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Mathematical Modeling: A Useful Tool For Engineering Research And Practice

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Abstract - *Engineering outcome in operations, experimentation, characterization and performance in machining applications are usually in different forms of trends. The trends vary with conditions. Data analysis is pivotal in modeling the various trends that is primarily linear or non linear. The models enable the engineer to identify the characteristic of a particular system. In this work several examples have been developed based on different trend pattern in relation to supply and demand output prediction including mechanical, physical and electrical processes. Models enable engineers and researchers to carry out predicting and controlling functions base on previously conducted experiment and observation. The validity of models is dependent on minimal level of existing errors. Modeling approach can help in scrutinizing complex and bogus system at ease with the aid of computer programming. The applications of models in software development bring about the reduction in storage requirement, due to the fact that parameters are generated mathematically.*

Keywords - *Development Engineering Applications, Mathematical Tools, Models, Trend*

I. INTRODUCTION

Modeling is considered more as an art other than a science and depends heavily on the experience and knowledge of the researchers involved. It requires both a good understanding of the nature of the process and a familiarity with the available models and methods. In some practical cases, the phenomena under study are of very high complexity, and any mathematical description is just a modest approximation. Unavoidable simplifications and approximations made during the modeling process can greatly alter the predicted behavior of the real-world phenomena. Applied mathematical modeling does not make sense without first defining the purpose of modeling. Before a model is developed, a specific existing problem should be described and the possible implications and benefits of the model need to be explored.

The modeling goals have to be realistic but not oversimplified. In general, the level of complexity of mathematical models varies substantially for different applied problems. There are no universal mathematical models and modeling methods in applied science and engineering. Numerous models of various types and complexity have been constructed to resolve specific practical problems. Some of these models appear to be appropriate for a wider area and, after thorough investigation and acceptance, contribute to the classical stock of mathematical modeling.

Many mathematical models of engineering systems have become standard calculation techniques, for example, in the study of the strength of materials, heat flows, resource consumption, construction design, and other engineering activities. Sometimes the same model describes different real-life processes in various areas. Finding analogies between mathematical models in different applied areas is always beneficial for the development of mathematical techniques and for all areas of modeling. Many good texts in applied mathematical modeling have been written during the last decades. It is worthwhile to describe a place of the present book among them.

Works on mathematical modeling can be considered in three categories. First, there are model-oriented where existing mathematical models are classified and listed by their categories. These books are characterized by brevity, wideness, an encyclopedic style, and extended collections of modeling examples and applications. The next type, application-oriented is represented by a larger number of publications, dedicated to different areas of engineering, for example, continuum mechanical systems, oscillation processes, elasticity theory, bioengineering, various industrial applications, and so on. The last and the most common category consist of method-oriented books, with examples of for modeling with differential equations, integral equations, for wave theory and so on.

The primary goal behind using the data models are:

Ensure that the data required by the database are accurately represented. A data model helps design the database at the conceptual, physical and logical levels. The structure of the data model helps to define the relational tables, primary and foreign keys and stored procedures.

It provides a clear picture of the database and can be used by database developers to create a physical database making it easy to identify missing and redundant data.

Advantages of Data Modeling:

Higher application quality

Data modeling process is the most effective way to gather correct and complete research/business data to ensure that the system will operate in the intended manner. The process generates more questions than any other modeling approach, leading to higher integrity. And its visual nature facilitates communication and collaboration between users and subject matter experts.

Reduced cost:

Data modeling catches errors and oversights early, when they are easy to fix. This is better than fixing errors once the software has been written or – worse yet – is in customer hands. In an Acute developing environment, development costs can be reduced significantly because a good data model will reveal the upfront otherwise, unknown or unanticipated requirements.

Quicker time to market:

Proper data modeling brings about automation of some tasks; design tools can take a model as an input and generate the initial database structure, as well as some data access code. The development by using data models as a guide for writing codes will develop database that is 10 times faster than conventional ETL (extract, transform, load) programming code.

Clearer scope:

A data model provides a focus for determining scope. It provides something tangible to help business sponsors and developers agree over precisely what is included with the software and what is omitted. The model provides a nucleus for reaching an agreement.

Improved data quality:

A good data model defines the metadata so the data itself can be properly understood, queried, and reported on. To truly leverage the power and flexibility of databases, it is important to ensure the enforcement of domain definitions, field constraints, editing rules and integrity of relationships.

Better performance:

A sound model simplifies database tuning. A well-constructed database typically runs fast, often quicker than expected. Data modeling provides DBAs with the means to understand the database and tune it for fast performance.

Business intelligence:

Data modeling would guide you to be successful in Business Intelligence (BI). It is predominant that the process is business-centered. It begins with the clear understanding of the business, its purposes, and how the data will be used to support the business.

Fewer errors:

A data model causes participants to well define concepts and resolve confusion. As a result, application development starts with a clear vision. Developers could still make detailed errors as they write application code, but they are less likely to make deep errors that are difficult to resolve (Michael, 2021).

Preliminary Consideration before Application of Modeling Tools.

First understand the types of data the researcher/enterprise intends to analyze and, by extension the data integration requirements. Data analysis begins with the data sources selection that is the table's rows and columns. Replicate the selected data to a data warehouse to create a single source of truth for analysis. Create room for assess data security and data governance as well. Shared data application between departments should have access control and permission systems to protect sensitive information.

Selection of Data Analysis Tool

Once you have data ready, analyzing can be carried out using different tools. A good fit for consideration is base on the research/company work. This commences with the research goals/organization's objectives. Analysts should consider the tool to ascertain its comprehensibility. Data sophistication should be considered in line with the data scientist's skill. Non technical users will need an intuitive interface to ease the usage of the propose tool

Check for platforms that provide an interactive experience for iterating on code development — typically using SQL. Point-and-click analysis can also be use for less technical users. The tool should also provide support for visualizations relevant to your enterprise. Consider a tool's data modeling capabilities. Finally, consider price and licensing. Some offerings are free, while others charge licensing or subscription fees. The most expensive tools are not necessarily the most feature-complete, and users should not ignore the many robust free solutions available.

