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An Inventory Model with Deteriorated items, Demand Fractionally Dependent on Time, Variable Holding Costs, and All Other Parameters Constant

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Abstract *- The study proposes an inventory model for degrading items with a demand rate that rises to the power m by n as time passes. The cost of holding is proportional to time. All other parameters, such as the rate of degradation and the cost of ordering, are held constant. Shortages are permitted and fully backlogged. With the use of numerical illustration, this model is numerically confirmed. The model is numerically and graphically verified using the Maple 18 software.*

Keywords — *Fractional Polynomial, deterioration, demand, inventory.*

I. INTRODUCTION

Inventory is a type of product that must be managed in order to optimize profit. Shipping, purchasing, packaging, tracking, and a variety of other services are all part of inventory control. Inventory consists of two sorts of goods: those that decay over time, such as bread and fruits, and those that do not deteriorate over time if stored in a pleasant atmosphere, such as paper and cotton. Several studies assumed that the demand rate was constant, while others assumed that the demand rate was a linear or quadratic function of time. In certain goods, demand rates were discovered to be dependent on selling price, while others found it to be dependent on time. Harish Kumar Yadav and T. P. Singh (2019) [1] built an inventory model with time-dependent holding cost, deterioration rate to be Weibull distribution function, and variable demand rate. They formulated the model without shortages. Harish Kumar Yadav, T. P. Singh, and Vinod Kumar (2020) [2] developed an inventory model with demand quadrate dependent on time, partially backlogged shortages, and Weibull distribution deterioration rate. Abu Hashan Md Mashud (2020) [3] built the model for deteriorated items with three types of demand rate, namely linear function dependent on stock, dependent on price having negative power of the constant, and exponentially dependent on price. This system included fully backlogged shortages. Mandal and S. Phaujdhar [4] formulated an inventory model for decaying products where they considered demand rate depends on stock. S. K. Ghosh AND K. S. Chaudhuri [5] constructed an inventory model for deteriorating items having instantaneous supply, demand was a quadratelly time-varying, and a two-parameter Weibull distribution is chosen to represent the time decaying rate. Kuo-Lung Hou [6] constructed a model with linear demand and constant decaying rate and is applied to on-hand inventory. In the paper, an inventory model is formulated for decaying items which deals with a single item with demand rate as a function of time raise to power m by n and decaying rate is constant. The holding cost is considered to be dependent on time, and the shortage is allowed and fully blocked.

II. Formulation and Solution of Mathematical Model:

Assumptions and Notations:

Notations:

- D - Demand Rate.
- C_D - Deterioration Cost.
- A - The ordering cost.
- $H(t)$ –Holding Cost.
- Q - Order Quantity
- θ- Deterioration Rate.

- C - Purchase Cost per unit
- L - Order Quantity
- Ω - is the time at which the level of inventory reaches to zero
- T - The length of a cycle time.
- C_S -Shortage cost per unit time.
- TIC -Total inventory cost.
- I(t) - Inventory level.

Assumptions:

The inventory Model uses the following assumptions: -

- The demand rate is fractionally dependent on time which is of type m by n

$$D(t) = \alpha + \beta t^{\frac{m}{n}} \quad \text{and} \quad \alpha \geq 0,\ \beta \geq 0.$$

- Holding cost per unit time is constant

H(t) = $\omega + \eta t$, where $\omega, \eta \geq 0$.

- The Order quantity per cycle is Q.
- Shortages are allowed and are fully backlogged.
- The lead time is zero.
- The deterioration rate is

$$\theta(t) = \gamma, 0 < \gamma < 1$$

- Ordering cost per item is A
- The shortage cost per unit time is C_s
- This inventory Model deals with single item.
- Ω is the time where the inventory level reaches zero and $\Omega \geq 0$.
- TIC is assumed as total inventory cost.
- I(t) is assumed as inventory level.
- C is assumed as the purchasing cost per unit.

Mathematical Formulation

The inventory behavior is depicted in Figure 1 at any given period. This graph depicts the order quantity Q verses. time T.

