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Disjoint Fair Domination in Graphs

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Abstract *- Let $G = (V(G), E(G))$ be a connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $u \in V(G) \setminus S$, there exists $v \in S$ such that $uv \in E(G)$. A dominating set S is called a fair dominating set if for each distinct vertices $u.v \in V(G) \setminus S$, $|N_G(u) \cap S| = |N_G(v) \cap S|$. Further, if D is a minimum fair dominating set of G, then a fair dominating set $S \subseteq V(G) \setminus D$ is called an inverse fair dominating set of G with respect to D. A disjoint fair dominating set of G is the set $C = D \cup S \subseteq V(G)$. In this paper, we investigate the concept and give some important results.*

Keywords: *dominating set, fair dominating set, inverse fair dominating set, disjoint fair dominating set*

I. INTRODUCTION

Suppose that $G = (V(G), E(G)$ is a simple graph with vertex set $V(G)$ and edge set $E(G)$. In simple graph, we mean, finite and undirected graph with neither loops nor multiple edges. For the general graph theoretic terminology, the readers may refer to [1].

A vertex v is said to dominate a vertex u if uv is an edge of G or $v = u$. A set of vertices $S \subseteq V(G)$ is called a dominating set of G if every vertex not in S is dominated by at least one member of S. The size of a set of least cardinality among all dominating sets for G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is called $\gamma-$set of G. Domination in a graph has been a huge area of research in graph theory. It was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. Domination in graphs has been studied in [3 - 15].

A dominating set S is called a fair dominating set [16] of G if all the vertices not in S are dominated by the same number of vertices from S, that is, $|N_{G(u)} \cap S| = |N_{G(v)} \cap S||$ for every two distinct vertices u and v from $V(G) \setminus S$ *and a subset* S *of* $V(G)$ is a k-fair dominating set in G if for every vertex $v \in V(G) \setminus S, N_{G(v)} \cap S = k$. The fair domination number of G, is the minimum cardinality of a fair dominating set of G and is denoted by $\gamma_{fd}(G)$. A fair dominating set of cardinality $\gamma_{fd}(G)$ is called γ_{fd}-*set of* G. Fair domination has been studied in [17 - 22].

A fair dominating set S is called an inverse fair dominating set of G if each $S \subseteq V(G) \setminus D$ with D is a $\gamma_{fd} - set\ of\ G$. The inverse fair domination number of G, is the minimum cardinality of an inverse fair dominating set of G and is denoted by $\gamma_{fd}^{-1}(G)$. An inverse fair dominating set of cardinality $\gamma_{fd}^{-1}(G)$ is called γ_{fd}^{-1}-*set of* G. The inverse domination has been studied in [23 - 31].

Motivated by the idea of inverse and disjoint domination in graphs [32,33], we initiate the study of disjoint fair dominating set. Let D be a minimum fair dominating set and S be an inverse fair dominating set of G with respect to D. A disjoint fair dominating set of G is the set $C = D \cup S \subseteq V(G)$. The disjoint fair domination number of G, is the minimum cardinality of a disjoint fair dominating set of G and is denoted by $\gamma\gamma_{fd}(G)$. A disjoint fair dominating set of cardinality $\gamma\gamma_{fd}(G)$ is called $\gamma\gamma_{fd}$-*set of* G. In this paper, we investigate the concept and give some important results.

II. RESULTS

Remark 2.1 *The set $S = V(G)$ is a fair dominating set of a graph G.*

Proof: If $S = V(G)$, then every vertex in $V(G) \setminus S = \emptyset$ vacuously satisfies the definitions of a fair dominating set. ∎

Remark 2.2 *Every graph G has a fair dominating set.*

Proof: By Remark 2.1, $S = V(G)$ is a fair dominating set. ∎
From the definitions of disjoint fair dominating set and Remark 2.2 the following is immediate.

Remark 2.3 *Let G be a nontrivial connected graph of order n. Then $2 \leq \gamma(G) \leq \gamma\gamma_{fd}(G) \leq n$.*

For a nontrivial connected graph G of order n, the following result says that $\gamma\gamma_{fd}(G)$ ranges over all intergers from 2 to n.

Theorem 2.4 *Given positive integers k, m and n such that $2 \leq k \leq n$, where $n \geq 2$ there exists a connected graph G with $|V(G)| = n$, and $\gamma\gamma_{fd}(G) = k$.*

Proof: Consider the following cases.