List of Top 25 Tools for Data Analysis
The top 25 data analysis tools are Microsoft Power BI, SAP BusinessObjects, Sisense, TIBCO Spotfire, Thoughtspot, Qlik, SAS Business Intelligence, Tableau, Google Data Studio, Grafana, Redash, Periscope Data, Metabase, Jupyter Notebook, IBM Cognos, Chartio, Mode, KNIME, Looker, RapidMiner, Domo, Oracle Analytics Cloud, R, Python and Excel.

This presentation is to acquaint the end users with mathematical model development based on data graphical trend. Samples of practical application of graphical trend in the models development have been highlighted. Creation of realistic assumptions is essential in developing models suitable for real life phenomena. Data analysis skills to a wide range of field such as banking, finance, epidemiology, medical, telecommunication, engineering, educational etc Application of models to solving real life problems as in stocking of goods, controlling of devices, calibrating measuring devices and so on. The applications of models in software development bring about the reduction in storage requirement, due to the fact that parameters are generated mathematically (Jigsawacadeny, 2021).

II. ENGINEERING PRACTICES

Engineering practice often drives investigation of new features of engineering systems, which requires either the development of new models or a substantial modification of known models (often, by using a new mathematical technique). The authors point out examples of slightly modified problems that lead to changes in the equation type. The mathematical nature of the derived equations is an important but secondary issue as compared with accounting for additional realistic assumptions. Most modeling methodology mainly related to continuous deterministic models, although some as to do with the combination of deterministic and stochastic models.

Deterministic vs. stochastic models
• **Deterministic** models, the output of the model is fully determined by the parameter values and the initial conditions.
• **Stochastic** models possess some inherent randomness. The same set of parameter values and initial conditions will lead to an ensemble of different outputs.
• Obviously, the natural world is buffeted by stochasticity. But, stochastic models are considerably more complicated.

The models use various mathematical tools, including differential, difference and integral equations, the calculus of variations and optimal control theory. Special attention is paid to the authors’ favorite mathematical tool – integral equations. Integral equations represent a more general tool than differential equations. The modification of an applied engineering problem often means a transformation from specific techniques based on differential equations to more general integral models. However, the authors are careful not to overestimate the importance of integral equations. The range of applications covered includes mechanical, physical and electrical processes. Varieties of models development are considered in some possible engineering applications. In this presentation our concentration is limited to linear and non linear modeling examples.

III. INSTRUMENTATION ENGINEERING DESIGN OF AN IMPROVED WEAR TESTING MACHINE

Performance evaluation of a wear testing equipment; The volume of wear debris generated as shown in Table 1 for a period of 450 seconds in conjunction with brinell hardness value, establishes a relationship of wear ratio in various materials (Ejiko et al., 2010).

Table.1: Relationship between Standard Brinell Hardness number and the volume of wear debris.

Materials	**Volume of wear (cm^3)**	***Standard Brinell Hardness number (HB)***
Stainless Steel	1.57	200
Mild Steel	3.35	120
Aluminum	15.67	16

Table.2: Comparison of standard test Ratios against test result ratio.

Test Ratio	**Standard Ratio**
1.57:3.35:15.67	0.005:0.0083:0.0625
1:2.13:10	5:8.3:62.5

Relationship between volume of wear and Standard Brinell hardness number.

Hardness is the resistance to wearing effect. The trend of the wearing effect increases with decrease to the hardness value. The test ratio of the selected materials is in ramp trend, to establish the volume of wear mathematically various divisible factors would be consider. Based on the ratio consideration an inversely proportional relationship to the hardness of the materials was considered most appropriate in relation to the single factor.

$\therefore$Vol. of wear, $W \alpha \frac{1}{H}$ this implies that $W = \frac{k}{H}$ (1)

Table 3: Ratio of standard test with test result.

Materials	Test Ratio	Standard Ratio
Stainless Steel	1	1
Mild Steel	2.13	1.66
Aluminum	10	12.5

Ejiko et. al., (2010) developed a model as given in equation 1 can be use to calibrate the wearing effect based on the materials hardness value.

IV. PREDICTING USING LEAST SQUARE MODEL FOR SALES OUTPUT OF 7UP BOTTLING COMPANY PLC

In establishing the least square model, equations 2 and 3 are applied in conjunction with the parameters of previous data generated from the company (Ejiko et al., 2015). The assumption taken is this scenario is that the sale trend is linearly related on quarterly basis, hence the application of least square regression and time series analysis

$$b = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2} \quad (2)$$

b = 11.05

$$a = \frac{\sum y\left(\sum x^2\right) - \left(\sum x\right)\left(\sum xy\right)}{n\sum x^2 - \left(\sum x\right)^2} \quad (3)$$

a = 419

The straight line equation as given by Oladebeye and Ejiko, (2007); Kareem and Ejiko, (2013), in equation 4 was established in equation 5.

$$y = a + bx \quad (4)$$

y = 419 + 11.05x (5)

where x is the indicator for quarter 1 to 12

Table 4: Parameters to established seasonal fluctuation factor

QUARTER	YR 1	YR 2	YR 3	SEASONAL FACTOR(C)
1	116.3	92.8	100.3	1.03
2	104.3	94.8	105.8	1.02
3	106.1	96.7	107.3	1.03
4	99.3	98.5	108.8	1.02

To predict the future sales output for the next four quarter, seasonal fluctuation factors were generated and used as a multiplying factors to determine the quarterly sales values from yc = 419+11.05x. This analysis is applicable for marketing situation especially in relation to linear and seasonal fluctuation occurrence.

V. DEVELOPMENT OF REGRESSION MODELS FOR APPROPRIATE BATTERY BANKING DETERMINATION IN SOLAR POWER SYSTEM (ELECTRICAL LOADING PURPOSE)

DEVELOPMENT OF MODELS

Table 5 was generated from the theoretical models base on specific load with an interval of 10 watt increment. This is achieved by using a mathematical formula to determine the specification of the component of a battery banking system

(Ejiko and Olaniyi, 2018):. It is such that, when the parameters needed for each formula are available, the capacity of each solar banking component can be determined using their respective equation as presented in Table 5.