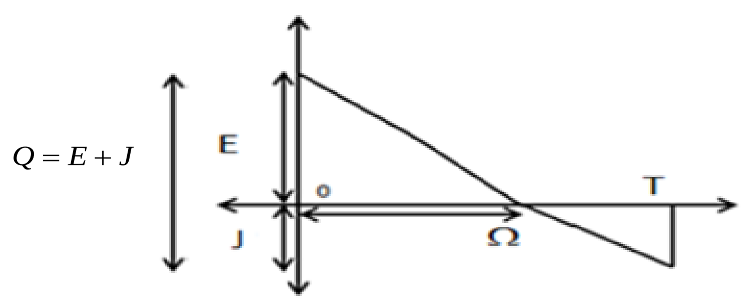

Figure 1. Inventory Level (Q) verses Time (T)

The inventory level is positive and decreasing in the period $(0,\Omega)$, but it hits zero at time $t=\Omega$, and a shortage begins, and inventory becomes negative in the period (Ω,T). The following is the differential equation for the instantaneous inventory level at time t:

$$\frac{dI_1(t)}{dt}+\gamma I(t)=-(\alpha+\beta t^{\frac{m}{n}});0\le t\le\Omega \quad (1)$$

$$\frac{dI_2(t)}{dt}=-(\alpha+\beta t^{\frac{m}{n}});\Omega\le t\le T \quad (2)$$

where m & n are natural numbers.

The model comprises of the following boundary conditions:

$I_1(t)=I_2(t)=0$, at $t=\Omega$

$I_2(t)=J$ at $t=T$ and $I_2(t)=E$ at t=0

When we solve equation (1) without taking into account higher powers of t, we get

$$I_1(t)=\begin{bmatrix}\alpha(\Omega-t)+\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}-t^{\frac{m+n}{n}}\right)+\frac{\alpha\gamma}{2}\left(\Omega^2-t^2\right)+\frac{n\beta\gamma}{m+2n}\left(\Omega^{\frac{m+2n}{n}}-t^{\frac{m+2n}{n}}\right)\\ +\frac{\alpha\gamma^2}{6}\left(\Omega^3-t^3\right)+\frac{n\beta\gamma^2}{2(m+3n)}\left(\Omega^{\frac{m+3n}{n}}-t^{\frac{m+3n}{n}}\right)-\alpha\gamma\left(\Omega t-t^2\right)-\frac{n\beta\gamma}{m+n}\left(\Omega^{\frac{m+n}{n}}t-t^{\frac{m+2n}{n}}\right)\\ -\frac{\alpha\gamma^2}{2}\left(\Omega^2t-t^3\right)-\frac{n\beta\gamma^2}{m+2n}\left(\Omega^{\frac{m+2n}{n}}t-t^{\frac{m+3n}{n}}\right)-\frac{\alpha\gamma^3}{6}\left(\Omega^3t-t^4\right)-\frac{n\beta\gamma^3}{2(m+3n)}\left(\Omega^{\frac{m+3n}{n}}t-t^{\frac{m+4n}{n}}\right)\\ +\frac{\alpha\gamma^2}{2}\left(\Omega t^2-t^3\right)+\frac{n\beta\gamma^2}{2(m+n)}\left(\Omega^{\frac{m+n}{n}}t^2-t^{\frac{m+3n}{n}}\right)+\frac{\alpha\gamma^3}{4}\left(\Omega^2t^2-t^4\right)\\ +\frac{n\beta\gamma^3}{2(m+2n)}\left(\Omega^{\frac{m+2n}{n}}t^2-t^{\frac{m+4n}{n}}\right)+\frac{\alpha\gamma^4}{12}\left(\Omega^3t^2-t^5\right)+\frac{n\beta\gamma^4}{4(m+3n)}\left(\Omega^{\frac{m+3n}{n}}t^2-t^{\frac{m+5n}{n}}\right)\end{bmatrix} \quad (3)$$

On solving (2), we get

$$I_2(t)=\alpha(\Omega-t)+\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}-t^{\frac{m+n}{n}}\right) \quad (4)$$

Now using the condition that $I_1(t)=E$ at $t=0$, it follows:

$$E=\alpha\Omega+\frac{n\beta}{m+n}\Omega^{\frac{m+n}{n}}+\frac{\alpha\gamma}{2}\Omega^2+\frac{n\beta\gamma}{m+2n}\Omega^{\frac{m+2n}{n}}+\frac{\alpha\gamma^2}{6}\Omega^3+\frac{n\beta\gamma^2}{2(m+3n)}\Omega^{\frac{m+3n}{n}}$$

Now using the condition that $I_2(t)=J$ at $t=0$, we get:

$$J=-\alpha(\Omega-T)-\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}-T^{\frac{m+n}{n}}\right)$$

Now the order quantity per cycle is given by:

Q = E +J

$$Q=\left[\begin{array}{l}\alpha\Omega+\frac{n\beta}{m+n}\Omega^{\frac{m+n}{n}}+\frac{\alpha\gamma}{2}\Omega^2+\frac{n\beta\gamma}{m+2n}\Omega^{\frac{m+2n}{n}}+\frac{\alpha\gamma^2}{6}\Omega^3\\+\frac{n\beta\gamma^2}{2(m+3n)}\Omega^{\frac{m+3n}{n}}-\alpha(\Omega-T)-\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}-T^{\frac{m+n}{n}}\right)\end{array}\right] \quad (5)$$

Now the inventory holding cost per cycle is given by:

$$IHC=\int_0^{\Omega}H(t)I_1(t)dt$$

$$IHC=\omega\left[\begin{array}{l}\frac{\alpha}{2}\Omega^2+\frac{n^2\beta}{(m+n)(m+2n)}\Omega^{\frac{m+2n}{n}}+\frac{\alpha\gamma}{6}\Omega^3+\frac{2n^2\beta\gamma}{(m+2n)(m+3n)}\Omega^{\frac{m+3n}{n}}+\frac{5\alpha\gamma^2}{12}\Omega^4\\+\frac{3n^2\beta\gamma^2}{2(m+3n)(m+4n)}\Omega^{\frac{m+4n}{n}}-\frac{n^2\beta\gamma}{(m+n)(m+3n)}\Omega^{\frac{m+3n}{n}}-\frac{2n^2\beta\gamma^2}{(m+2n)(m+4n)}\Omega^{\frac{m+4n}{n}}\\-\frac{1}{15}\alpha\gamma^3\Omega^5-\frac{3n^2\beta\gamma^3}{2(m+3n)(m+5n)}\Omega^{\frac{m+5n}{n}}+\frac{n^2\beta\gamma^2}{2(m+n)(m+4n)}\Omega^{\frac{m+4n}{n}}\\+\frac{n^2\beta\gamma^3}{(m+2n)(m+5n)}\Omega^{\frac{m+5n}{n}}+\frac{\alpha\gamma^4}{72}\Omega^6+\frac{3n^2\beta\gamma^4}{4(m+3n)(m+6n)}\Omega^{\frac{m+6n}{n}}\end{array}\right]$$

$$+\eta\left[\begin{array}{l}\frac{\alpha\Omega^3}{6}+\frac{n\beta\Omega^{\frac{m+3n}{n}}}{2(m+3n)}+\frac{\alpha\gamma\Omega^4}{8}+\frac{n\beta\gamma\Omega^{\frac{m+4n}{n}}}{2(m+4n)}+\frac{\alpha\gamma^2\Omega^5}{20}+\frac{n\beta\gamma^2\Omega^{\frac{m+5n}{n}}}{4(m+5n)}-\frac{\alpha\gamma\Omega^4}{12}\\-\frac{n\beta\gamma\Omega^{\frac{m+4n}{n}}}{3(m+4n)}-\frac{\alpha\gamma^2\Omega^5}{15}-\frac{n\beta\gamma^2\Omega^{\frac{m+5n}{n}}}{3(m+5n)}-\frac{\alpha\gamma^3\Omega^6}{36}-\frac{n\beta\gamma^3\Omega^{\frac{m+9n}{n}}}{6(m+6n)}\\+\frac{\alpha\gamma^2\Omega^5}{40}+\frac{n\beta\gamma^2\Omega^{\frac{m+5n}{n}}}{8(m+5n)}+\frac{\alpha\gamma^3\Omega^6}{48}+\frac{n\beta\gamma^3\Omega^{\frac{m+6n}{n}}}{8(m+6n)}+\frac{\alpha\gamma^4\Omega^7}{112}+\frac{n\beta\gamma^4\Omega^{\frac{m+7n}{n}}}{16(m+7n)}\end{array}\right] \quad (6)$$

Total amount of inventory shortage cost per unit time during the period (Ω,T) is given by:

$$ISC=-C_S\int_{\Omega}^{T}I_2(t)dt$$

$$ISC=-C_S\int_{\Omega}^{T}\left[\alpha(\Omega-t)+\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}-t^{\frac{m+n}{n}}\right)\right]dt$$

$$ISC=-C_S\left[\alpha\left(\Omega T-\frac{T^2}{2}-\Omega^2+\frac{\Omega^2}{2}\right)+\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}T-\frac{n}{m+2n}T^{\frac{m+2n}{n}}-\Omega^{\frac{m+2n}{n}}+\frac{m}{2n}\Omega^{\frac{m+2n}{n}}\right)\right]$$

$$ISC=-C_S\left[\alpha\left(\Omega T-\frac{T^2}{2}-\frac{\Omega^2}{2}\right)+\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}T-\frac{n}{m+2n}T^{\frac{m+2n}{n}}-\frac{(m+n)}{(m+2n)}\Omega^{\frac{m+2n}{n}}\right)\right] \quad (7)$$