Case 1. Suppose that $2 \leq k < n$.

Let $G = \langle x \rangle + K_{r,s}$ where $r > s \geq 1$ (see Figure 1).

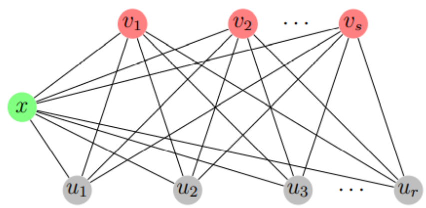

Figure 1: A graph G with $2 \leq k < n$.

Let $k = s + 1$ and $n = k + r$. The set $D = \{x\}$ is a $\gamma_{fd} - set\ of\ G, the\ set\ S = \{v_i : i = 1,2, \dots, s\}$ is a $\gamma_{fd}^{-1} - set\ of\ G$ with respect to D. Thus, $C = D \cup S = \{x, v_i : i = 1,2, \dots, s\}$ is a $\gamma\gamma_{fd}$ –set of G, that is, $\gamma\gamma_{fd}(G) = 1 + s = k$ and $|V(G)| = |V(\langle x \rangle + K_{r,s})| = 1 + s + r = k + r = n$

Case 2. Suppose that $2 < k = n$.

Let $G = P_m \circ K_1\ where\ m \geq 2$ (see Figure 2).

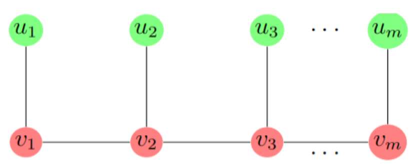

Figure 2: A graph G with $2 < k = n$.

Let $k = 2m$. The set $D = \{u_i : i = 1,2, \dots, m\}$ is a γ_{fd}-set and the set $S = \{v_i : i = 1,2, \dots, m\}$ is a γ_{fd}^{-1}-*set of* G. Thus, $C = D \cup S == \{u_i, v_i : i = 1,2, \dots, m\}$ is a $\gamma\gamma_{fd} - set\ of\ G$, that is, $\gamma\gamma_{fd}(G) = |C| = 2m = k\ and\ |V(G)| = |V(P_m \circ K_1)| = m + m = k = n$. ∎

Corollary 2.5 *The difference between $\gamma\gamma_{fr}(G) - \gamma_{fd}(G)$ can be made arbitrarily large.*

Proof: Let n be a positive integer. By Theorem 2.4, there exists a connected graph G such that $\gamma_{fd}(G) = 1\ and\ \gamma\gamma_{fd}(G) = n + 1$. Thus, $\gamma\gamma_{fd}(G) - \gamma_{fd}(G) = (n + 1) - 1 = n$. Hence, the difference between $\gamma_{pr}^{-1}(G) - \gamma_r(G)$ can be made arbitrarily large.∎

Let $C_n = [v_1, v_2, \dots, v_n]$ such that $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $E(C_n) = \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1\}$. The next result shows the disjoint fair domination number of a cycle graph C_n.

Theorem 2.6 Let $G = C_n$ of order $n \geq 3$. Then,

$$\gamma\gamma_{fd}(G) = \begin{cases} \frac{2n}{3}, & if\ n \equiv 0(mod\ 3) \\ \frac{2n+4}{3}, & if\ n \equiv 1(mod\ 3) \\ \frac{2n+8}{3}, & if\ n \equiv 2(mod\ 3), n \neq 5. \end{cases}$$

Proof: Case 1: Suppose that $n \equiv\ 0(mod\ 3)$. Consider the graph $G = C_n$ where $n\ \neq 5$ (see Figure 3).

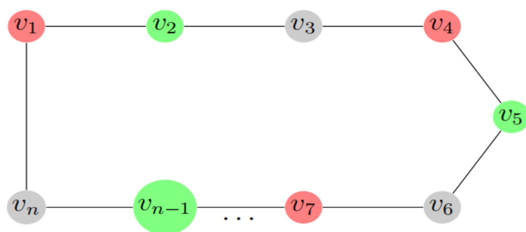

Figure 3: A graph with $\gamma\gamma_{fd}(G) = \frac{2n}{3}$.