Calculations by Formula

$C_p = R[\frac{C}{R}]^n$ mmmm (6)

where

C = capacity of battery at specified hour rating

n = Peukert constant (1.3)

R = The hour rating (10hrs)

Table 5: Load against PV panel

Load	50	60	70	80	90	100
PV panel (W)	120	144	168	192	216	240

load × peak sun hour × shading × derate (7)

where, Peak sun = 4hrs

Shading = 0.8

Derate = 0.75

Table 6: Load against charge controller

Load	50	60	70	80	90	100
Charge Con(A)	156	187.2	218.4	249.6	280.8	312

Total short circuit current of PV array × 1.3 (8)

Table 7: Load against inverter

Load	50	60	70	80	90	100
Inverter (w)	14.58	15.45	16.1	16.76	17.4	17.79

$B = 2.37L – 39.1666667$ (9)

$I = 0.06688571L + 11.3585714$ (10)

$CC = 3.12L$ (11)

$PV = 2.4L$ (12)

$P = 2.108L + 9.2$ (13)

Where:

B = Battery specification

I = Inverter specification

CC = Charge controller specification

PV = Photovoltaic panel specification

P = Price

L = Load

VI. CARBOHYDRATE BASED OPTIMIZATION MODEL DEVELOPMENT FOR BIOGAS PRODUCTION USING TRIPARTITE MIXTURE OF COWDUNG, PIGDUNG AND POLTRY DROPING (AGRICULTURAL SCIENCES)

Table 8: Quantity of Methane Gas Produced for ANOVA Analysis

Cow	Poul	Pig	Days	CH_2O	Protein	Q m^3
1	2	3	17	31.17	9.2	0.00126
1	2	3	24	31.17	9.2	0.00101
2	3	1	25	26.33	7.8	0.00164
2	3	1	28	26.33	7.8	0.000882
3	1	2	29	19.5	8	0.000882
3	1	2	31	19.5	8	0.001387

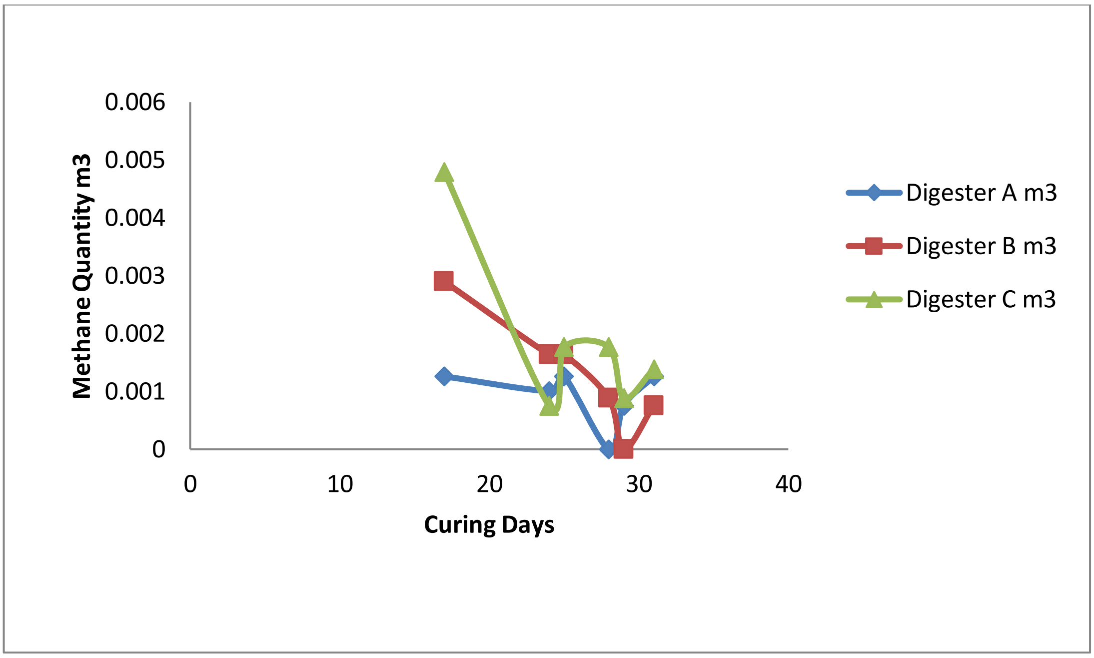

Fig 1: Ignitable Methane Gas Production Data Day 17 to 31

SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.757684194
R Square	0.574085337
Adjusted R Square	0.268532195
Standard Error	0.000780466
Observations	18

Methane Quant. (m^3) = 0.00001399Pi + 0.0001529D – 0.00008402Ch + 0.0074253 (14)

Here an application of a multiple regression was utilized so as to capture all the weighty parameters that has effect in the quantity of biogas (methane) produced. Equation shows that gas yield is dependent on the protein, carbohydrate and curing days accommodated in the digester (Ejiko *et al.*, 2020b).

VII. DEVELOPMENT OF A FOURIER SERIES FORECASTING MODEL FOR PREDICTING THE SALES VOLUME OF SELECTED MANUFACTURING COMPANY

This is a case of marketing that requires the application of Linear and non linear models, which led to the development of a Fourier series forecasting model.

The graph in Fig. 2 is the representation of sales volume against time in quarterly basis for a period of 7 years.

