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To develop an Deterministic Inventory Model for Deteriorating Items with Biquadratic Demand Rate And Constant Deterioration Rate

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Abstract *- This work investigates a deterministic inventory mathematical model for decaying items with a biquadratic demand function over time. Shortages are also allowed in the model. It also demonstrates that the biquadratic demand function is convex and provides the best solution. The convexity of this model is demonstrated via a three-dimensional graphical representation. To double-check the model, an illustration is made. The ideal solution has been subjected to a sensitivity analysis with regard to main parameters, and the results have been presented.*

Keywords - *Deterioration, Biquadratic demand, Shortages, Total inventory cost.*

I. INTRODUCTION

Inventory is defined as the stock of items kept on hand to ensure the efficient and smooth operation of a trade or business. It also aids in the development of the company. Manufacturers, schools, farms, hospitals, and higher education institutions all rely on it. Both merchants and wholesalers must keep their inventory of things or products at a minimum. It is made up of a number of diverging conditions that can be turned into models. These circumstances could include deterministic demand, demand changes over time, degradation, and so forth.

The mathematical model is distributed into two kinds, i.e., Deterministic and Stochastic models. Deterministic and stochastic mathematical models are the two types of models available. The demand rate is constant in the Deterministic model. In inventory, demand is quite crucial. The demand rate was expected to remain constant in the basic inventory model, however this isn't always possible.

For many inventory products or items, such as fashionable items, dairy goods, electronic items, fruits and vegetables, and so on, the assumption of a constant demand rate is not suitable; the demand rate may be time-dependent, price-dependent, and stock-dependent. Fruits, vegetables, medications, dairy products, and other items have a finite shelf life. They deteriorate over time. Items or things that deteriorate are referred to as degrading items. The inventory system is plagued by issues caused by the deterioration of items or products.

The change in demand rate could also be linear, i.e., demand might increase or decrease linearly with time. We employ a linear polynomial as a demand function for linear demand. It should occasionally be a significant change in demand, i.e., demand is rapidly increasing with relevance time. A biquadratic polynomial demand function can be utilized to achieve this increment. Biquadratic demand can help you reduce inventory costs and grow your business.

S.K. Ghosh and K.S. Chaudhuri (2004) created an order-level inventory model for a decaying item with Weibull distribution degradation. An EOQ inventory mathematical model for deteriorating items with exponentially declining demand was proposed by Liang-Yuh Ouyang, Kun-Shan WU, and Mei-Chuan CHENG (2005). In 2008, Ajanta Roy created an inventory model for decaying items with price-dependent demand. He'd also added a time-based holding cost. When the deterioration rate follows Weibull two-parameter distributions, C. K. Tripathy* and U. Mishra (2010) devised a listing model. A list model for decaying products was presented by R. Amutha and Dr. E. Chandrasekaran (2012). Demand was supposed to be a straight line in this model. The cost of holding is calculated as a linear function of time. Volume Flexibility in Production Model with Cubic Demand Rate and Weibull Deterioration with Partial Backlog is shown by Dr. Ravish Kumar Yadav and Ms. Pratibha Yadav (2013). R. Venkateswarlu and R. Mohan (2014) created a list model for deteriorating products based on the assumption that demand is a quadratic function of time. This concept was created with the goal of lowering total inventory costs (TIC). Shortages are permitted and are partially backlogged. Garima Sharma and Bhawna Vyas (2018) suggested a deterministic inventory model for decaying products with a Weibull distribution. Demand is considered to be a linear function of time, with shortages allowed and partially backlogged. Ganesh Kumar, Sunita, and Ramesh Inaniyan (2020) proposed a listing model with time-dependent demand rate employing various factors. It is assumed that the demand rate is a cubical polynomial of your time.

The demand rate is treated as a biquadratic polynomial of time in this paper, and the deterioration rate fluctuates with time. The cost of ordering is believed to be constant and does not fluctuate over time. This model's convexity is checked using a three-dimensional graphical representation. To validate the model, an illustration is also created. The ideal solution has been subjected to a sensitivity analysis with regard to important factors, and the results are displayed.

