\frac{d^n}{ds^n}[\overline{f}(s)]$

Theorem 3.7: $If \ A[f(t)] = \overline{f}(s), \ then \ \ A\left[\frac{1}{t}f(t)\right] = (\log a)\int_s^\infty \overline{f}(s)ds$

Theorem 3.8(Convolution Theorem)[2] :

$If \ A[f_1(t)] = \overline{f}_1(s) \ \ and \ \ A[f_2(t)] = \overline{f}_2(s), \ then$

$$A\left\{\int_0^t f_1(x) f_2(t-x)dx\right\} = \overline{f_1}(s).\overline{f_2}(s)$$

III.MAIN RESULTS

The Inverse Exponential Transform

Definition 3.1 : If the Exponential Transform of a function $f(t)$ is $\overline{f}(s)$ i.e. $A[f(t)] = \overline{f}(s)$ then $f(t)$ is called an Inverse Exponential Transform of $\overline{f}(s)$ .And is written as $f(t) = A^{-1}[\overline{f}(s)]$

A^{-1} is called the Inverse Exponential Transformation operator

Inverse Exponential Transform of some functions

Using definition of inverse exponential transform we get

I] $A^{-1}\left[\frac{1}{(s\log a)}\right]=1, a>1, (s\log a)>0$

II] $A^{-1}\left[\frac{1}{(s\log a)^{n+1}}\right]=\frac{t^n}{n!}, a>1, n=0,1,2,3,$

III] $A^{-1}\left[\frac{1}{(s\log a)-k}\right]=e^{kt}, a>1, (s\log a)>k$

IV] $A^{-1}\left[\frac{(s\log a)}{(s\log a)^2-k^2}\right]=\cosh hkt, a>1, (s\log a)^2>k^2$

V] $A^{-1}\left[\frac{1}{(s\log a)^2-k^2}\right]=\frac{1}{k}(\sinh kt), a>1, (s\log_{k>0} a)^2>k^2$

VI] $A^{-1}\left[\frac{1}{(s\log a)^2+k^2}\right]=\frac{1}{k}(\sin kt)k>0$

VII] $A^{-1}\left[\frac{(s\log a)}{(s\log a)^2+k^2}\right]=\cos kt$

Properties of Inverse Exponential Transform

Linearity property

Theorem 3.2 : If $\overline{f_1}(s)$ and $\overline{f_2}(s)$ are Exponential Transform of functions $f_1(t)$ and $f_2(t)$ respectively, then $A^{-1}\left[k_1\overline{f_1}(s)+k_2\overline{f_2}(s)\right]=k_1A^{-1}\left[\overline{f_1}(s)\right]+k_2A^{-1}\left[\overline{f_2}(s)\right]$,where k_1, k_2 are any constants

Proof: We have

$$A[f_1(t)]=\overline{f_1}(s), \therefore f_1(t)=A^{-1}[\overline{f_1}(s)]$$

$$A[f_2(t)]=\overline{f_2}(s), \therefore f_2(t)=A^{-1}[\overline{f_2}(s)]$$

$$A\left[k_1f_1(t)+k_2f_2(t)\right]=k_1A[f_1(t)]+k_2A[f_2(t)]$$

$$A\left[k_1f_1(t)+k_2f_2(t)\right]=k_1\overline{f_1}(s)+k_2\overline{f_2}(s)$$

$$\therefore k_1f_1(t)+k_2f_2(t)=A^{-1}\left[k_1\overline{f_1}(s)+k_2\overline{f_2}(s)\right]$$

$$\therefore k_1A^{-1}[\overline{f_1}(s)]+k_2A^{-1}[\overline{f_2}(s)]=A^{-1}\left[k_1\overline{f_1}(s)+k_2\overline{f_2}(s)\right]$$

$$\therefore A^{-1}\left[k_1\overline{f_1}(s)+k_2\overline{f_2}(s)\right]=k_1A^{-1}\left[\overline{f_1}(s)\right]+k_2A^{-1}\left[\overline{f_2}(s)\right]$$

Corollary 3.3: [First shifting property]

If $A^{-1}[\overline{f}(s)]=f(t)$ then $A^{-1}\left[\overline{f}\left(s-\frac{k}{\log a}\right)\right]=e^{kt}f(t)$

