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Original Article

Solution of Fingering Phenomenon in Double Phase Flow through Heterogeneous Porous Media for Vertically Downward Direction

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Abstract - *The present paper discusses the fingering phenomenon in the vertical direction via heterogeneous porous media. Governing equation of this phenomenon is a nonlinear second order partial differential equation. It is analysed with suitable initial condition by Reduced differential transform method (RDTM). The obtained solutions are represented numerically as well as graphically.*

Keywords — *Fingering phenomenon, Secondary oil recovery process, Heterogeneous porous medium, Reduced Differential Transform Method.*

I. INTRODUCTION

Since last few decades, flow displacement in a porous material has been an important topic of research. Instead of regular displacement of the whole front, when a fluid contained in a porous medium is displaced by another of lesser viscosity, protuberances may occur which shoot through the porous medium at relatively great speed, is called fingering phenomenon [8]. Many researchers and mathematician worked on this phenomenon with and without capillary pressure in homogeneous as well as heterogeneous porous medium [1,3,15]. Different mathematical techniques are used to solve this phenomenon. Phenomenon of instability occurring in inclined porous media is solved using Optimal Homotopy analysis method [9]. This phenomenon through fracture porous media with inclination and gravitational effect by is also analysed by applying Adomian Decomposition Method [10]. The phenomenon of instability also has been discussed and obtained solutions using Crank-Nicolson Scheme [17] and using Variational iteration method [16]. Similarity solution obtained for instabilities in double-phase flow through porous media [4].

Very less work is done on this phenomenon in downward direction. Recently, this model is analysed using Generalized separable method [6] as well as by Variational iteration method [7]. The governing equation of this model is a nonlinear second order partial differential equation that is solved by using Reduce differential transform method [5,12-14,19,21-23] with appropriate initial condition.

Our primary objective in this investigation is to ascertain the saturation level of injecting water in well-developed fingers as a result of water injection, which assists in pushing oil toward the oil production well. We apply RDTM method to obtain the approximate analytical solutions for this model. Numerical and graphical results are also discussed. The method explained in this paper are expected to be used for more non-linear models. This phenomenon takes place in secondary oil recovery process.

II. MATHEMATICAL MODEL FORMULATION

For the ease of mathematical study, we assume the oil-saturated heterogeneous porous matrix as vertical pipe shaped part whose surface is impermeable except its two ends, one is the uppermost part that is the common interface ($z = 0$) and the other is lowest part which is linked with oil production well. When water is injected at the common interface ($z = 0$), due to force of injecting water and gravitational effect, small protuberance occurs instead of regular displacement of the whole front and then fluid (water) flows vertically downward via interconnected capillaries to push native fluid (oil) towards the end of the pipe shaped part of porous matrix and the irregular fingers are formed with irregular size and shapes as shown in fig. 1 [7]. For the propose work, schematic fingers of rectangular size as shown in fig. 2 [1] are considered in place of irregular fingers. It is hard to obtain saturation of the injected water for any given $t > 0$ at any z. Therefore, average cross section region of all rectangular fingers of average length have been considered. $S_i(z, t)$ determines the saturation in the z-direction with injection time t.

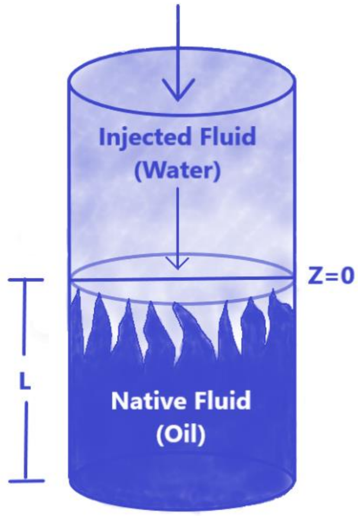

Figure- 1: Formation of fingers in pipe shaped porous matrix [7]

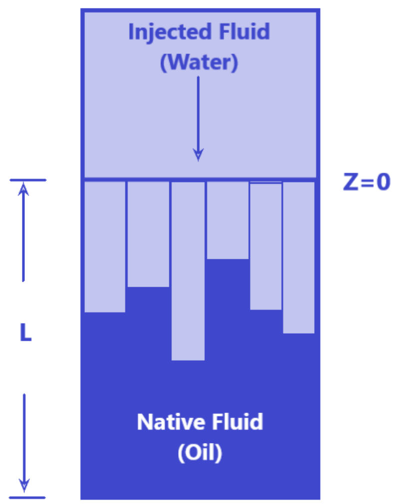

Figure-2: Schematic view of fingers [7]