NORMS AND REPRESENTATIONS:

The following assumptions and notations are used to explain this mathematical model.

Norms

1. a, b, c, d, and e are constant.
2. The demand rate $f(t)$ at time t is assumed as $f(t) = a + bt + ct^2 + dt^3 + et^4$;a, b, c, d, e is constant.
3. Replenishment takes place.
4. Shortages are permitted.
5. $\theta(t) = \theta t$ Denotes the deterioration rate.

Representations

C_{SC}	Shortage cost per unit per unit time.
C_{OC}	Ordering cost per order.
C_{DC}	Deterioration cost.
C_{HC}	Holding Cost.
W	The maximum inventory level for each ordering cycle.
S	Shortage level for each cycle.
Q	The order quantity ($Q = W + s$).
$Q(t)$	Inventory level at time t.
t_1	Time at which shortages start.
T	The total length of each ordering cycle.
TC	Total inventory cost over the period (0, T).

MATHEMATICAL FORMULATION:

The graph below (Figure 1) shows how inventory changes over time. The ideal order quantity, Q, and the total optimal inventory cost, TC, are shown in this diagram.

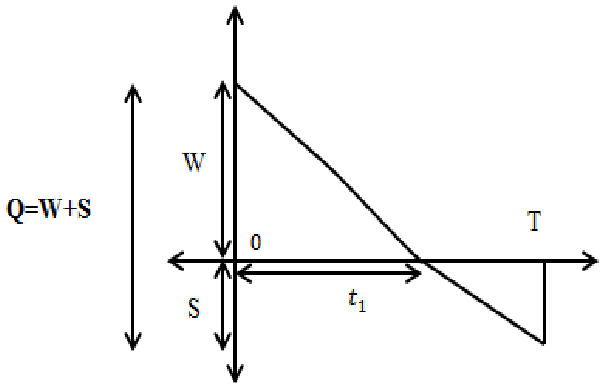

Figure 1: Inventory level (Q) vs Time

At t=0, the inventory level reaches its maximum and replenishment begins. Following that, the inventory level lowers over the time range [0, t], until falling to zero at t=t_1. Further, at $t = t_1$, shortages occur during the time interval $[t_1, T]$.

Now till the shortages are allowed at interval $[0, t_1]$, the differential equation is given by:

$$\frac{dQ_1(t)}{dt} + \theta t Q_1(t) = -(a + bt + ct^2 + dt^3 + et^4);\ 0 \le t \le t_1 \quad (1)$$

And during the interval $[t_1, T]$, the shortage occurs, so the differential equation is given by:-

$$\frac{dQ_2(t)}{dt} = -(a + bt + ct^2 + dt^3 + et^4);\ t_1 \le t \le T \quad (2)$$

With the boundary conditions: t=0, Q(0)=W

T=t_1; Q(t_1)=0

T=T; Q(T)=S

Now, by solving the above equations (1), we get:

$$Q_1(t) = \Big[a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4) + \frac{e}{5}(t_1^5 - t^5) + \frac{a\theta}{6}(t_1^3 - t^3) + \frac{b\theta}{8}(t_1^4 - t^4) + \frac{c\theta}{10}(t_1^5 - t^5) + \frac{d\theta}{12}(t_1^6 - t^6) + \frac{e\theta}{14}(t_1^7 - t^7) - \Big(\frac{a\theta}{2}(t_1 t^2 - t^3) + \frac{b\theta}{4}(t_1^2 t^2 - t^4) + \frac{c\theta}{6}(t_1^3 t^2 - t^5) + \frac{d\theta}{8}(t_1^4 t^2 - t^6) + \frac{e\theta}{10}(t_1^5 t^2 - t^7) + \frac{a\theta^2}{12}(t_1^3 t^2 - t^5) + \frac{b\theta^2}{16}(t_1^4 t^2 - t^6) + \frac{c\theta^2}{20}(t_1^5 t^2 - t^7) + \frac{d\theta^2}{24}(t_1^6 t^2 - t^8) + \frac{e\theta^2}{28}(t_1^7 t^2 - t^9)\Big)\Big] \quad (3)$$