The set $D = \{v_{3i-2}: i = 1,2,\ldots,\frac{n}{3}\}$ is a γ_{fd}-set of G, the set $S = \{v_{3i-1}: i = 1,2,\ldots,\frac{n}{3}\}$ is a γ_{fd}^{-1}-set of G with respect to D. Thus, $C = D \cup\ S\ = \{v_{3i-2}, v_{3i-1}: i = 1,2,\ldots,\frac{n}{3}\}$ is a $\gamma\gamma_{fd} - set\ of\ G$, that is, $\gamma\gamma_{fd}(G) = |C| = \frac{n}{3} + \frac{n}{3} = \frac{2n}{3}$.

Case 2: Suppose that $n \equiv\ 1(mod\ 3)$. Consider the graph G (see Figure 4).

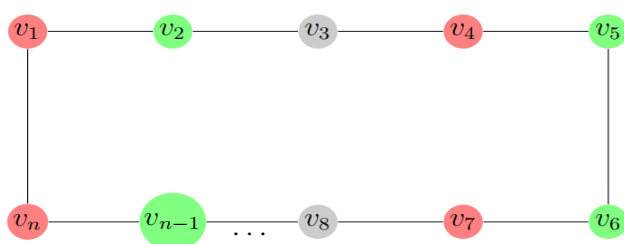

Figure 4: A graph with $\gamma\gamma_{fd}(G) = \frac{2n+4}{3}$.

The set $D = \left\{v_{3i-2}: i = 1,2,\ldots,\frac{n-1}{3}\right\} \cup \{v_n\}$ is a γ_{fd}-set of G, the set $S = \{v_{3i-1}: i = 1,2,\ldots,\frac{n-1}{3}\} \cup\ \{v_{n-1}\}$ is a γ_{fd}^{-1}-set of G. Thus, $C = D \cup\ S = \left\{v_{3i-2}: i = 1,2,\ldots,\ \frac{n-1}{3}\right\} \cup\ \{v_n\} \cup\ \left\{v_{3i-1}: i = 1,2,\ldots,\frac{n-1}{3}\right\} \cup\ \{v_{n-1}\}$ is a $\gamma\gamma_{fd}$-set of G, that is,

$$\gamma\gamma_{fd}(G) = |C| = \frac{n-1}{3} + 1 + \frac{n-1}{3} + 1 = \frac{2n+4}{3}.$$

Case 3: Suppose that $n \equiv\ 2(mod\ 3)$ and $n\ \neq 5$. Consider the graph G (see Figure 5).

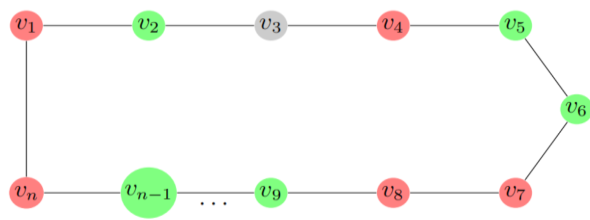

Figure 5: A graph with $\gamma_{pr}^{-1}(G) = \frac{2n+8}{3}$.

The set $D = \left\{v_{3i-2}: i = 1,2, \dots, \frac{n-2}{3}\right\} \cup \{v_{n-3}, v_n\}$ is a γ_{fd}-set of G, the set $S = \{v_{3i-1} : i = 1,2, \dots, \frac{n-5}{3}\} \cup \{v_{n-1}, v_{n-2}, v_{n-5}\}$ is a γ_{fd}^{-1}-set of G. Thus, $C = D \cup S = \left\{v_{3i-2}: i = 1,2, \dots, \frac{n-2}{3}\right\} \cup \{v_{n-3}, v_n\} \cup \left\{v_{3i-1}: i = 1,2, \dots, \frac{n-5}{3}\right\} \cup \{v_{n-1}, v_{n-2}, v_{n-5}\}$ is a $\gamma\gamma_{fd}$-set of G, that is, $\gamma\gamma_{fd}(G) = |C| = \frac{n-2}{3} + 2 + \frac{n-5}{3} + 3 = \frac{2n+8}{3}\}$. ■

Theorem 2.7 *Let $G = \langle x \rangle + H$ and H is a nontrivial connected graph. Then $\gamma\gamma_{fd}(G) = 2$ if and only if $\gamma(H) = 1$.*

Proof: Suppose that $\gamma\gamma_{fd}(G) = 2$. Let $D = \{x\}$ and $S = \{v\}$ be a fair dominating set and an inverse fair dominating set of G respectively. Since H is a $S \subseteq V(G) \setminus D = V(H)$, it follows that $\gamma(H) = 1$.