Now, Inventory deterioration cost per item is given by:

$$IDC=C_D\left[E-\int_0^{\Omega}\left(\alpha+\beta t^{\frac{m}{n}}\right)dt\right]$$

$$IDC = C_D\left[\frac{\alpha\gamma\Omega^2}{2}+\frac{n\beta\gamma\Omega^{\frac{m+2n}{n}}}{m+2n}+\frac{\alpha\gamma^2\Omega^3}{2\times 3}+\frac{n\beta\gamma^2\Omega^{\frac{m+3n}{n}}}{2(m+3n)}\right] \quad (8)$$

The inventory ordering cost per order during (0,Ω) is given by:

IOC = A (9)

Now, the total cost per unit time for the cycle is given by:

$$TIC = \frac{1}{T}\left[\omega\left[\begin{array}{l}\frac{\alpha}{2}\Omega^2+\frac{n^2\beta}{(m+n)(m+2n)}\Omega^{\frac{m+2n}{n}}+\frac{\alpha\gamma}{6}\Omega^3+\frac{2n^2\beta\gamma}{(m+2n)(m+3n)}\Omega^{\frac{m+3n}{n}}+\frac{5\alpha\gamma^2}{12}\Omega^4\\ +\frac{3n^2\beta\gamma^2}{2(m+3n)(m+4n)}\Omega^{\frac{m+4n}{n}}-\frac{n^2\beta\gamma}{(m+n)(m+3n)}\Omega^{\frac{m+3n}{n}}-\frac{2n^2\beta\gamma^2}{(m+2n)(m+4n)}\Omega^{\frac{m+4n}{n}}\\ -\frac{1}{15}\alpha\gamma^3\Omega^5-\frac{3n^2\beta\gamma^3}{2(m+3n)(m+5n)}\Omega^{\frac{m+5n}{n}}+\frac{n^2\beta\gamma^2}{2(m+n)(m+4n)}\Omega^{\frac{m+4n}{n}}\\ +\frac{n^2\beta\gamma^3}{(m+2n)(m+5n)}\Omega^{\frac{m+5n}{n}}+\frac{\alpha\gamma^4}{72}\Omega^6+\frac{3n^2\beta\gamma^4}{4(m+3n)(m+6n)}\Omega^{\frac{m+6n}{n}}\end{array}\right]\right]$$

$$+\frac{1}{T}\left[\eta\left[\begin{array}{l}\frac{\alpha\Omega^3}{6}+\frac{n\beta\Omega^{\frac{m+3n}{n}}}{2(m+3n)}+\frac{\alpha\gamma\Omega^4}{8}+\frac{n\beta\gamma\Omega^{\frac{m+4n}{n}}}{2(m+4n)}+\frac{\alpha\gamma^2\Omega^5}{20}+\frac{n\beta\gamma^2\Omega^{\frac{m+5n}{n}}}{4(m+5n)}-\frac{\alpha\gamma\Omega^4}{12}\\ -\frac{n\beta\gamma\Omega^{\frac{m+4n}{n}}}{3(m+4n)}-\frac{\alpha\gamma^2\Omega^5}{15}-\frac{n\beta\gamma^2\Omega^{\frac{m+5n}{n}}}{3(m+5n)}-\frac{\alpha\gamma^3\Omega^6}{36}-\frac{n\beta\gamma^3\Omega^{\frac{m+9n}{n}}}{6(m+6n)}\\ +\frac{\alpha\gamma^2\Omega^5}{40}+\frac{n\beta\gamma^2\Omega^{\frac{m+5n}{n}}}{8(m+5n)}+\frac{\alpha\gamma^3\Omega^6}{48}+\frac{n\beta\gamma^3\Omega^{\frac{m+6n}{n}}}{8(m+6n)}+\frac{\alpha\gamma^4\Omega^7}{112}+\frac{n\beta\gamma^4\Omega^{\frac{m+7n}{n}}}{16(m+7n)}\end{array}\right]\right]$$

$$+\frac{1}{T}\left[\begin{array}{l}-C_S\left[\alpha\left(\Omega T-\frac{T^2}{2}-\frac{\Omega^2}{2}\right)+\frac{n\beta}{m+n}\left(\Omega^{\frac{m+n}{n}}T-\frac{n}{m+2n}T^{\frac{m+2n}{n}}-\frac{(m+n)}{(m+2n)}\Omega^{\frac{m+2n}{n}}\right)\right]\\ +A+C_D\left[\frac{\alpha\gamma\Omega^2}{2}+\frac{n\beta\gamma\Omega^{\frac{m+2n}{n}}}{m+2n}+\frac{\alpha\gamma^2\Omega^3}{2\times 3}+\frac{n\beta\gamma^2\Omega^{\frac{m+3n}{n}}}{2(m+3n)}\right]\end{array}\right] \quad (10)$$