By solving equation (2) we get:

$$Q_2(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4) + \frac{e}{5}(t_1^5 - t^5) \quad (4)$$

Now, at t=0, the maximum inventory level for each cycle is given by:

Q(0)=W, t=0

$$W = Q_1(0) = at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{et_1^5}{5} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14} \quad (5)$$

And at t=T, the maximum amount of cubic demand per cycle is given by:

t=T, Q_2(t)= -S

$$S = -\Big(a(t_1 - T) + \frac{b}{2}(t_1^2 - T^2) + \frac{c}{3}(t_1^3 - T^3) + \frac{d}{4}(t_1^4 - T^4) + \frac{e}{5}(t_1^5 - T^5)\Big)$$

Now, the order quantity per cycle is:

$$Q = W + S = at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{et_1^5}{5} + \frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14} - \Big(a(t_1 - T) + \frac{b}{2}(t_1^2 - T^2) + \frac{c}{3}(t_1^3 - T^3) + \frac{d}{4}(t_1^4 - T^4) + \frac{e}{5}(t_1^5 - T^5)\Big)$$

Holding cost per unit per unit time is given by:

$$\text{Holding Cost per cycle} = C_{HC}\int_0^{t_1} Q_1(t)dt$$

$$\text{Holding Cost per cycle} = C_{HC}\Big(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \Big(\frac{c}{4} + \frac{a\theta}{12}\Big)t_1^4 + \Big(\frac{d}{5} + \frac{b\theta}{15}\Big)t_1^5 + \Big(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\Big)t_1^6 + \Big(\frac{d\theta}{21} - \frac{b\theta^2}{84}\Big)t_1^7 - \Big(\frac{e\theta}{24} - \frac{c\theta^2}{96}\Big)t_1^8 - \frac{d\theta^2 t_1^9}{108} - \frac{e\theta^2 t_1^{10}}{120}\Big) \quad (6)$$

Shortages cost per unit per unit time is given by:

$$\text{Shortage Cost per cycle} = (-)\, C_{SC} \int_{t_1}^{T} Q_2(t)$$

$$\text{Shortage Cost per cycle} = -C_{SC}\left[a\left(t_1T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right) + e\left(\frac{t_1^5T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] \quad (7)$$

Ordering cost per order is given by:

$$\text{Ordering cost per order} = C_{OC} \quad (8)$$

Now, the deteriorating cost is given by:

$$\text{Cost due to Deterioration} = C_{DC}\left[W - \int_0^{t_1} Q(t)dt\right]$$

$$\text{Cost due to Deterioration} = C_{DC}\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14}\right] \quad (9)$$

Therefore, the total cost per unit time per unit cycle is given by:

$$TC = \frac{1}{T}(\text{Holding Cost per cycle} + \text{Shortage Cost per cycle} + \text{Ordering Cost per cycle} + \text{Cost due to Deterioration})$$

$$TC = \frac{1}{T}\Bigg(C_{HC}\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \left(\frac{c}{4} + \frac{a\theta}{12}\right)t_1^4 + \left(\frac{d}{5} + \frac{b\theta}{15}\right)t_1^5 + \left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)t_1^6 + \left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)t_1^7 - \left(\frac{e\theta}{24} - \frac{c\theta^2}{96}\right)t_1^8 - \frac{d\theta^2t_1^9}{108} - \frac{e\theta^2t_1^{10}}{120}\right) + C_{SC}\left[a\left(t_1T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right) + e\left(\frac{t_1^5T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] + C_{OC} + C_{DC}\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14}\right]\Bigg) \quad (10)$$

This is the essential condition to minimize the total cost of inventory.