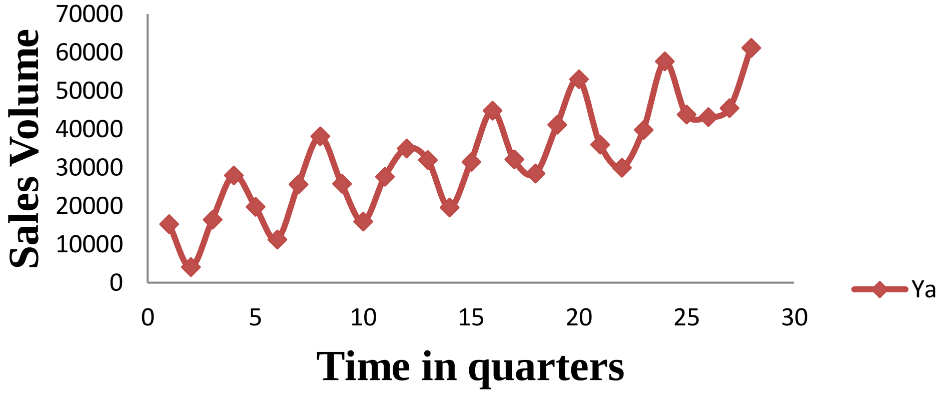

Fig.2: Actual sales volume(Ya) against time for year 2001 to 2007

The first period gives a Upper limit = 28000 represent ‘ d ’ and Lower limit = 4000 represent ‘ b ’
From the observed graph the occurrence of sales tends to replicate itself in an upward trend. The maximum point for the first periods is ‘ d ’ and minimum point is ‘ b ’.
The graph shows a sinusoidal function therefore for continuous modeling of the system. The cosine x was integrated using Fourier series method.

Equation $\int \cos x \, dx$ where, x = (-3,-2,-1,0,1,2,3) , y = (0,-1,0,1,0,-1,0)

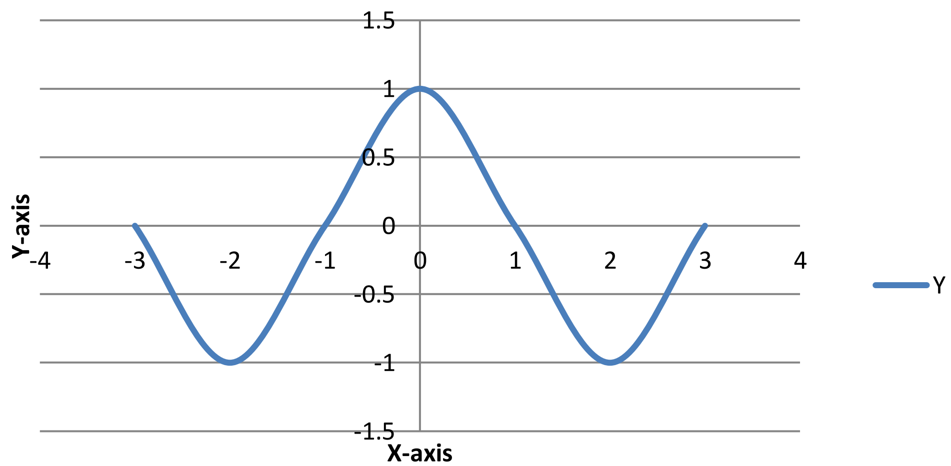

Fig.3: Graph Of Cosine x Function Of Fourier Series

The upper boundary = 1 and lower boundary = -1 on y-axis
In this section, $f(x)$ denotes a function of the real variable x. This function is usually taken to be periodic, of period 2π, which is to say that $f(x + 2\pi) = f(x)$, for all real numbers x. We will attempt to write such a function as an infinite sum, or series of simpler 2π–periodic functions. We will start by using an infinite sum of sine and cosine functions on the interval $[-\pi, \pi]$, as Fourier did.

Fourier's formula for 2π-periodic functions using sines and cosines
For a periodic function $f(x)$ that is integral on $[-\pi, \pi]$, the numbers

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\partial x$, n ≥ 0 and $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\partial x$, n ≥ 1 are called the Fourier coefficients of *f*. One introduces the partial sums of the Fourier series for *f*, often denoted by (SnF) = $\frac{a_0}{2} + \sum_{n=1}^{N}\left[a_n \cos(nx) + b_n \sin(nx)\right]$, N ≥ 0.

The partial sums for *f* are trigonometric polynomial. One expects that the function (SnF) approximate the function *f*, and that the approximation improves as N tends to infinity. The infinite sum $\frac{a_0}{2} + \sum_{n=1}^{N}\left[a_n \cos(nx) + b_n \sin(nx)\right]$ is called the Fourier series of *f*.

This constant falls within the positive and negative range. To bring the constant magnitude to the positives portion of the graph, a magnitude greater than the midpoint value will be added to balance the negative effect.

Let the value be "U "

Therefore

$$u + y = d$$

$$u - y = b$$

Combining the simultaneous equations above 'U 'becomes

$$U = \frac{d+b}{2} \qquad (15)$$

$$c = u + \left(\frac{d-b}{2}\right)\sum_{n=i=0}^{\infty\leftrightarrow 0}\sin nx \qquad (16)$$

$$c = \left(\frac{d+b}{2}\right) + \left(\frac{d-b}{2}\right)\sum_{n=i=0}^{\infty\leftrightarrow 0}\sin nx \qquad (17)$$

Therefore, the straight line equation is

$$Y = mx + c$$

$$Y = mx + \left(\frac{d+b}{2}\right) - \left(\frac{d-b}{2}\right)\sum_{n=i=0}^{\infty\leftrightarrow 0}\sin nx \qquad (18)$$

Where, m = slope of observed graph

$$m = \frac{e-a}{x_5 - x_1} \qquad (19)$$

Hence, the developed model is $Y_m = \left[\frac{e-a}{x_5 - x_1}\right]x + \left(\frac{d+b}{2}\right) - \left(\frac{d-b}{2}\right)\sum_{n=i=0}^{\infty\leftrightarrow 0}\sin nx$

(20)

From the observed graph in fig.2,

a = 15200 e = 19815 $x_1 = 1$

d = 28000 b = 4000 $x_5 = 5$

The slope $m = \frac{e-a}{x_5 - x_1}$

$$m = 1153.75 \approx 1153.80$$

So, $Y = 1153.8x + (\frac{2800+4000}{2}) - (\frac{2800-4000}{2}) \sum_{n=i=0}^{\infty \leftrightarrow 0} \sin nx$

$$Y_m = 1153.8x + 1600 - 12000 \sum_{n=i=0}^{\infty \leftrightarrow 0} \sin nx \quad (21)$$

Based on the data collected from the HONDA Motorcycle sales outlet in Apapa and the interaction with the sales manager it was observed that accurate forecasting have not been practiced. The application of the developed model in comparison with the actual sales volume (Y_a) and (Y_m) was found to be closely related. To further confirm the veracity of the developed model the correlation coefficient for the developed model and the theoretical sales volume was found to be 0.98 and 0.79 respectively. This signifies that the developed model is highly dependable than the previous theoretical model.

