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Inverse Perfect Secure Domination in Graphs

Cristina S. Castañares[1], Enrico L. Enriquez[2]

[1]*Faculty, Mathematics and Statistics Department, Eastern Visayas State University, Tacloban City, Philippines*
[2]*Faculty, Department of Computer, Information Science and Mathematics, University of San Carlos, Cebu City, Philippines*

Abstract - *Let $G = (V(G), E(G))$ be a connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $u \in V(G) \setminus S$, there exists $v \in S$ such that $uv \in E(G)$. A dominating set S is called a secure dominating set if for each $u \in V(G) \setminus S$ there exists $v \in S$ such that u is adjacent to v and $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. A secure dominating set S is called a perfect secure dominating set of G if each $u \in V(G) \setminus S$ is dominated by exactly one element of S. Further, if D is a minimum perfect secure dominating set of G, then a perfect secure dominating set $S \subseteq V(G) \setminus D$ is called an inverse perfect secure dominating set of G with respect to D. In this paper, we investigate the concept and give some important results.*

Keywords - *dominating set, secure dominating set, perfect secure dominating set, inverse perfect secure dominating set*

I. INTRODUCTION

Suppose that $G = (V(G), E(G))$ is a simple graph with vertex set $V(G)$ and edge set $E(G)$. In simple graph, we mean, finite and undirected graph with neither loops nor multiple edges. For the general graph theoretic terminology, the readers may refer to [1].

A vertex v is said to dominate a vertex u if uv is an edge of G or $v = u$. A set of vertices $S \subseteq V(G)$ is called a dominating set of G if every vertex not in S is dominated by at least one member of S. The size of a set of least cardinality among all dominating sets for G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is called γ-set of G. Domination in a graph has been a huge area of research in graph theory. It was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. Domination in graphs has been studied in [3-7].

A dominating set S is called a secure dominating set of G if for each $u \in V(G) \setminus S$ there exists $v \in S$ such that u is adjacent to v and $(S \setminus \{v\}) \cup \setminus\{u\}$ is a dominating set. The secure domination number of G, is the minimum cardinality of a secure dominating set of G and is denoted by $\gamma_s(G)$. A secure dominating set of cardinality $\gamma_s(G)$ is called γ_s-set of G. Secure domination has been studied in [8-17].

A secure dominating set S is called a perfect secure dominating set of G if each $u \in V(G) \setminus S$ is dominated by exactly one element of S. The perfect secure domination number of G, is the minimum cardinality of a perfect secure dominating set of G and is denoted by $\gamma_{ps}(G)$. A perfect secure dominating set of cardinality $\gamma_{ps}(G)$ is called γ_{ps}-set of G. Perfect secure domination has been studied in [18]. Variants of perfect domination in graphs are studied in [19-21].

Motivated by [18] and the inverse domination in graphs [22-28], we initiate the study of an inverse perfect secure dominating set. Let D be a minimum perfect secure dominating set of G. A perfect secure dominating set $S \subseteq V(G) \setminus D$ is called an inverse perfect secure dominating set of G with respect to D.The inverse perfect secure domination number of G, is the minimum cardinality of an inverse perfect secure dominating set of G and is denoted by $\gamma_{ps}^{-1}(G)$. An inverse perfect secure dominating set of cardinality $\gamma_{ps}^{-1}(G)$ is called γ_{ps}^{-1}-set of G.

In this paper, we investigate the concept and give some important results. We further give the characterization of an inverse perfect secure dominating set in the join and corona of two graphs.

II. RESULTS

Remark 2.1 *The set $S = V(G)$ is a secure dominating set and a perfect dominating set.*

Proof: If $S = V(G)$, then every vertex in $V(G) \setminus S = \emptyset$ vacuously satisfies the definitions of a secure dominating set and a perfect dominating set. ■

Remark 2.2 *Every graph G has a secure dominating set and a perfect dominating set.*

Proof: By Remark 2.1, $S = V(G)$ is a secure dominating set and a perfect dominating set. ∎

From the definitions of inverse perfect secure dominating set and Remark 2.2 the following is immediate.