$$\frac{d(TC)}{dT} = -\frac{1}{T^2}\Bigg[C_{HC}\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \left(\frac{c}{4} + \frac{a\theta}{12}\right)(t_1^4) + \left(\frac{b\theta}{15} + \frac{d}{5}\right)(t_1^5) + \left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)(t_1^6) + \left(\frac{d\theta}{21} - \frac{x\theta^2}{84}\right)(t_1^7) + \left(\frac{e\theta}{24} - \frac{c\theta^2}{96}\right)t_1^8 - \frac{d\theta^2t_1^9}{108} - \frac{e\theta^2t_1^{10}}{120}\right) - C_{SC}\left[a\left(t_1T - \frac{T^2}{2} - t_1\right) + b\left(\frac{t_1^2T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right) + e\left(\frac{t_1^5T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] + C_{OC} + C_{DC}\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14}\right]\Bigg] - \frac{1}{T}\left[C_{SC}\left(a(t - T) + b\left(\frac{t^2}{2} - \frac{T^2}{2}\right) + c\left(\frac{t^3}{3}\ \ T^3\right. d\left(\frac{t^4}{4} - \frac{T^4}{4}\right) + e\left(\frac{t^5}{5} - \frac{T^5}{5}\right)\right)\right] \quad (11)$$

$$\frac{d(TC)}{dt_1} = \frac{1}{T}\Bigg\{\Bigg(C_{HC}\left(at_1 + bt_1^2 + 4\left(\frac{c}{4} + \frac{a\theta}{12}\right)(t_1^3) + 5\left(\frac{b\theta}{15} + \frac{d}{5}\right)(t_1^4) + 6\left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)(t_1^5) + 7\left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)(t_1^6) - 8\left(\frac{e\theta}{24} - \frac{c\theta^2}{96}\right)t_1^7 - \frac{d\theta^2t_1^8}{12} - \frac{e\theta^2t_1^9}{12}\right) - C_{SC}[a(T - t_1) + b(t_1T - t_1^2) + (t_1^2T - t_1^3) + d(t_1^3T - t_1^4)] + e(t_1^4T - t_1^5) + C_{DC}\left[\frac{a\theta t_1^2}{2} + \frac{b\theta t_1^3}{2} + \frac{c\theta\, t_1^4}{2} + \frac{d\theta\, t_1^5}{2} + \frac{e\theta\, t_1^6}{2}\right]\Bigg\} \quad (12)$$

We get the optimal values of t_1 and T by solving equation (11) & (12) by using MAPLE 15.

NUMERICAL ILLUSTRATION:

Now check the optimality of model with the help of numerical illustration and solve the illustration with the help of Maple 15.

To explain the model numerically, assume the following parameters of the inventory system are:

a=10, b=4, c=4, d=3, e=1, C_{HC}=4, C_{SC}=15, C_{OC}=100, C_{DC}=10, θ=0.01

Under the above-given parameters, by using Maple 15 get the optimal shortage value t_1 = 1.037414372 per unit time and the optimal length of the ordering cycle is T=1.318536282 unit time. The total inventory cost is TC=114.8780155

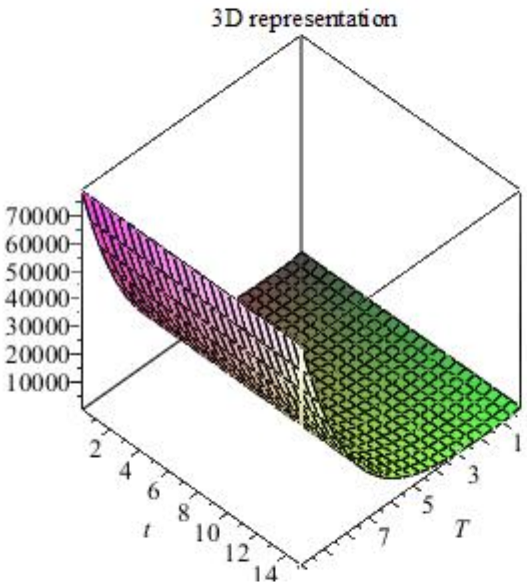

Figure 2: 3-Dimensional Graphical representation of TC of Inventory Model

SENSITIVITY ANALYSIS:

Here we study the effect of changes in the model parameters such that $a, b, c, d, e, C_{HC}, C_{SC}, C_{OC}, C_{DC}$, and θ.The outcome is given in the below table:

Parameter	% change	Change in			
		T	t_1	Q	TC
a	+20%	1.298678880	1.021844024	24.78481732	119.0440829
	+10%	1.308572915	1.029602025	23.80734555	116.9690262
	-10%	1.328566032	1.045278809	21.81079968	112.7709394
	-20%	1.338659113	1.053193015	20.79139215	110.6476908
b	+20%	1.306183237	1.027729503	23.10455655	116.1862892
	+10%	1.312317785	1.032539069	22.96144154	115.5352456
	-10%	1.324839934	1.042356357	22.66835465	114.2144838

	-20%	1.331229934	1.047365966	22.51824742	113.5445333
c	+20%	1.302371797	1.024741798	22.89028737	116.0297644
	+10%	1.310332616	1.030982984	22.85347013	115.4592332
	-10%	1.326995693	1.044046064	22.77802668	114.2856425
	-20%	1.335724691	1.050888904	22.73931690	113.6816136
d	+20%	1.303315202	1.025481991	22.76350839	115.6962820
	+10%	1.310774674	1.031329850	22.78883528	115.2919079
	-10%	1.326624294	1.043754561	22.84537099	114.4540330
	-20%	1.335065989	1.050371780	22.87696231	114.0193327
e	+20%	1.312128901	1.032406731	22.76615440	115.1520315
	+10%	1.315298199	1.034883752	22.79074800	115.0158581
	-10%	1.321846215	1.040000971	22.84213537	114.7384500
	-20%	1.325231270	1.042646096	22.86900316	114.5971053
C_{HC}	+20%	1.278837419	.9657937607	21.56317032	119.8902808
	+10%	1.297450129	.9999202468	22.14290714	117.4868420
	-10%	1.342685363	1.078963534	23.60899658	112.0292607
	-20%	1.370706930	1.125494840	24.55949979	108.8960382
C_{SC}	+20%	1.296604943	1.057676639	22.12341571	116.8658988
	+10%	1.306665462	1.048355479	22.43881374	115.9469022
	-10%	1.332756088	1.024389978	23.27534164	113.6191280
	-20%	1.350101848	1.008623224	23.84668000	112.1143168
C_{OC}	+20%	1.384613950	1.089193853	25.03443058	129.6690645
	+10%	1.352868577	1.064320225	23.94610650	122.3638112
	-10%	1.281115353	1.008081698	21.63745281	107.1858159
	-20%	1.239937796	.9757967341	20.40135224	99.25409940
C_{DC}	+20%	1.317862357	1.036325002	22.79428591	114.9283552
	+10%	1.318198828	1.036869008	22.80515633	114.9032076
	-10%	1.318874724	1.037961099	22.82700634	114.8527788
	-20%	1.319214158	1.038509196	22.83798629	114.8274969
θ	+20%	1.317685572	1.036047932	22.79519292	114.9385364
	+10%	1.318110094	1.036729996	22.80560586	114.9083118
	-10%	1.318964150	1.038101076	22.82656496	114.8476476
	-20%	1.319393707	1.038790123	22.83711178	114.8172077

- With an increases in a, b, c, TC and Q will decrease.
- With the increase in d, e, TC increase and Q decrease.
- If C_{SC} (Shortage Cost), C_{DC} (Deterioration Cost) and C_{HC} (Holding Cost) increases, TC increases and Q will decreases.
- If C_{OC} (Ordering Cost) increases, Q and TC will increase.
- If θ increases, Q will decrease and TC will increase.

On the basis of supposition of biquadratic demand and constant deterioration function different cases are arise:

CASE 1: Variable Holding Cost

- **Holding cost:**

$$HC = \int_0^{t_1} (A + Bt)\, Q_1(t) dt$$

$$HC = A\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \left(\frac{c}{4} + \frac{a\theta}{12}\right)t_1^4 + \left(\frac{d}{5} + \frac{b\theta}{15}\right)t_1^5 + \left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)t_1^6 + \left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)t_1^7 + \left(\frac{e\theta}{24} - \frac{c\theta^2}{96}\right)t_1^8 - \frac{d\theta^2 t_1^9}{108} - \frac{e\theta^2 t_1^{10}}{120}\right) + B\left(\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \left(\frac{c}{10} + \frac{a\theta}{40}\right)t_1^5 + \left(\frac{d}{12} + \frac{b\theta}{48}\right)t_1^6 + \left(\frac{e}{12} + \frac{c\theta}{56} - \frac{a\theta^2}{112}\right)t_1^7 + \left(\frac{d\theta}{64} - \frac{b\theta^2}{128}\right)t_1^8 + \left(\frac{e\theta}{72} - \frac{c\theta^2}{144}\right)t_1^9 - \frac{d\theta^2 t_1^{10}}{160} - \frac{e\theta^2 t_1^{11}}{176}\right)$$