VIII. MATLAB MODELING OF ROBOTIC MANIPULATIVE MOVEMENT FOR EFFECTIVE WELDING PERFORMANCE AND PROCESS AUTOMATION

Developed numerical model

Models are generated by analyzing various parameters, this involve using the available parameters. Mathematical model was developed with the aid of Matlab software to determine the weld time in minutes (Oladebeye et al., 2020). In this work the dependent and independent parameters needs to be clearly stated.

Dependent parameter

- Time

Independent parameter

- Speed of weld
- Depth of weld
- Width of weld
- Power required
- Gauge of electrode

Ƿ = Density in kg/mm^3
A= Weld area for a single pass in mm^2
S= Speed in mm/min
ἰ = Welding current in amps
d= Diameter of electrode in mm
L_1= Stick out of electrode in mm
L_2= Weld length in mm
T= Weld time or time required to perform the welding operation in min
MR= Melting rate in kg/min
the melting rate is given by Singh (2010) in equation 3

$$\mathbf{MR} = \frac{1}{10^3} 0.35 + \frac{d^2}{645} 2.08 \times 10^{-7} \left(\frac{ἰ L_1 * 25.4}{d^2}\right)^{1.22} \mathbf{kg/min} \quad (22)$$

since the melting rate is given in kg/min a model is required to establish the time in minutes (min)

MR= $\frac{kg}{min}$

Recall mass (m) is in kg

density (Ƿ) = $\frac{m}{v}$ (23)

Mass (m) = ǷV

Therefore, MR = $\frac{ǷV}{T} = \frac{kg}{min}$ (24)

But V = AL_2 (25)

where

A = Cross section of weld to be determined by weld type and thickness,

L_2 = Length of weld or weld length (Taking all dimensions for length in mm)

MR=$\frac{ǷV}{T}$=$\frac{\text{Weld volume} * \text{Density}}{\text{Time}}$ (26)

MR=$\frac{ǷAL_2}{T}$ (27)

But L_2/t= Speed (S) in mm/min

$$\frac{L_2}{T}=\frac{MR}{\rho A} \qquad (28)$$

But we are looking for time in minutes therefore, from $L_2/t=\frac{MR}{\rho A}$

$$\frac{MR}{\rho A}=\frac{10^{-3}\left[0.35+\frac{d^2}{645}+2.08\times10^{-7}\left(\frac{iL_1\times25.4}{d^2}\right)^{1.22}\right]}{\rho A} \qquad (29)$$

Therefore, $MRT=\rho AL_2$, where

$$T=\frac{\rho AL_2}{MR}$$

$$T=\frac{\rho AL_2}{10^{-3}\left[0.35+\frac{d^2}{645}+2.08\times10^{-7}\left(\frac{iL_1\times25.4}{d^2}\right)^{1.22}\right]} \qquad (30)$$

IX. EVALUATION OF ADO-EKITI METROPOLIS SANDY SOIL COMPRESSIVE STRENGTH USING DEVELOPED HYPERBOLIC REGRESSION MODEL (CIVIL ENGINEERING APPLICATION)

The data collected from running the experiment as highlighted by Olubayode and Akinwamide, (2014) were given in Table 9.

Table 9: Summary of Laboratory Work

Sand deposite location	Silt performed on samples %	Density (Kg/m^3)	Impact %	Abrasion%	Absorption%	Compressive Strength (N/mm^2)		
						3	7	14
Iworoko	6.7	1820	10.3	1.75	11.5	1.3	1.8	2.0
Ikere	6.5	1650	12.27	1.99	13.2	1.0	1.2	1.4
Ijan	5.3	1800	13.24	2.34	12.9	1.0	1.3	1.5
Afao	5	1950	9.57	1.73	10.4	2.1	2.4	2.6
Ilawe	8	1640	18.7	2.56	13.9	0.5	1.0	1.3