For the converse, suppose that $\gamma(H) = 1$. Let $S = \{v\}$ be a dominating set of H and hence a dominating set in $G = \langle x \rangle + H$. Hence, S is a $1-$fair dominating set of G. Similarly, $D = \{x\}$ is a $1-$fair dominating set of G. Thus, $C = D \cup S = \{x, v\}$ is a $\gamma\gamma_{fd}-$set of G, that is, $\gamma\gamma_{fd}(G) = |C| = 2$. ■

A complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices. The following result is an immediate consequence of Theorem 2.7.

Corollary 2.8 *Let $G = K_n$ of order $n \geq 2$. Then $\gamma\gamma_{fd}(G) = 2$.*

Proof: Let $G = K_n$. Then $G = K_1 + K_{n-1}$ where $\gamma(K_{n-1}) = 1$. By Theorem 2.7, $\gamma\gamma_{fd}(G) = 2$. ■

The complete graph $G = K_n$ of order $n \geq 3$ is shown below (see Figure 6).

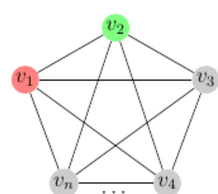

Figure 6: A graph with $\gamma_{pr}^{-1}(G) = 1$.

The set $D = \{v_1\}$ is a γ_{fd}-*set* of G and the set $S = \{v_2\}$ is γ_{fd}^{-1}-set of G with respect to D. Thus, $C = D \cup S = \{v_1, v_2\}$ is a $\gamma\gamma_{fd}$-*set* of G.

Corollary 2.9 *Let $G = P_2 + H$ where H be any graph. Then $\gamma\gamma_{fd}(G) = 2$.*

Proof: Let $V(P_2) = \{x, y\}$ and $I = \langle y \rangle + H$. Since $G = P_2 + H = \langle \{x, y\} \rangle + H = \langle x \rangle + (\langle y \rangle + H) = \langle x \rangle + I$, it follows that $\gamma\gamma_{fd}(G) = 2$ by Theorem 2.7. ■

Theorem 2.10 *Let $G = P_2 \circ H$ where H is a connected graph. Then $\gamma\gamma_{fd}(G) = 4$ if and only if $\gamma(H) = 1$.*

Proof: Suppose that $\gamma\gamma_{fd}(G) = 4$. Let $\gamma_{fd} - set\ of\ G$ be $D = V(P_2) = \{x, y\}$ and let γ_{fd}^{-1}-set of G with respect to D be $S = \bigcup_{v \epsilon V(P_2)} S_v$ where $S_v \subset V(H^v)$ is a fair dominating set of $v + H^v$ for all $v \in V(P_2)$. Since $|C| = \gamma\gamma_{fd}(G) = 4$, it follows that $4 = |C| = |D \cup S| = |D| + |S| = |V(P_2)| + |S| = 2 + |S|$, that is, $|S| = 2$. Hence, $2 = |S| = |\bigcup_{v \epsilon V(P_2)} S_v| = |S_x \cup S_y| = |S_x| + |S_y|$. This implies that $|S_x| = |S_y| = 1$. Since $S_x \subset V(H^x)$ is a fair dominating set of $x + H^x$ and hence a dominating set of H^x, it follows that $\gamma(H^x) = |S_x| = 1$ $(and\ \gamma(H^y) = 1)$. Thus, $\gamma(H) = 1$.

For the converse, suppose that $\gamma(H) = 1$. Let $S_v = \{v\}$ be a dominating set of H. Clearly, $S_v\ is\ a\ 1-$fair dominating set of $v + H^v$ where $v \in V(P_2)$. Thus $S = \bigcup_{v \epsilon V(P_2)} S_v = \{v_1, v_2\}$ is a fair dominating set of G. Similarly, $D = V(P_2)$ is a fair dominating set of G. Since $S \subseteq V(G) \setminus D$, it follows that S is an inverse fair dominating set of G with respect to D. Hence, $\gamma_{fd}(G) = |D| = 2$ and $\gamma_{fd}^{-1}(G) = |S| = 2$. Thus, $\gamma\gamma_{fd}(G) = |C| = |D \cup S| = |D| + |S| = 2 + 2 = 4$.■

The next results follow immediately from Theorem 2.10.