Remark 2.3 *Let G be a nontrivial graph. Then* $1 \leq \gamma_p(G) \leq \gamma_{ps}^{-1}(G) \leq n$.

For a nontrivial connected graph G, the following result says that $\gamma_{ps}^{-1}(G)$ ranges over all integers from 1 to $\frac{n}{2}$.

Theorem 2.4 *Given positive integers k, m and n such that $1 \leq k \leq m \leq \frac{n}{2}$, where $n \geq 2$ there exists a connected graph G with $|V(G)| = n$, $\gamma_p(G) = k$, and $\gamma_{ps}^{-1}(G) = m$.*

Proof: Consider the following cases.

Case 1. Suppose that $1 = k = m \leq \frac{n}{2}$.

Let $G = K_n$ (see Figure 1). Then the set $D = \{v_1\}$ is a $\gamma_p\text{-}set$ of G, the set $S = \{v_2\}$ is a $\gamma_{ps}^{-1}\text{-}set$ of G. Thus $\gamma_p(G) = 1 = k$, $\gamma_{ps}^{-1}(G) = 1 = m$ and $|V(G)| = |V(K_n)| = n$.

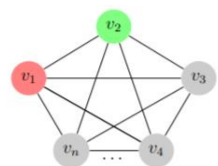

Figure 1: A graph G with $1 = k = m \leq \frac{n}{2}$.

Case 2. Suppose that $1 = k \leq m < \frac{n}{2}$.

Let $G = K_1 + \langle \bigcup_{i=1}^{m} A_I \rangle$ where $A_i = K_r$ for all $i \in \{1,2,\ldots,m\}$ and an integer $r \geq 2$ and let $n = 1 + mr$ (see Figure 2).

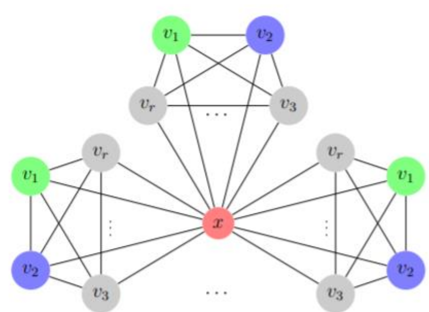

Figure 2: A graph G with $1 = k \leq m < \frac{n}{2}$.

The set $V(K_1) = \{x\}$ is a $\gamma_p\text{-}set$ of G is clear. Let $u \in V(K_r)$. Since $r \geq 2, V(K_r) \setminus \{u\} = \emptyset$. Let $v \in V(K_r) \{u\}$. The set $D = \bigcup_{i=1}^{m} D_i$ where $D_i = \{v\} \subset V(A_i)$ for all $i \in \{1,2,\ldots,m\}$ is a $\gamma_{ps}\text{-}set$ of G and the set $S = \bigcup_{i=1}^{m} S_i$where $S_i = \{u\} \subset V(A_i)$ for all $i \in \{1,2,\ldots,m\}$ is a $\gamma_{ps}^{-1}\text{-}set$ of G. Thus, $\gamma_p(G) = 1 = k$, $\gamma_{ps}^{-1}(G) = |S| = |\bigcup_{i=1}^{m} S_i| = m \cdot 1 = m$, and

$$|V(G)| = |V(K_1 + \langle \bigcup_{i=1}^{m} A_i \rangle |$$

$$= |V(K_1)| + |V(\langle \cup_{i=1}^{m} A_i \rangle)|$$
$$= 1 + |\cup_{i=1}^{m} A_i| = 1 + mr = n.$$

Case 3. Suppose that $1 < k = m < \frac{n}{2}$.
Let $G = P_k \circ K_r$ where $k \geq 2$ and $r \geq 2$ (see Figure 3).