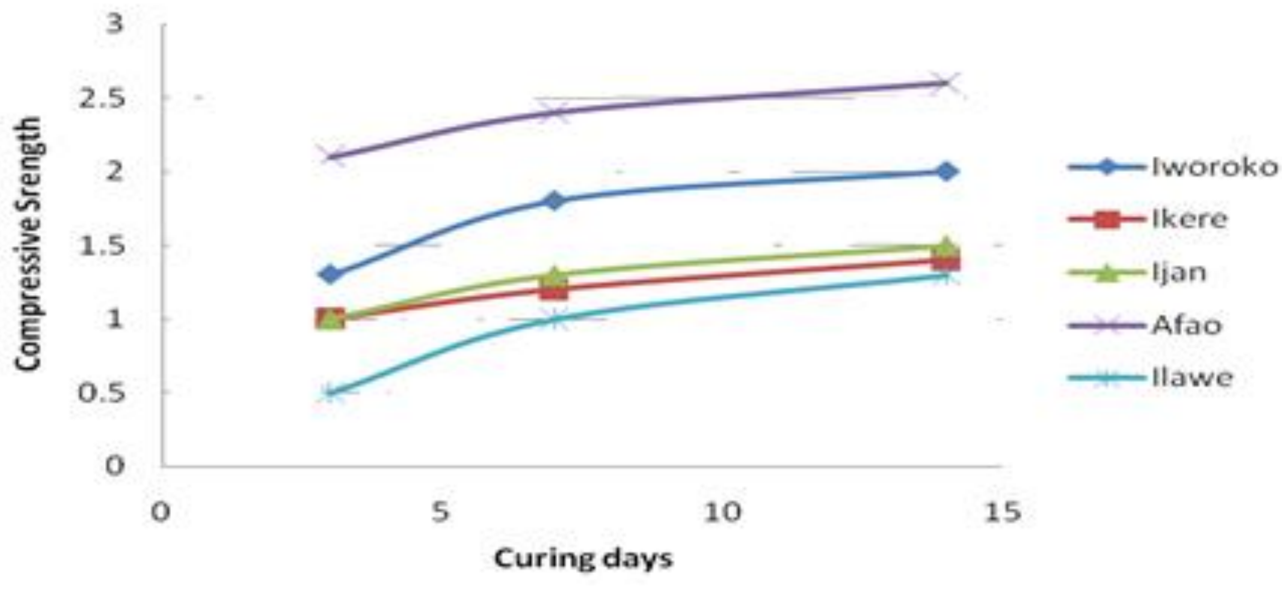

Figure 4: Graph of compressive strength against curing Age (days)

The graph presented in Figure. 4 of Ado-Iworoko soil shows the relationship that exist between Age of curing to the compressive strength of sandcrete block. These relationships can be use to establish mathematically from the following graphs of Figures 4 and 5.

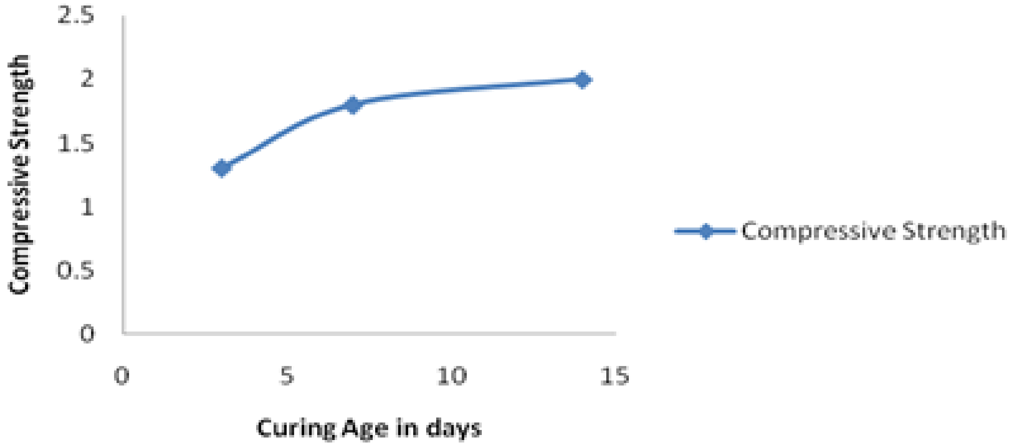

Figure 5: Ado-Iworoko Soil Relationship between Curing Days and Compressive Strength

The trend of the compressive strength is synonymous to the graph of half side tanh x, (hyperbolic function). The full presentation of a hyperbolic function can be seen in Fig. 6 with the inclusion of a lower and upper half.

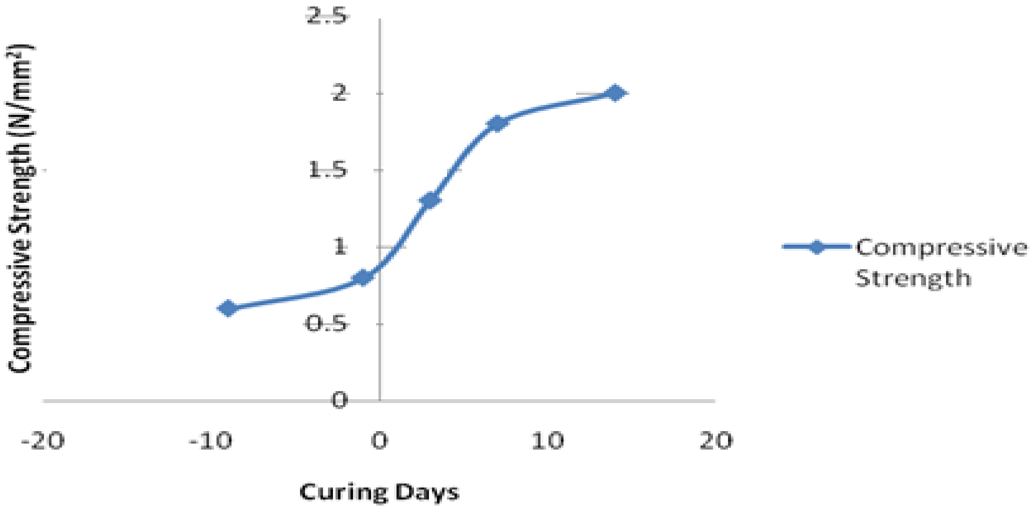

Figure 6: Graph of Hyperbolic Function with Specified Magnitude

The trend $Y = \tanh x$ (Bird, 2002; Ejiko *et al.*, 2019; Ejiko *et al.*, 2020a)

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \tag{31}$$

by representing Y as a function of x that is f(x) and x as nx in order to accommodate the boundary condition the equation becomes;

$$f_{(x)} = \tanh nx$$

This function has an upper and lower boundary of +1 and -1 which follow after the compressive strength format that is deficient of the magnitude. To complement the magnitude Y is given as a function of trend and magnitude.

$$Y = AC \tag{32}$$

Where,
C = arbitrary constant as tanh nx
A= magnitude of the center of compressive strength occurrence
The magnitude lies below the upper limit, c and a
Where the midpoint is at “A”

$$A = \frac{c - a}{2} \tag{33}$$

$$Y = \frac{c-a}{2}\tanh nx \tag{34}$$

To establish the constant value in the positive side, a magnitude greater than the mid point will be to balance the negative effect (Ejiko *et al.*, 2015a).
This value therefore becomes

$u + y = c$,

$u - y = a$,

$2u = c + a$,

$u = \frac{c+a}{2}$

$$\sigma = \left[\left(\frac{c+a}{2}\right)+\left(\frac{c-a}{2}\right)\right]\tanh nx \tag{35}$$

f(x) = tanh nx is for a period of 22 days (11days/section)
22days = 6x (from -3 to +3)