Proof: We have by first shifting property of Exponential Transform

If $A[f(t)]=\overline{f}(s)$ then $A\left[e^{kt}f(t)\right]=\overline{f}\left(s-\frac{k}{\log a}\right)$

$\therefore$ by the definition of Inverse Exponential Transform, we get $A^{-1}\left[\overline{f}\left(s-\frac{k}{\log a}\right)\right]=e^{kt}f(t)$

Remark: The result of the corollary also expressible as

If $A^{-1}[\overline{f}(s)]=f(t)$, then $A^{-1}\left[\overline{f}\left(s+\frac{k}{\log a}\right)\right]=e^{-kt}f(t)$

Remark: With the help of first shifting property of Inverse Exponential Transform, we have the following Important Results

I] $A^{-1}\left[\frac{1}{(s\log k-1)^{n+1}}\right]=\frac{e^{kt}t^n}{n!}, n=0,1,2,.......$

II] $A^{-1}\left[\frac{s\log a-k}{(s\log k-k)^2-b^2}\right]=e^{kt}\cosh bt$

III] $A^{-1}\left[\frac{1}{(s\log k-k)^2-b^2}\right]=\frac{1}{b}(e^{kt}\cosh bt), b>0$

IV] $A^{-1}\left[\frac{1}{(s\log k-k)^2+b^2}\right]=\frac{1}{b}(e^{kt}\cosh bt), b>0$

V] $A^{-1}\left[\frac{s\log a-k}{(s\log k-k)^2-b^2}\right]=e^{kt}\cos bt$

Corollary 3.4: [Second shifting property]

If $A^{-1}\left[\overline{f}(s)\right]=f(t)$ then $A^{-1}\left[a^{-ks}\overline{f}(s)\right]=G(t)$ where

$$G(t)=\begin{cases} f(t-k), t>k \\ 0, t<k \end{cases}$$

Proof: We have by second shifting property of Exponential Transform,

If $A\left[f(t)\right]=\overline{f}(s)$, then

$$G(t)=\begin{cases} f(t-k), t>k \\ 0, t<k \end{cases}$$

$\therefore$ by the definition of Inverse Exponential Transform,

we get $A\left[G(t)\right]=a^{-ks}\overline{f}(s)$

$\therefore A^{-1}\left[a^{-ks}\overline{f}(s)\right]=G(t)$

Remark: Second shifting theorem can also be stated as

If $A^{-1}\left[\overline{f}(s)\right]=f(t)$ then $A^{-1}\left[a^{-ks}\overline{f}(s)\right]=f(t-k)H(t-k)$ where

$$H(t)=\begin{cases} 1 if, t>k \\ 0 if t<k \end{cases}$$

Theorem 3.5: If $A^{-1}\left[\overline{f}(s)\right]=f(t)$ and $f(0)=0$, then $A^{-1}\left[(s\log a)\overline{f}(s)\right]=\frac{d}{dt}f(t)$

In general, $A^{-1}\left[(s\log a)^n\overline{f}(s)\right]=\frac{d^n}{dt^n}\left[f(t)\right]$ provided $f(0)=f^1(0)=.....=f^{n-1}(0)=0$

Proof: We have $A\left[f(t)\right]=\overline{f}(s)$ and $f(0)=0$,

$$A\left[\frac{d}{dt}f(t)\right]=(s\log a)\overline{f}(s)-f(0)$$

$$\therefore A\left[\frac{d}{dt}f(t)\right]=(s\log a)\overline{f}(s)$$

$$\therefore A^{-1}\left[(s\log a)\overline{f}(s)\right]=\frac{d}{dt}f(t)$$

also, we have $f(0)=f^1(0)=.....=f^{n-1}(0)=0$

$\therefore$ by using Exponential Transform of derivative of nth order of a function $f(t)$

$$A\left[\frac{d^n}{dt^n}f(t)\right]=(s\log a)^n\overline{f}(s)$$

$$\therefore A^{-1}\left[(s\log a)^n\overline{f}(s)\right]=\frac{d^n}{dt^n}f(t)$$

Corollary 3.6: If $A^{-1}\left[\overline{f}(s)\right]=f(t)$ then $A^{-1}\left[\frac{\overline{f}(s)}{s\log a}\right]=\int_0^t f(t)dt$