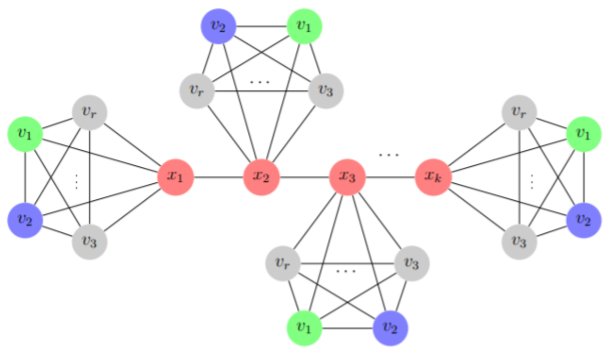

Figure 3: A graph G with $1 < k = m < \frac{n}{2}$.

Let $n = k(r+1)$. The set $V(P_k)$ is a $\gamma_p - set$ of G, the set $D = \cup_{x \in V(P_k)} S_x$ where $S_x = \{v_1\}$ for all $x \in V(P_k)$ is a γ_{ps}-set of G, and the set $S = \cup_{\{y \in V(P_k)\}} S_y$ where $S_y = \{v_2\}$ for all $y \in V(P_k)$ is a γ_{ps}^{-1}-set of G with respect to D. Thus, $\gamma_p(G) = |V(P_k)| = k$, $\gamma_{ps}^{-1}(G) = |S| = |\cup_{x \in V(P_k)} S_y| = k \cdot 1 = k = m$, and $|V(G)| = |V(P_k \circ K_r)| = k + kr = k(1+r) = n$.

Case 4. Suppose that $1 < k < m = \frac{n}{2}$.

Let $G = P_2 \square P_m$ where $m \geq 3$ and $m \equiv 1(\,mod\ 4)$ (see Figure 4).

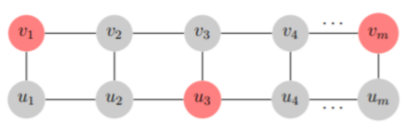

Figure 4: A graph G with $1 < k < m = \frac{n}{2}$.

Let $k = \frac{2(m+1)}{4}$ and let $n = 2m$. The set $A = \left\{v_{4i-3} : i = 1,2,\ldots,\frac{m+3}{4}\right\} \cup \{u_{4i-1} : i = 1,2,\ldots,\frac{m+1}{4}\}$ is a $\gamma_p - set$ of G, the set $D = \{u_i : i = 1,2,\ldots,m\}$ is a $\gamma_{ps} - set$ of G, and the set $S = \{v_i : i = 1,2,\ldots,m\}$ is a γ_{ps}^{-1}-set of G. Hence, $\gamma_p(G) = |A| = \frac{m+3}{4} + \frac{m-1}{4} = \frac{2(m+1)}{4} = k$, $\gamma_{ps}^{-1}(G) = |S| = m$, and $|V(G)| = |V(P_2 \square P_m)| = 2m = n$. ■

Corollary 2.5 *The difference between $\gamma_{ps}^{-1}(G) - \gamma_p(G)$ can be made arbitrarily large.*

Proof: By Theorem 2.5, there exists a connected graph G such that $\gamma_p(G) = 1$ and $\gamma_{ps}^{-1}(G) = n + 1$. Then $\gamma_{ps}^{-1}(G)$-$\gamma_p(G) = (n+1) - 1 = n$. Hence, the difference between $\gamma_{ps}^{-1}(G)$-$\gamma_p(G)$ can be made arbitrarily large.■

Let $P_n = [v_1, v_2, \ldots, v_n]$ such that $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(P_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-1}v_n\}$. The next result shows the inverse perfect secure domination number of a path graph P_n.

Theorem 2.6 *Let $n \geq 3$. If n is an odd integer, then $\gamma_{ps}(P_n) = \frac{n+1}{2}$.*

Proof: Case 1: Suppose that $n \equiv 1(mod\ 4)$. Consider the graph P_n where $n \geq 5$ (see Figure 5).