$$TC = \frac{1}{T}(HC + SC + OC + DC)$$

$$TC = \frac{1}{T}\Bigg(A\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \left(\frac{c}{4} + \frac{a\theta}{12}\right)t_1^4 + \left(\frac{d}{5} + \frac{b\theta}{15}\right)t_1^5 + \left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)t_1^6 + \left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)t_1^7 - \frac{e\theta}{24} - \frac{c\theta^2}{96})t_1^8 - \frac{d\theta^2 t_1^9}{108} - \frac{e\theta^2 t_1^{10}}{120}\right) + B\left(\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \left(\frac{c}{10} + \frac{a\theta}{40}\right)t_1^5 + \left(\frac{d}{12} + \frac{b\theta}{48}\right)t_1^6 + \left(\frac{e}{12} + \frac{c\theta}{56} - \frac{a\theta^2}{112}\right)t_1^7 + \left(\frac{d\theta}{64} - \frac{b\theta^2}{128}\right)t_1^8 + \left(\frac{e\theta}{72} - \frac{c\theta^2}{144}\right)t_1^9 - \frac{d\theta^2 t_1^{10}}{160} - \frac{e\theta^2 t_1^{11}}{176}\right) + C_{SC}\left[a\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4 T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right) + e\left(\frac{t_1^5 T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] + C_{DC}\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14}\right] + C_{OC}\Bigg)$$

This is the essential condition to minimize the total cost of inventory.

$$\frac{\mathrm{d(TC)}}{\mathrm{dT}} = -\frac{1}{T^2}\Bigg[A\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \left(\frac{c}{4} + \frac{a\theta}{12}\right)t_1^4 + \left(\frac{d}{5} + \frac{b\theta}{15}\right)t_1^5 + \left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)t_1^6 + \left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)t_1^7 - \frac{e\theta}{24} - \frac{c\theta^2}{96})t_1^8 - \frac{d\theta^2 t_1^9}{108} - \frac{e\theta^2 t_1^{10}}{120}\right) + B\left(\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \left(\frac{c}{10} + \frac{a\theta}{40}\right)t_1^5 + \left(\frac{d}{12} + \frac{b\theta}{48}\right)t_1^6 + \left(\frac{e}{12} + \frac{c\theta}{56} - \frac{a\theta^2}{112}\right)t_1^7 + \left(\frac{d\theta}{64} - \frac{b\theta^2}{128}\right)t_1^8 + \left(\frac{e\theta}{72} - \frac{c\theta^2}{144}\right)t_1^9 - \frac{d\theta^2 t_1^{10}}{160} - \frac{e\theta^2 t_1^{11}}{176}\right) - C_{SC}\left[a\left(t_1 T - \frac{T^2}{2} - t_1\right) + b\left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4 T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right) + e\left(\frac{t_1^5 T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] + C_{OC} + C_{DC}\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14}\right]\Bigg] - \frac{1}{T}\left[C_{SC}\left(a(t - T) + b\left(\frac{t^2}{2} - \frac{T^2}{2}\right) + c\left(\frac{t^3}{3} - \frac{T^3}{3}\right) + d\left(\frac{t^4}{4} - \frac{T^4}{4}\right) + e\left(\frac{t^5}{5} - \frac{T^5}{5}\right)\right)\right]$$