Proof: We have by theorem

If $A\left[f(t)\right]=\overline{f}(s)$ then $A^{-1}\left[\int_0^t f(t)dt\right]=\frac{\overline{f}(s)}{(s\log a)}$

$\therefore$ by using definitions of Inverse Exponential Transform

we get $A^{-1}\left[\frac{\overline{f}(s)}{(s\log a)}\right]=\int_0^t f(t)dt$

Inverse Exponential Transform of Derivatives

Theorem: If $A^{-1}\left[\overline{f}(s)\right]=f(t)$ then $A^{-1}\left[\frac{d^n}{ds^n}\overline{f}(s)\right]=(-1)^n(\log a)^n\left[t^n f(t)\right]$, for n=1,2, ….

Proof: We have by theorem, If $A^{-1}\left[f(t)\right]=\overline{f}(s)$ then

$$A\left[t^n.f(t)\right]=(-1)^n\frac{1}{(\log a)^n}.\frac{d^n}{ds^n}\left[\overline{f}(s)\right]$$

for $n=1,2,3,.........$

$\therefore$ by using definition of Inverse Exponential Transform, we get

$$A^{-1}\left[\frac{d^n}{ds^n}[\overline{f}(s)]\right]=(-1)^n(\log a)^n.\left[t^n f(t)\right]$$

Inverse Exponential Transform of Integrals

Corollary 3.7: If $A^{-1}[\overline{f}(s)]=f(t)$, then $A^{-1}\left[\int_s^\infty \overline{f}(s)ds\right]=\frac{f(t)}{(t.\log a)}$

Proof: We have by theorem,

If $A[f(t)]=\overline{f}(s)$ then $A\left[\frac{f(t)}{t}\right]=(\log a)\int_s^\infty \overline{f}(s)ds$

$\therefore$ by using definition of Inverse Exponential Transform,

we get $A^{-1}\left[\int_s^\infty \overline{f}(s)ds\right]=\frac{f(t)}{(t.\log a)}$

Theorem3.8: [Convolution Theorem for Inverse Exponential Transform]

If $A^{-1}\left[\overline{f_1}(s)\right]=f_1(t), A^{-1}\left[\overline{f_2}(s)\right]=f_2(t)$, then

$$A^{-1}\left[\overline{f_1}(s)\overline{f_2}(s)\right]=\int_0^t f_1(x), f_2(t-x)dx$$

Proof: We have by convolution theorem of Exponential Transform,

If $A\left[f_1(t)\right]=\overline{f_1}(s)$, $A\left[f_2(t)\right]=\overline{f_2}(s)$,then

$$A\left[\int_0^t f_1(x), f_2(t-x)dx\right]=\overline{f_1}(s)\overline{f_2}(s)$$

$\therefore$ by using definition of Inverse Exponential Transform,

we get $A^{-1}\left[\overline{f_1}(s).\overline{f_2}(s)\right]=\int_0^t f_1(x)f_2(t-x)dx$

Applications of Exponential Transform to finding solution of Initial value problems

Example 1: Consider the initial value problem

$y'' -4y'+4y=64\sin 2t, y(0)=0, y'(0)=1$

$$y''-4y'+4y=64\sin 2t \tag{1}$$

taking exponential transform of both sides of (1), we get

$$\left[(s\log a)^2\overline{y}-(s\log a)y(0)-y'(0)\right]-4\left[(s\log a)\overline{y}-y(0)\right]+4\overline{y}=\frac{64\times 2}{(s\log a)^2+4}$$

on putting the values of $y(0)$ and $y'(0)$ in above equation, we get

$$(s\log a)^2\overline{y}-1-4(s\log a)\overline{y}+4\overline{y}=\frac{128}{(s\log a)^2+4}$$

$$\left[(s\log a)^2-4(s\log a)+4\right]\overline{y}=1+\frac{128}{(s\log a)^2+4}$$

$$\left[(s\log a)-2\right]^2\overline{y}=1+\frac{128}{(s\log a)^2+4}$$

$$\therefore \overline{y}=\frac{1}{[(s\log a)-2]^2}+\frac{128}{[(s\log a)-2]^2[(s\log a)^2+4]}$$