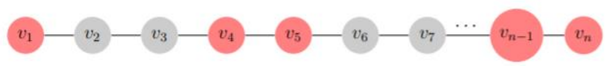

Figure 5: A graph with $\gamma_{ps}(P_n) = \frac{n+1}{2}$.

The set $D = \left\{v_{4i-3}: i = 1,2,\ldots,\frac{n+3}{4}\right\} \cup \left\{v_{4i}: i = 1,2,\ldots,\frac{n-1}{4}\right\}$ is a γ_{ps}-set of P_n. Thus, $\gamma_{ps}(P_n) = |D| = \frac{n+3}{4} + \frac{n-1}{4} = \frac{n+1}{4}$.

Case 2: Suppose that $n \equiv 3(mod\ 4)$. Consider the graph P_n where $n \geq 3$ (see Figure 6).

Figure 6: A graph with $\gamma_{ps}(P_n) = \frac{n+1}{2}$.

The set $D = \left\{v_{4i-3}: i = 1,2,\ldots,\frac{n+1}{4}\right\} \cup \left\{v_{4i-2}: i = 1,2,\ldots,\frac{n+1}{4}\right\}$ is a γ_{ps}-set of P_n. Thus, $\gamma_{ps}(P_n) = |D| = \frac{n+1}{4} + \frac{n+1}{4} = \frac{n+1}{2}$. ■

Consider a path graph P_n (see Figure 6). The set $S = V(P_n) \setminus D$ is an inverse dominating set of P_n. However,

$$|S| = |V(P_n) \setminus D| = |V(P_n)| - |D| = n - \left(\frac{n+1}{2}\right) = \frac{n-1}{2} < \frac{n+1}{2} = \gamma_{ps}(P_n),$$

contrary to the definition of an inverse dominating set. Thus, $|S|$ is not an inverse perfect secure dominating set of P_n whenever n is an odd integer greater than or equal to 3.

However, if n is an even integer, the following result shows the inverse perfect secure domination number of P_n.

Theorem 2.8 *Let $n \geq 2$. If n is an even integer, then $\gamma_{ps}^{-1}(P_n) = \frac{n}{2}$.*
Proof: Case 1: Suppose that $n \equiv 0(mod\ 4)$. Consider the graph P_n where $n \geq 5$ (see Figure 7).

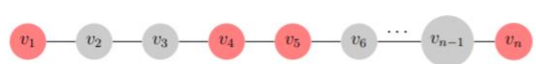

Figure 7: A graph with $\gamma_{ps}^{-1}(P_n) = \frac{n}{2}$.

The set $D = \left\{v_{4i-2}: i = 1,2,\ldots,\frac{n}{4}\right\} \cup \left\{v_{4i-1}: i = 1,2,\ldots,\frac{n}{4}\right\}$ is a $\gamma_{ps} - set$ of P_n, the set $S = \left\{v_{4i-3}: i = 1,2,\ldots,\frac{n}{4}\right\} \cup \left\{v_{4i}: i = 1,2,\ldots,\frac{n}{4}\right\}$ is a γ_{ps}^{-1}-set of P_n with respect to D. Thus, $\gamma_{ps}^{-1}(P_n) = |S| = \frac{n}{4} + \frac{n}{4} = \frac{n}{2}$.

Case 2: Suppose that $n \equiv 2(mod\ 4)$. Consider the graph P_n where $n \geq 3$ (see Figure 8).

Figure 8: A graph with $\gamma_{ps}^{-1}(P_n) = \frac{n}{2}$.