For $-8 \le x \le 14$,

$nx = -3$,

where $x = -8$,

and $\tanh(-3) = -1$

$22n = 6x$,

$n = \frac{6x}{22}$

Since $= -3$, $\boldsymbol{and}\ \mathbf{tanh}(-\mathbf{3}) = -\mathbf{1}, \rightarrow \boldsymbol{nx} = -\mathbf{3}$

$\frac{6x}{22} = -3$

Substituting x as -8 in the above equation

$n = \frac{6 \times -8}{22} = -3$

To attain a value that will bring nx to -3 an addition of +3 is required

$-2.18 + 3 = 0.82$

For $\boldsymbol{nx\ to\ be\ equal\ to} - \mathbf{3}$

$\frac{\boldsymbol{6x}}{\mathbf{22}} - \mathbf{0.82}$

$$\boldsymbol{\sigma} = \frac{\boldsymbol{c+a}}{\mathbf{2}} + \frac{\boldsymbol{c-a}}{\mathbf{2}}\mathbf{tanh}\left(\frac{\boldsymbol{6x}}{\mathbf{22}} - \mathbf{0.82}\right) \tag{36}$$

$\frac{\mathbf{2+0.6}}{\mathbf{2}} + \frac{\mathbf{2-0.6}}{\mathbf{2}}$

$\boldsymbol{\sigma} = \mathbf{1.3} + \mathbf{0.7}\boldsymbol{tanh}\left(\frac{\boldsymbol{6x}}{\mathbf{22}} - \mathbf{0.82}\right)$

where $\boldsymbol{x}$ is given as day D

$$\sigma = 1.3 + 0.7\tanh\left(\frac{6D}{22} - 0.82\right) \tag{37}$$

Table 10: Density against Compressive Strength

Density (x)	Compressive strength N/mm^2
1820	1.3
1650	1
1800	1
1950	2.05
1640	0.5

Using regressive model according to Oladebeye and Ejiko, 2007 and 2015

$$y = ax + b$$

where,
a is the coefficient of x
b is the constant (Point of interception)

$$y = 0.0039848X - 5.8911$$

where,
X is the density
Y is the compressive strength $\left(N/mm^2 \right)$

By regression analysis at 95% confidential level the correction coefficient is 0.906 which implies a high degree if correlation. The final equation to determine the compressive strength is given in equation. The regression equation introduced establishes the impact of density against the compressive strength. This therefore eliminates the impact of magnitude in equation 9 that is the removal of 1.3. The new equation can now be written as

$$\sigma = 0.003948x - 5.8911 + 0.75\tanh\left(\frac{6D}{22} - 0.82\right) \quad (38)$$

X. AZIMUTH AND ALTITUDE DETERMINATION FOR SOLAR TRACKING

One method of deriving of the sun-tracking formula is based on the sun's position vector description and the collector's normal vector in the same coordinate reference frame, which is the collector-center frame. Nevertheless, the unit vector of the sun's position is usually described in the earth-center frame due to the sun's daily and yearly rotational movements relative to the earth. Ejiko *et. al.,* (2015) and (2019) utilized the data collated to developed seven equations in determining the sun's position, where equations (39) - (43), (44) - (45) are to Azimuth and Altitude concerning the hourly, daily conditions on yearly basis respectively for Ekiti state.

The azimuth and altitude angle obtained at 7.00am was plotted against the days to establish the trend on daily basis in consideration of year 2014, while the subsequent years were considered as having similar periodic trend as presented in Fig. 7.

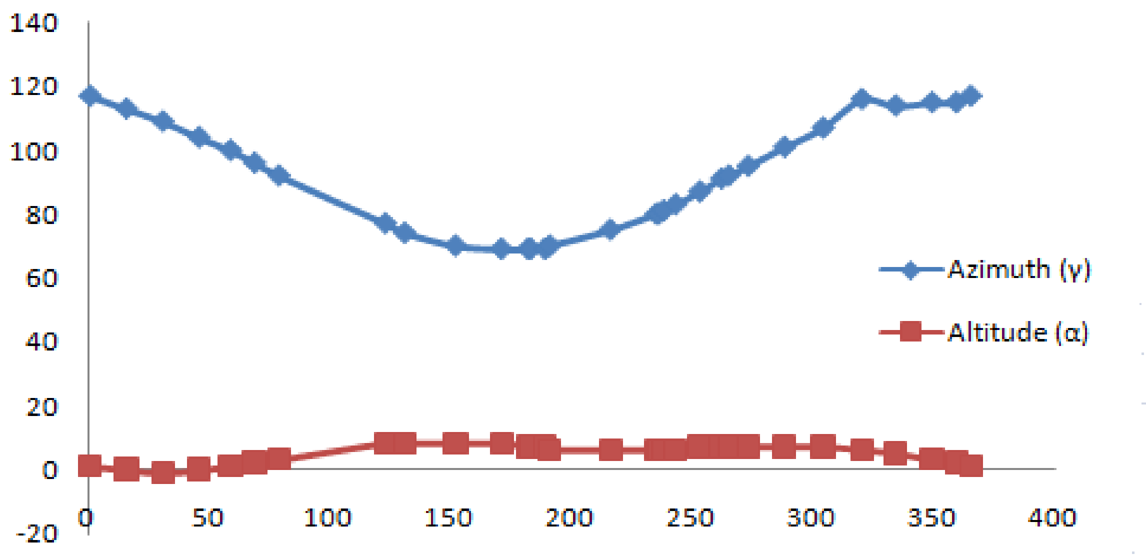

Fig 7: Azimuth and Altitude Angle for 2014 Jan. 1 to Dec. 2014 from Experiment.

The period from September 22 (September equinox) to March 21, (March equinox) no insolation is received at 90 degrees North. During this period the Sun slips below the horizon as the Northern axis of the earth has an orientation that is tilted away from the Sun. The azimuth at this period takes a special format that is unique to his season.