$$\therefore \overline{y}=\frac{1}{[(s\log a)-2]^2}-\frac{8}{(s\log a-2)}+\frac{16}{(s\log a-2)^2}+\frac{8(s\log a)}{[(s\log a)^2+4]}$$

$$\therefore \overline{y}=\frac{17}{[(s\log a)-2]^2}-\frac{8}{(s\log a-2)}+\frac{8(s\log a)}{[(s\log a)^2+4]}$$

$$\therefore y = A^{-1}\left[\frac{17}{[(s\log a)-2]^2} - \frac{8}{(s\log a-2)} + \frac{8(s\log a)}{[(s\log a)^2+4]}\right]$$

$$\therefore y = A^{-1}\left[\frac{17}{[(s\log a)-2]^2} - +\right] - A^{-1}\left[\frac{8}{(s\log a-2)}\right] + A^{-1}\left[\frac{8(s\log a)}{[(s\log a)^2+4]}\right]$$

$$\therefore y = 17A^{-1}\left[\frac{1}{[(s\log a)-2]^2} - +\right] - 8A^{-1}\left[\frac{1}{(s\log a-2)}\right] + 8A^{-1}\left[\frac{(s\log a)}{[(s\log a)^2+4]}\right]$$

$$\therefore y = 17t.e^{2t} - 8e^{2t} + 8\cos 2t$$

Example 2: Consider the Initial Value problem

$$y'' + y = \sin 3t, y(0) = 0, y'(0) = 0$$

$$y'' + y = \sin 3t \qquad (2)$$

taking exponential transform on both sides of equation (2) we get

$$\left[(s\log a)^2\overline{y} - (s\log a)y(0) - y'(0)\right] + \overline{y} = \frac{3}{(s\log a)^2+9}$$

on putting the values $y(0)$ and $y'(0)$ in above equation, we get

$$(s\log a)^2\overline{y} + \overline{y} = \frac{3}{(s\log a)^2+9}$$

$$\left[(s\log a)^2+1\right]\overline{y} = \frac{3}{(s\log a)^2+9}$$

$$\overline{y} = \frac{3}{[(s\log a)^2+1][(s\log a)^2+9]}$$

$$\overline{y} = \frac{3}{8}\left[\frac{1}{[(s\log a)^2+1]} - \frac{1}{[(s\log a)^2+9]}\right]$$

taking inverse exponential transform

$$y = \frac{3}{8}A^{-1}\left[\frac{1}{[(s\log a)^2+1]}\right] - \frac{3}{8}A^{-1}\left[\frac{1}{[(s\log a)^2+9]}\right]$$

$$y = \frac{3}{8}\sin t - \frac{3}{8}(\frac{1}{3}\sin 3t)$$

$$\therefore y = y = \frac{3}{8}\sin t - \frac{1}{8}\sin 3t$$

Example 3: Consider the Initial value problem

$$y''' + 2y'' - y' - 2y = 0, y(0) = y'(0) = 0,\ y''(0) = 6$$

$$y''' + 2y'' - y' - 2y = 0 \qquad (3)$$

taking the exponential transform of both sides, we get

$$\left[(s\log a)^3\overline{y} - (s\log a)^2 y(0) - (s\log a)y'(0) - y''(0)\right] + 2\left[(s\log a)^2\overline{y} - (s\log a)y(0) - y'(0)\right] - \left[(s\log a)\overline{y} - y(0)\right] - 2\overline{y} = 0$$

Using the given conditions, we get

$$\left[(s\log a)^3\overline{y} + (s\log a)^2\overline{y} - (s\log a) - 2\overline{y}\right] = 6$$

$$\left[(s\log a)^3 + 2(s\log a) - (s\log a) - 2\right]\overline{y} = 6$$

$$\therefore \overline{y} = \frac{6}{\left[(s\log a-1)(s\log a+1)(s\log a+2)\right]}$$

$$\bar{y}=\frac{6}{6[(s\log a)-1]}+\frac{6}{(-2)[s\log a+1]}+\frac{6}{3\left[s\log a+2\right]}$$

taking inverse exponential transform

$$y=A^{-1}\left[\frac{1}{s\log a-1}\right]-3A^{-1}\left[\frac{1}{s\log a-1}\right]+2A^{-1}\left[\frac{1}{s\log a-2}\right]$$

$$y=e^{t}-3e^{-t}+2e^{-2t}$$

Example 4: Consider the Initial value problem

$$y''+25y=10\cos 5t,\ y(0)=2,\ y'(0)=0$$

$$y''+25y=10\cos 5t \tag{4}$$

taking exponential transform on both sides, we get

$$\left[(s\log a)^2\bar{y}-(s\log a)y(0)-y'(0)\right]+25\bar{y}=\frac{10(s\log a)}{(s\log a)^2+25}$$