$$\frac{\mathrm{d(TC)}}{\mathrm{dt_1}} = \frac{1}{\mathrm{T}}\Bigg\{A\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \left(\frac{c}{4} + \frac{a\theta}{12}\right)t_1^4 + \left(\frac{d}{5} + \frac{b\theta}{15}\right)t_1^5 + \left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)t_1^6 + \left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)t_1^7 - \frac{e\theta}{24} - \frac{c\theta^2}{96})t_1^8 - \frac{d\theta^2 t_1^9}{108} - \frac{e\theta^2 t_1^{10}}{120}\right) + B\left(\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \left(\frac{c}{10} + \frac{a\theta}{40}\right)t_1^5 + \left(\frac{d}{12} + \frac{b\theta}{48}\right)t_1^6 + \left(\frac{e}{12} + \frac{c\theta}{56} - \frac{a\theta^2}{112}\right)t_1^7 + \left(\frac{d\theta}{64} - \frac{b\theta^2}{128}\right)t_1^8 + \left(\frac{e\theta}{72} - \frac{c\theta^2}{144}\right)t_1^9 - \frac{d\theta^2 t_1^{10}}{160} - \frac{e\theta^2 t_1^{11}}{176}\right) - \mathrm{C_{SC}}[\,\mathrm{a(T - t_1) + b(t_1 T - t_1^2) + (t_1^2 T - t_1^3)} + d(\mathrm{t_1^3 T - t_1^4})] + \mathrm{e(t_1^4 T - t_1^5) + C_{DC}}\left[\frac{\mathrm{a\theta t_1^2}}{2} + \frac{\mathrm{b\theta t_1^3}}{2} + \frac{\mathrm{c\theta\, t_1^4}}{2} + \frac{\mathrm{d\theta\, t_1^5}}{2} + \frac{\mathrm{e\theta\, t_1^6}}{2}\right]\Bigg\}$$

NUMERICAL ILLUSTRATION:

Now check the optimality of model with the help of numerical illustration and solve the illustration with the help of Maple 15.

To explain the model numerically, assume the following parameters of the inventory system are:

a=10, b=4, c=4, d=3, e=1, C_{SC}=15, C_{OC}=100, C_{DC}=10, θ=0.01,A=0.5,B=0.5

Under the above-given parameters, by using Maple 15 get the optimal shortage value t_1 = 1.668388711per unit time and the optimal length of the ordering cycle is T=1.765832726unit time. The total inventory cost is TC=76.84492665

CASE 2: Variable Ordering Cost

- **Ordering Cost:**

$$OC = \frac{C_{OC}}{T}$$

$$TC = \frac{1}{T}\left(C_{HC}\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \left(\frac{c}{4} + \frac{a\theta}{12}\right)t_1^4 + \left(\frac{d}{5} + \frac{b\theta}{15}\right)t_1^5 + \left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)t_1^6 + \left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)t_1^7 - \left(\frac{e\theta}{24} - \frac{c\theta^2}{96}\right)t_1^8 - \frac{d\theta^2 t_1^9}{108} - \frac{e\theta^2 t_1^{10}}{120}\right) + C_{SC}\left[a\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + b\left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4 T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right) + e\left(\frac{t_1^5 T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] + \frac{C_{OC}}{T} + C_{DC}\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14}\right]\right)$$

This is the essential condition to minimize the total cost of inventory.

$$\frac{d(TC)}{dT} = -\frac{1}{T^2}\left(C_{HC}\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \left(\frac{c}{4} + \frac{a\theta}{12}\right)(t_1^4) + \left(\frac{b\theta}{15} + \frac{d}{5}\right)(t_1^5) + \left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)(t_1^6) + \left(\frac{d\theta}{21} - \frac{x\theta^2}{84}\right)(t_1^7) + \left(\frac{e\theta}{24} - \frac{c\theta^2}{96}\right)t_1^8 - \frac{d\theta^2 t_1^9}{108} - \frac{e\theta^2 t_1^{10}}{120}\right) - C_{SC}\left[a\left(t_1 T - \frac{T^2}{2} - t_1\right) + b\left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}\right) + c\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + d\left(\frac{t_1^4 T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5}\right) + e\left(\frac{t_1^5 T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] + \frac{C_{OC}}{T} + C_{DC}\left[\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8} + \frac{c\theta t_1^5}{10} + \frac{d\theta t_1^6}{12} + \frac{e\theta t_1^7}{14}\right]\right) - \frac{1}{T}\left(C_{SC}\left(a(t - T) + b\left(\frac{t^2}{2} - \frac{T^2}{2}\right) + c\left(\frac{t^3}{3} - \frac{T^3}{3}\right) + d\left(\frac{t^4}{4} - \frac{T^4}{4}\right) + e\left(\frac{t^5}{5} - \frac{T^5}{5}\right)\right) - \frac{C_{OC}}{T^2}\right)$$