The Darcy's law is used to determine velocities of the injected water (wetting fluid) (V_i) and native oil (non-wetting fluid) (V_n) for low Reynolds numbers [7]. It is assumed that the porous medium is heterogeneous. As a result, porosity (P) and permeability (K) are chosen in the variable form. As water and oil flow vertically downward, gravitational forces play a significant role in increasing the velocity of water and oil by an additional term ρg in Darcy's law [11]. Let the lowest point of the vertical pipe-shaped part of the heterogeneous porous matrix be at $z = L$, and the top point of the vertical pipe-shaped part be at $z = 0$.

Applying the law of Darcy, the seepage velocity of water V_i and oil V_n can be written as [11]:

$$V_i = -\left(\frac{K_i}{\delta_i}\right) K \left(\frac{\partial P_i}{\partial z} + \rho_i g\right) \tag{1}$$

$$V_n = -\left(\frac{K_n}{\delta_n}\right) K \left(\frac{\partial P_n}{\partial z} + \rho_n g\right) \tag{2}$$

The continuity equations for these two fluids are given by,

$$P\left(\frac{\partial S_i}{\partial t}\right)+\frac{\partial V_i}{\partial z}=0 \tag{3}$$

$$P\left(\frac{\partial S_n}{\partial t}\right)+\frac{\partial V_n}{\partial z}=0 \tag{4}$$

Using result for phase saturation [1],

$$S_i+S_n=1. \tag{5}$$

Further capillary pressure plays an important role for the instability phenomenon. Due to the pressure difference in native oil and injected water, the wetting fluid can flow via interconnected capillaries. Therefore, P_c described as [2],

$$P_c(S_i)=P_n-P_i. \tag{6}$$

The capillary pressure is in the opposite direction. Also it is a linear function of displacing fluid saturation [6]. so,

$$P_c=-\beta S_i. \tag{7}$$

The relation between the phase saturation and relative permeability defined as [1]:

$$K_i=S_i\ ,\ K_n=1-\lambda S_i. \tag{8}$$

We choose $\lambda\approx 1$,

$$K_n\approx 1-S_i=S_n \quad (\because S_i+S_n=1)$$

For heterogeneous porous medium, according to the variation law, the relation between $P(z)$ and $K(z)$ is described as [3],

$$P(z)=\frac{1}{a-bz}$$

$$K(z)=K_n(1+a_1 z). \tag{9}$$

As $P(z)$ cannot exceed beyond unity, we consider that $a-bz\geq 1$.

For simplicity, $K\propto P$ [24]

Therefore, $K=K_cP$. (10)

From equations (1) to (4), we get

$$P\left(\frac{\partial S_i}{\partial t}\right)=\frac{\partial}{\partial z}\left(\frac{K_i}{\delta_i}K\frac{\partial P_i}{\partial z}\right)-\frac{\partial}{\partial z}\left(\frac{K_i}{\delta_i}K\rho_i g\right), \tag{11}$$

$$P\left(\frac{\partial S_n}{\partial t}\right)=\frac{\partial}{\partial z}\left(\frac{K_n}{\delta_n}K\frac{\partial P_n}{\partial z}\right)-\frac{\partial}{\partial z}\left(\frac{K_n}{\delta_n}K\rho_n g\right). \tag{12}$$

Using P_i from (6) into (11), we have

$$P\left(\frac{\partial S_i}{\partial t}\right)=\frac{\partial}{\partial z}\left(\frac{K_i}{\delta_i}K\left(\frac{\partial P_n}{\partial z}-\frac{\partial P_c}{\partial z}\right)\right)-\frac{\partial}{\partial z}\left(\frac{K_i}{\delta_i}K\rho_i g\right). \tag{13}$$

From equation (5),

$$\frac{\partial S_i}{\partial t}+\frac{\partial S_n}{\partial t}=0\,.$$

Thus,

$$\frac{\partial S_i}{\partial t}=-\frac{\partial S_n}{\partial t}\,.$$

Using the above result in (12) and compare with (11),

$$\frac{\partial}{\partial z}\left(\left(\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}\right)K\frac{\partial P_n}{\partial z}-\frac{K_i}{\delta_i}K\frac{\partial P_c}{\partial z}\right)=\frac{\partial}{\partial z}\left(\left(\frac{K_i}{\delta_i}\rho_i+\frac{K_n}{\delta_n}\rho_n\right)Kg\right). \tag{14}$$