The set $D = \left\{v_{4i-2}: i = 1,2, \dots, \frac{n+2}{4}\right\} \cup \left\{v_{4i-1}: i = 1,2, \dots, \frac{n-2}{4}\right\}$ is a γ_{ps}-*set* of P_n, the set $S = \left\{v_{4i-3}: i = 1,2, \dots, \frac{n+2}{4}\right\} \cup \left\{v_{4i}: i = 1,2, \dots, \frac{n-2}{4}\right\}$ is a γ_{ps}^{-1}-*set* of P_n with respect to D. Thus, $\gamma_{ps}^{-1}(P_n) = |S| = \frac{n+2}{4} + \frac{n-2}{4} = \frac{n}{4}$. ■

Let $C_n = [v_1, v_2, \dots, v_n]$ such that $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $E(C_n) = \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1\}$. The next result shows the super inverse domination number of a cycle graph C_n.

Theorem 2.9 *Let* $n \geq 3$. *If* n *is an odd integer, then*

$$\gamma_{ps}(C_n) = \begin{cases} \frac{n+1}{4}, & if\ n \equiv 1(\ mod\ 4) \\ \frac{n+3}{2}, & if\ n \equiv 3(\ mod\ 4), n \neq 3 \\ 1 & if\ n = 3. \end{cases}$$

Proof: Case 1: Suppose that $n \equiv 1(\ mod\ 4)$. Consider the graph C_n where $n \geq 5$ (see Figure 9).

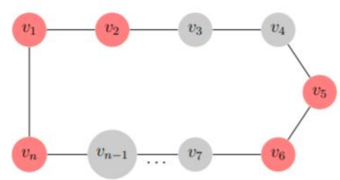

Figure 9: A graph with $\gamma_{ps}(C_n) = \frac{n+1}{2}$.

The set $D = \left\{v_{4i-3}: i = 1,2, \dots, \frac{n-1}{4}\right\} \cup \left\{v_{4i-2}: i = 1,2, \dots, \frac{n-1}{4}\right\} \cup \{v_n\}$ is a γ_{ps}-*set* of P_n. Thus, $\gamma_{ps}(C_n) = |D| = \frac{n-1}{4} + \frac{n-1}{4} + 1 = \frac{n+1}{2}$.

Case 2: Suppose that $n \equiv 3(\ mod\ 4)$, $n \neq 3$. Consider the graph C_n where $n > 3$ (see Figure 10).

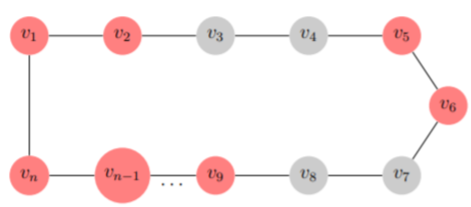

Figure 10: A graph with $\gamma_{ps}(C_n) = \frac{n+3}{2}$.

The set $D = \left\{v_{4i-1}: i = 1,2, \dots, \frac{n+1}{4}\right\} \cup \left\{v_{4i-2}: i = 1,2, \dots, \frac{n+1}{4}\right\} \cup \{v_n\}$ is a γ_{ps}-*set* of C_n. Thus, $\gamma_{ps}(C_n) = |D| = \frac{n+1}{4} + \frac{n+1}{4} + 1 = \frac{n+3}{2}$.

Case 3: Suppose that $n = 3$. Clearly, $\gamma_{ps}(C_3) = 1$.■

Now, consider a cycle graph C_n (see Figure 9). The set $S = V(C_n) \setminus D$ is an inverse dominating set of C_n. However,

$$|S| = |V(C_n) \setminus D| = |V(C_n)| - |D| = n - \left(\frac{n+1}{2}\right) = \frac{n-1}{2} < \frac{n+1}{2} = \gamma_{ps}(C_n),$$

contrary to the definition of an inverse dominating set. Thus, $|S|$ is not an inverse perfect secure dominating set of C_n whenever n is an odd integer greater than or equal to 3. Similarly, if $n \equiv 2(\ mod\ 4)$, then $|S|$ is not an inverse perfect secure dominating set of C_n.

However, if $n \equiv 0(\, mod\ 4)$, the following result shows the inverse perfect secure domination number of C_n.