Corollary 2.11 Let $G = P_2 \circ K_n$ of order $n \geq 1$. Then $\gamma\gamma_{fd}(G) = 4$.

Proof: *Let* $G = P_2 \circ K_n$. Since K_n is a connected graph and $\gamma(K_n) = 1$, by Theorem 2.10, $\gamma\gamma_{fd}f(G) = 4$. ■

A star graph S_n is a tree of order $n + 1$ with maximum diameter 2; in which case a star of $n > 2$ has $n - 1$ leaves (see Figure 7).

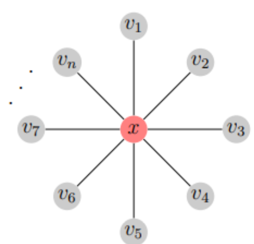

Figure 7: A star graph with S_n.

Corollary 2.12 *Let* $G = P_2 \circ S_n$ *where* $n \geq 2$. *Then* $\gamma\gamma_{fd}(G) = 4$.

Proof: Let $G = P_2 \circ S_n$. Since S_n is a connected graph and $\gamma(S_n) = 1$, by Theorem 2.10, $\gamma\gamma_{fd}(G) = 4$. ■

A wheel graph $W_n = \langle x \rangle + C_n$ is a graph formed by connecting a single universal vertex x to all vertices of a cycle C_n (see Figure 8).

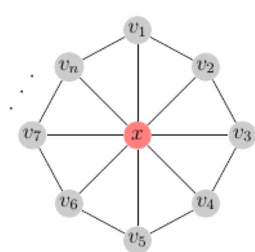

Figure 8: A star graph with $W_n = \langle x \rangle + C_n$.

Corollary 2.13. *Let* $G = P_2 \circ W_n$ *where* $n \geq 3$. *Then* $\gamma\gamma_{fd}(G) = 4$.

Proof: *Let* $G = P_2 \circ W_n$. Since W_n is a connected graph and $\gamma(W_n) = 1$, by Theorem 2.10, $\gamma\gamma_{fd}(G) = 4$.■

A complete bipartite $K_{m,n}$ is a graph whose vertices can be partitioned into two subsets V_1 of order m and V_2 or order n such that no edge has both endpoints in the same subset, and every possible edge that could connect vertices in different subsets is part of the graph (see Figure 9).

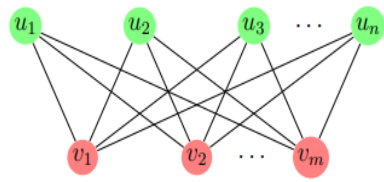

Figure 9: A star graph with $K_{m,n} = \overline{K}_m + \overline{K}_n$.

Theorem 2.14 *Let* $G = K_{m,n}$. *Then* $\gamma\gamma_{fd}(G) = |V(G)|$.

Proof: Refer to Figure 9, if $m < n$ then the set $D = \{v_i: i = 1,2,\ldots,m\}$ is a $\gamma_{fd} - set\ of\ G$ and the set $S = \{u_i: i = 1,2,\ldots,n\}$ is the $\gamma_{fd}^{-1} - set\ of\ G$. Thus, $\gamma\gamma_{fd}(G) = |C| = |D \cup S| = |D| + |S| = m + n = |V(G)|$. Similarly, if $m \geq n$, then $\gamma\gamma_{fd}(G) = |C| = |D \cup S| = |D| + |S| = n + m = |V(G)|$. ∎

III. CONCLUSION

In this paper, we introduced the concept of disjoint fair domination in graphs and prove that given positive integers k, m and n such that $2 \leq k \leq n$, where $n \geq 2$ there exists a connected graph G with $|V(G)| = n$, and $\gamma\gamma_{fd}(G) = k$.

Further, we prove the disjoint fair domination number of a cycle C_n graph and a complete bipartite $K_{m,n}$ graph. Prove the characterization of the disjoint fair domination number of a graph $G = \langle x \rangle + H\ and\ G = P_2 \circ H$ with $\gamma(H) = 1$. Some related problems on disjoint fair domination in graphs are still open for research.

1. Characterize the disjoint fair dominating sets of the join, corona, Cartesian product, and lexicographic product of two graphs.
2. Find the disjoint fair domination number of the join, corona, Cartesian product, and lexicographic product of two graphs.

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