Integrating equation (14),

$$\left(\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}\right)K\frac{\partial P_n}{\partial z}-\frac{K_i}{\delta_i}K\frac{\partial P_c}{\partial z}=\left(\frac{K_i}{\delta_i}\rho_i+\frac{K_n}{\delta_n}\rho_n\right)Kg+C(t)\,. \tag{15}$$

By simplifying the above equation (15),

$$\frac{\partial P_n}{\partial z}=\frac{\frac{K_i}{\delta_i}K\frac{\partial P_c}{\partial z}+\left(\frac{K_i}{\delta_i}\rho_i+\frac{K_n}{\delta_n}\rho_n\right)g+\frac{C(t)}{K}}{\left(\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}\right)}\,. \tag{16}$$

Substituting the values of $\frac{\partial P_n}{\partial z}$ from equation (16) into (13),

$$\frac{\partial S_i}{\partial t}=\frac{\partial}{\partial z}\left(\frac{\frac{K_i}{\delta_i}K\left(\left(\frac{K_i}{\delta_i}\rho_i+\frac{K_n}{\delta_n}\rho_n\right)g+\frac{C(t)}{K}-\frac{\partial P_c}{\partial z}\left(\frac{K_n}{\delta_n}\right)\right)}{\left(\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}\right)}\right)-\frac{\partial}{\partial z}\left(\frac{K_i}{\delta_i}K\rho_i g\right). \tag{17}$$

Pressure (P_n) can be defined as [1],

$$P_n=\frac{P_n-P_i}{2}+\frac{P_n+P_i}{2}=\frac{1}{2}P_c+\bar{P}\,. \tag{18}$$

By differentiating the above equation (18) with respect to z ,

$$\frac{\partial P_n}{\partial z}=\frac{1}{2}\frac{\partial P_c}{\partial z}\,. \tag{19}$$

Using (19) into (16), we get

$$\frac{1}{2}\frac{\partial P_n}{\partial z}\left(\frac{K_n}{\delta_n}-\frac{K_i}{\delta_i}\right)-\left(\frac{K_i}{\delta_i}\rho_i+\frac{K_n}{\delta_n}\rho_n\right)g=\frac{C(t)}{K} \tag{20}$$

Now, from equation (17) we get the result,

$$P\left(\frac{\partial S_i}{\partial t}\right)=\frac{\partial}{\partial z}\left(\frac{K_i}{\delta_i}K\left(-\frac{1}{2}\frac{\partial P_c}{\partial z}\right)\right)-\frac{\partial}{\partial z}\left(\frac{K_i}{\delta_i}K\rho_i g\right). \tag{21}$$

Using standard results (7), (8) and (10) in equation (21),

$$\frac{\partial S_i}{\partial t}=\frac{\beta K_c}{2\delta_i}\left(\frac{\partial}{\partial z}\left(S_i\frac{\partial S_i}{\partial z}\right)+S_i\left(\frac{\partial S_i}{\partial z}\right)\frac{b}{a}\right)-\frac{K_i}{\delta_i}\rho_i g\left(\frac{\partial S_i}{\partial z}+S_i\frac{b}{a}\right), \tag{22}$$

$$\left(\because \frac{1}{P}\frac{\partial P}{\partial z}=\frac{\partial(\log P)}{\partial z}=\frac{\partial}{\partial z}\left(-\log a+\frac{b}{a}\right)=\frac{b}{a}(9)\right)$$

(Neglecting second and higher power of z)

Equation (22) represents governing equation of this model.
For the simplicity, we use dimensionless variables

$$Z=\frac{z}{L}\ ,\ T=\frac{K_c\beta t}{2\delta_i L^2}\ ,\ 0\le Z\le 1\ ,\ 0\le T\le 1$$

in equation (22) and it can be reduced as below

$$\frac{\partial S_i}{\partial T}=\frac{\partial}{\partial z}\left(S_i\frac{\partial S_i}{\partial z}\right)+BS_i\frac{\partial S_i}{\partial z}-A\frac{\partial S_i}{\partial z}-ABS_i\ , \tag{23}$$

Where $A=\dfrac{2L\rho_i g}{\beta}$, $B=\dfrac{b}{a}L$ and $S_i(z,t)=S_i(Z,T)$.