Putting $y(0)=2,\ y'(0)=0$

$$(s\log a)^2\bar{y}-2(s\log a)y+25\bar{y}=\frac{10(s\log a)}{(s\log a)^2+25}$$

$$\left[(s\log a)^2+25\right]\bar{y}=2(s\log a)+\frac{10(s\log a)}{(s\log a)^2+25}$$

$$\bar{y}=\frac{2(s\log a)}{(s\log a)^2+25}+\frac{10(s\log a)}{\left[(s\log a)^2+25\right]^2}$$

taking inverse exponential transform we get,

$$y=A^{-1}\left[\frac{2(s\log a)}{(s\log a)^2+25}\right]+A^{-1}\left[\frac{10(s\log a)}{\left[(s\log a)^2+25\right]^2}\right] \tag{5}$$

Now

$$A[t\sin 5t]=\frac{5}{(s\log a)^2+25}$$

$$\therefore A[t\sin 5t]=\frac{-1}{\log a}\frac{d}{ds}\left[\frac{5}{(s\log a)^2+25}\right]$$

$$\therefore A[t\sin 5t]=\frac{-1}{\log a}\left[\frac{-5\times 2(s\log a).\log a}{\left[(s\log a)^2+25\right]^2}\right]$$

$$\therefore A[t\sin 5t]=\frac{10(s\log a)}{\left[(s\log a)^2+25\right]^2}$$

$$\therefore A^{-1}\left[\frac{10(s\log a)}{\left[(s\log a)^2+25\right]^2}\right]=t\sin 5t$$

Putting this value in equation (5) we get

$$y=2\cos 5t+t\sin 5t$$

Example 5: Consider the Initial Value problem

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=e^{-x}\sin x \quad , y(0)=0, y'(0)=1$$

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=e^{-x}\sin x \tag{6}$$

taking the exponential transform of both sides we get,

$$\left[(s\log a)^2\bar{y}-(s\log a)y(0)-y'(0)\right]+2\left[(s\log a)\bar{y}-y(0)\right]+5\bar{y}=A\left[e^{-x}\sin x\right]$$

Using conditions

$$\left[(s\log a)^2\bar{y}-1\right]+2\left[(s\log a)\bar{y}\right]+5\bar{y}=\frac{1}{(s\log a+1)^2+1}$$

$$\left[(s\log a)^2+2(s\log a)+5\right]\bar{y}=1+\frac{1}{(s\log a)^2+2(s\log a)+2}$$

$$\left[(s\log a)^2+2(s\log a)+5\right]\bar{y}=\frac{(s\log a)^2+2(s\log a)+3}{(s\log a)^2+2(s\log a)+2}$$

$$\therefore \bar{y}=\frac{(s\log a)^2+2(s\log a)+3}{\left[(s\log a)^2+2(s\log a)+5\right]\left[(s\log a)^2+2(s\log a)+2\right]}$$

on solving using partial function, we get

$$\bar{y}=\frac{2}{3}\frac{1}{\left[(s\log a+1)^2+2^2\right]}+\frac{1}{3}\frac{1}{\left[(s\log a+1)^2+1^2\right]}$$

taking inverse exponential transform, we get

$$y=\frac{1}{3}\left[\frac{2}{\left[(s\log a+1)^2+2^2\right]}\right]+\frac{1}{3}A^{-1}\left[\frac{1}{\left[(s\log a+1)^2+1^2\right]}\right]$$

$$y=\frac{1}{3}e^{-x}.\sin 2x+\frac{1}{3}e^{-x}\sin x$$

$$y=\frac{1}{3}e^{-x}\left[\sin x+\sin 2x\right]$$

IV. Conclusions

In this work I introduced inverse exponential transform and proved some properties like linearity, shifting and some important results with application to initial value problem have been discussed. Moreover, convolution theorem for inverse exponential transform is verified.

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