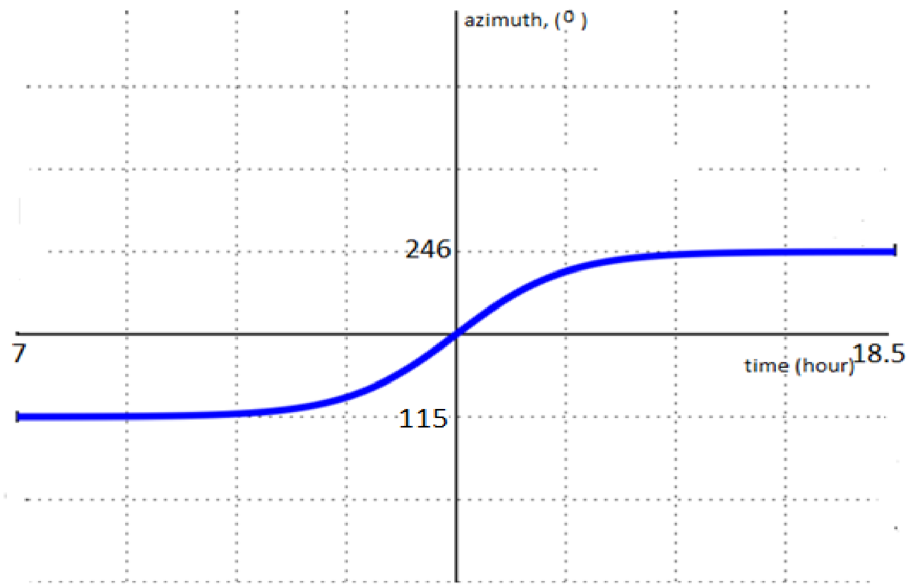

Fig. 8: January 1, 2014 Daily Azimuth Angle

The trend of this day azimuth is presented in Fig. 8.

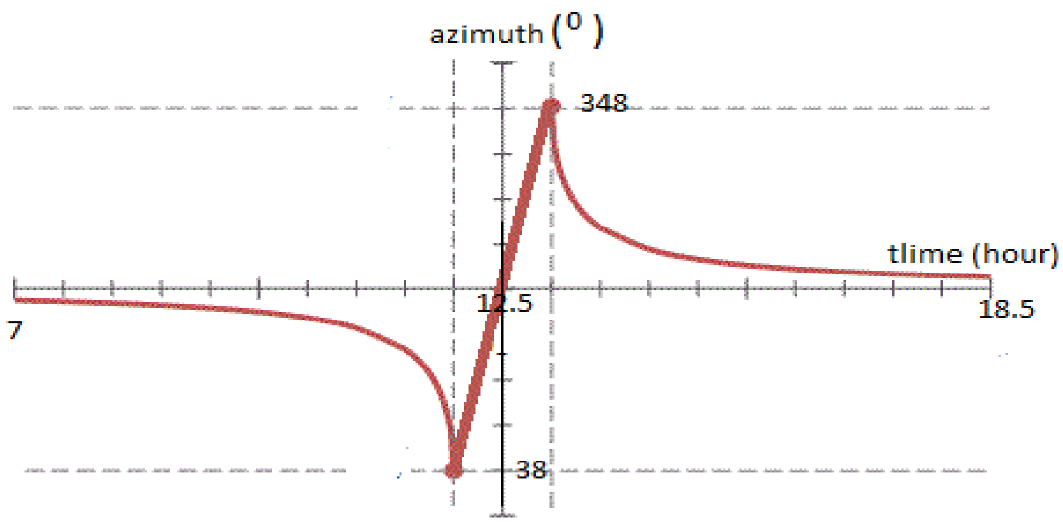

Fig. 9: Day 191, 2014 Daily Azimuth Angle

$$\gamma_1 = 180.5 + 65.5\tanh\left(\frac{6T}{11.5} - 6.65\right) + |22.325 + 24\cos(0.984D)| \tag{39}$$

$$\gamma_2 = 180.5 + 65.5\tanh\left(\frac{6T}{11.5} - 6.65\right) - |22.325 + 24\cos(0.984D)| \tag{40}$$

$$\gamma_3 = 38 - 32\tanh a - |-23 + 24\cos(-0.984D)| \tag{41}$$

$$\gamma_4 = 348 - 56\tanh a - |-23 + 24\cos(-0.984D)| \tag{42}$$

$$\gamma_5 = 93 + 24\cos(0.984D) + 310T - 3720 \tag{43}$$

$$\alpha_1 = 9.667T - 66.669 + 1.8\ell^{(\sin 0.492D + 0.3t - 2.1)} \quad (44)$$

$$\alpha_2 = -10T + 198.2 + 1.8\ell^{(\sin 0.492D - 0.3t + 5.7)} \quad (45)$$

where, γ_1 to γ_5 are for the azimuth angle at various periods of the year, D is the number of days of the year, and T is the time in an hour per day. γ_1 is the azimuth angle for days 1 to 78 and 273 to 366; $7 \le T < 13$, γ_2 is the azimuth angle on the same days as γ_1; $13 \le T \le 18.5$, γ_3 and γ_4 are for azimuth angle on day 79 to 272; $7 \le T < 12$ and $13 < T \le 18.5$ respectively, while γ_5 is the azimuth angle for day 79 to 272 at 12hours to 13hours. a is a function of T as in γ_1. α_1, is the altitude angle on either 365 or 366 days for $7 \le T < 13$ and, α_2 is the altitude angle on either 365 or 366 days for $13 < T \le 18.5$. The developed models were compared with the actual data of the first six months in the year 2015.

XI. CONCLUSION

Mathematical modeling is an important technique that is of utmost priority in engineering application. The models enable the engineer to identify the characteristic of a particular system. Models enable engineers and researchers to carry out predicting and controlling functions base on previously conducted experiment and observation. The validity of models is dependent on minimal level of existing errors. Modeling approach can help in scrutinizing complex and bogus system at ease with the aid of computer programming.

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