In order to minimize the total inventory cost, we find the optimal values of T and Ω. The optimal values of T and Ω can be obtained by equating the partial derivatives of total inventory cost with respect to T and Ω respectively to zero.

For optimal value of Ω and T we have

$$\frac{\partial(TIC)}{\partial\Omega}=0 \quad (11)$$

and

$$\frac{\partial(TIC)}{\partial T}=0 \quad (12)$$

The total minimum cost per unit time TIC (T, Ω) satisfy by sufficient condition

$$\frac{\partial^2(TIC)}{\partial T^2}>0 \quad (13)$$

and

$$\frac{\partial^2(TIC)}{\partial\Omega^2} > 0 \qquad (14)$$

and

$$\frac{\partial^2(TIC)}{\partial T^2} * \frac{\partial^2(TIC)}{\partial\Omega^2} - \frac{\partial^2(TIC)}{\partial\Omega\partial T} > 0 \qquad (15)$$

We get the value of Ω by solving equation (11), and we can get the value of T by solving equation (12) and putting these values in equation (10). We get the values that fulfill the essential conditions (13), (14), and (15) to have the lowest cost per unit of time (15).

III. NUMERICAL ILLUSTRATION

Now, we'll look at a numerical example to see how the total inventory cost may be optimized. To solve the case, we utilized Maple 18 Mathematical software.

The inventory system's following parameters are used to quantitatively describe the model:

α= 14, β= 10, ϒ= 12, ω=10, η=4, C_S = 15, A = 500, C_D =10, m=3, n=4.

We get the optimal shortage value by the above given parameters by using Maple 18, Ω=0.2600697826, T = 8.005935389.

Finally, the calculated total optimum cost is TIC=30933.57638.

The graph is as shown in figure 1 and its three-dimensional representation is shown in figure 2.

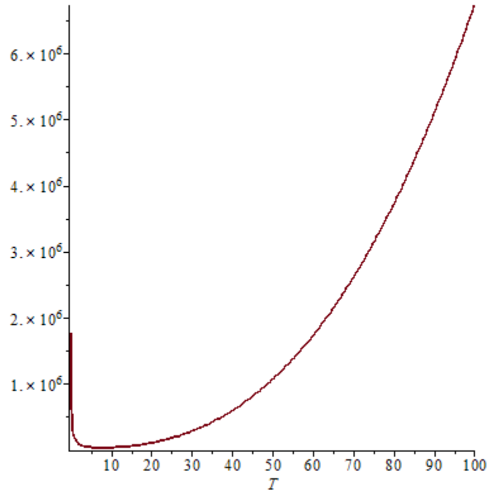

Figure1. shows total cost function verse time

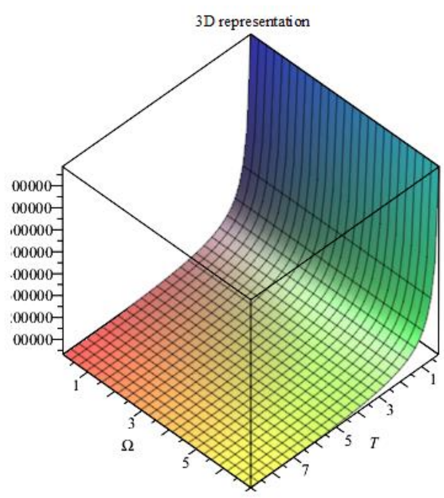

Figure 2. shows total cost function verses Ω and T

IV. CONCLUSIONS

A deterministic inventory model with demand dependent on time raised to the power m by n, constant decline rate, and flexible holding cost is constructed in this paper. The total optimal cost has been evaluated for the values of Ω and T, satisfying the necessary condition. The model has been tested using numerical illustrations and graphical diagrams. This model is helpful for the problems in which demand changes fractionally over time as commodity prices increase day by day. This model is more realistic to the environment. As a result, future cities and businesses will benefit greatly from the paradigm.

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