Theorem 2.10 *If* $n \equiv 0(\, mod\ 4)$, *then* $\gamma_{ps}^{-1}(C_n) = \frac{n}{2}$.

Proof: Consider the graph C_n where $n \equiv 0(\, mod\ 4)$ (see Figure 11).

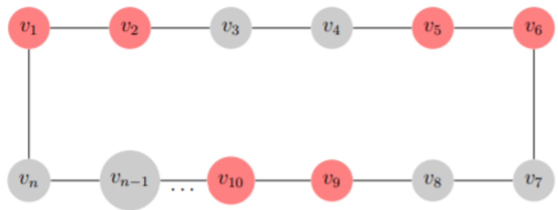

Figure 11: A graph with $\gamma_{ps}^{-1}(C_n) = \frac{n}{2}$.

The set $D = \left\{v_{4i-1}: i = 1,2, \dots, \frac{n}{4}\right\} \cup \left\{v_{4i}: i = 1,2, \dots, \frac{n}{4}\right\}$ is a γ_{ps}-set of C_n, the set $S = \left\{v_{4i-3}: i = 1,2, \dots, \frac{n}{4}\right\} \cup \left\{v_{4i-2}: i = 1,2, \dots, \frac{n}{4}\right\}$ is a γ_{ps}^{-1}-set of $C_n$with respect to D. Thus, $\gamma_{ps}^{-1}\ (C_n) = |S| = \frac{n}{4} + \frac{n}{4} = \frac{n}{2}$. ■

A complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices

Theorem 2.11 *Let* G *be a nontrivial connected graph. The* $\gamma_{ps}^{-1}(G) = 1$ *if and only if* $G = K_n$.

Proof: Suppose that $\gamma_{ps}^{-1}(G) = 1$. Let $D = \{x\}$ be a perfect secure dominating set of G. Since G in nontrivial, $V(G) \setminus D \neq \emptyset$. Let $u \in V(G) \setminus D$. Suppose that $G \neq K_n$. Then there exists $v \in V(G) \setminus D$ distinct from u such that $uv \notin E(G)$. Thus, D is a dominating set but $(D \setminus \{x\}) \cup \{v\} = \{v\}$ is not a dominating set of G contrary to our assumption that D is a secure dominating set of G. Thus, G must be equal to K_n.

For the converse, suppose that $G = K_n$. Let $D = \{x\}$. Then D is a dominating set of G. Since $V(G) \setminus D \neq \emptyset$, let $u \in V(G) \setminus D$. Since D is a dominating set and $(D \setminus \{x\}) \cup \{u\} = \{u\}$, a dominating set of G, it follows that D is a secure dominating set of G. Clearly, every $u \in V(G) \setminus D$ is dominated by only $x \in D$. Thus D is a perfect secure dominating set of G, that is, γ_{ps}-set of G. Let $S = \{v\}$ with $v \neq u$. Since G is complete, S is a dominating set of G Similarly, S is a perfect secure dominating set of G, that is, a γ_{ps}^{-1}-set of G. Hence, $\gamma_{ps}^{-1}(G) = |S| = 1$.■

III. CONCLUSIONS

In this paper, we introduced the concept of inverse perfect secure domination in graphs and prove that given positive integers k, m and n such that $1 \leq k \leq m \leq \frac{n}{2}$, where $n \geq 2$ there exists a connected graph G with $|V(G)| = n$, $\gamma_p(G) = k$, and $\gamma_{ps}^{-1}(G) = m$. Further, we prove the inverse perfect secure domination number of a path graph P_n and a cycle C_n graph. Prove the characterization of the inverse perfect secure domination number of a complete graph. Some related problems on inverse perfect secure domination in graphs are still open for research.

1. Characterize the inverse perfect secure dominating sets of the join, corona, Cartesian product, and lexicographic product of two graphs.

2. Find the inverse perfect secure domination number of the join, corona, Cartesian product, and lexicographic product of two graphs.

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