$$\frac{d(TC)}{dt_1} = \frac{1}{T}\left\{\left(C_{HC}\left(at_1 + bt_1^2 + 4\left(\frac{c}{4} + \frac{a\theta}{12}\right)(t_1^3) + 5\left(\frac{b\theta}{15} + \frac{d}{5}\right)(t_1^4) + 6\left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)(t_1^5) + 7\left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)(t_1^6) - 8\left(\frac{e\theta}{24} - \frac{c\theta^2}{96}\right)t_1^7 - \frac{d\theta^2 t_1^8}{12} - \frac{e\theta^2 t_1^9}{12}\right) - C_{SC}[a(T - t_1) + b(t_1 T - t_1^2) + (t_1^2 T - t_1^3) + d(t_1^3 T - t_1^4)] + e(t_1^4 T - t_1^5) + C_{DC}\left[\frac{a\theta t_1^2}{2} + \frac{b\theta t_1^3}{2} + \frac{c\theta\, t_1^4}{2} + \frac{d\theta\, t_1^5}{2} + \frac{e\theta\, t_1^6}{2}\right]\right\}\frac{d(TC)}{dt_1} = \frac{1}{T}\left\{\left(C_{HC}\left(at_1 + bt_1^2 + 4\left(\frac{c}{4} + \frac{a\theta}{12}\right)(t_1^3) + 5\left(\frac{b\theta}{15} + \frac{d}{5}\right)(t_1^4) + 6\left(\frac{e}{6} + \frac{c\theta}{18} - \frac{a\theta^2}{72}\right)(t_1^5) + 7\left(\frac{d\theta}{21} - \frac{b\theta^2}{84}\right)(t_1^6) - 8\left(\frac{e\theta}{24} - \frac{c\theta^2}{96}\right)t_1^7 - \frac{d\theta^2 t_1^8}{12} - \frac{e\theta^2 t_1^9}{12}\right) - C_{SC}[a(T - t_1) + b(t_1 T - t_1^2) + (t_1^2 T - t_1^3) + d(t_1^3 T - t_1^4)] + e(t_1^4 T - t_1^5) + C_{DC}\left[\frac{a\theta t_1^2}{2} + \frac{b\theta t_1^3}{2} + \frac{c\theta\, t_1^4}{2} + \frac{d\theta\, t_1^5}{2} + \frac{e\theta\, t_1^6}{2}\right]\right\}$$

NUMERICAL ILLUSTRATION:

Now check the optimality of model with the help of numerical illustration and solve the illustration with the help of Maple 15.

To explain the model numerically, assume the following parameters of the inventory system are:

$$a = 10, b = 4, c = 4, d = 3, e = 1, C_{SC} = 15, C_{OC} = 100, C_{DC} = 10, \theta = 0.01$$

Under the above-given parameters, by using Maple 15 get the optimal shortage value t_1 =1.131847710 per unit time and the optimal length of the ordering cycle is $T = 1.439065451$ unit time. The total inventory cost is $TC = 94.82011160$

CONCLUSION

The sensitivity analysis found that in the case of biquadratic demand (a function of time), the model predicts that the rate of deterioration will alter over time. It can be shown that the parameters a, b, and c are inversely proportional to TC and Q, whereas the parameters d, e, C_{HC}, C_{SC}, and C_{DC} are inversely proportional to Q. It also demonstrates that this technique can be used to determine the total inventory cost. This model also concluded that Total cost minimize in case of variable holding cost. Finally, a graphical depiction is used to verify the model. The model's stability is determined by the obtained findings. This model can be expanded or replaced with a different demand rate.

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