With initial condition $S_i(Z,0)=S_{i0}(Z)$, where $Z>0$.

III. BASICS OF REDUCED DIFFERENTIAL TRANSFORM METHOD

This section contains some basic definition and properties.

Definition: If function $\phi(\xi,\eta)$ is analytic and differentiated continuously with respect to time η and space ξ in the domain of interest, then let

$$\Phi_k(\xi)=\frac{1}{k!}\left[\frac{\partial^k}{\partial\eta^k}\phi(\xi,\eta)\right]_{\eta=0} \tag{24}$$

where the η -dimensional spectrum function $\Phi_k(\xi)$ is the transformed function [22].

The differential inverse transform of $\Phi_k(\xi)$ is defined as follows:

$$\phi(\xi,\eta)=\sum_{k=0}^{\infty}\Phi_k(\xi)\eta^k \tag{25}$$

Hence from equations (24) and (25), we write

$$\phi(\xi,\eta)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{\partial^k}{\partial\eta^k}\phi(\xi,\eta)\right]_{\eta=0}\eta^k \tag{26}$$

Therefore, we can say that the concept of the reduced differential transform is derived from the power series expansion.

Table I: Reduced differential transformation [14]

Functional Form	Transformed Form
$\phi(\xi,\eta)$	$\frac{1}{k!}\left[\frac{\partial^k}{\partial\eta^k}\phi(\xi,\eta)\right]_{\eta=0}$
$\chi(\xi,\eta)=\phi(\xi,\eta)\pm\psi(\xi,\eta)$	$\Omega_k(\xi)=\Phi_k(\xi)\pm\Psi_k(\xi)$
$\chi(\xi,\eta)=\alpha\phi(\xi,\eta)$	$\Omega_k(\xi)=\alpha\Phi_k(\xi)$ (α is a constant)
$\chi(\xi,\eta)=\xi^m\eta^n$	$\Omega_k(\xi)=\xi^m\varepsilon(k-n),\varepsilon(k)=\begin{cases}1;k=0\\0;k\neq 0\end{cases}$
$\chi(\xi,\eta)=\xi^m\eta^n\phi(\xi,\eta)$	$\Omega_k(\xi)=\xi^m\Phi(k-n)$
$\chi(\xi,\eta)=\phi(\xi,\eta)\psi(\xi,\eta)$	$\Omega_k(\xi)=\sum_{r=0}^{k}\Phi_r(\xi)\Psi_{k-r}(\xi)=\sum_{r=0}^{k}\Psi_r(\xi)\Phi_{k-r}(\xi)$
$\chi(\xi,\eta)=\frac{\partial^r}{\partial\eta^r}\phi(\xi,\eta)$	$\Omega_k(\xi)=(k+1)...(k+r)\Omega_{k+1}(\xi)=\frac{(k+r)!}{k!}\Omega_{k+r}(\xi)$
$\chi(\xi,\eta)=\frac{\partial}{\partial\xi}\phi(\xi,\eta)$	$\Omega_k(\xi)=\frac{\partial}{\partial\xi}\Phi_k(\xi)$

IV. SOLUTION PROCEDURE

Implementing the aforesaid method to (23) and from Table 1, the transformed form written as [5,12-14,19,21-23]

$$\begin{aligned}(k+1)S_{k+1}(Z,T)=&\sum_{r=0}^{k}\frac{\partial}{\partial Z}S_{k-r}(Z,T)\frac{\partial}{\partial Z}S_r(Z,T)+\sum_{r=0}^{k}\frac{\partial}{\partial Z}S_{k-r}(Z,T)\frac{\partial^2}{\partial Z^2}S_r(Z,T)\\&+B\cdot\sum_{r=0}^{k}\frac{\partial}{\partial Z}S_{k-r}(Z,T)\frac{\partial}{\partial Z}S_r(Z,T)-A\cdot\frac{\partial}{\partial Z}S_k(Z,T)-AB\cdot S_k(Z,T)\end{aligned}\tag{27}$$

Using the initial condition [6],

$$S_{i0}(Z,0)=e^{-Z}\tag{28}$$

Now, substituting (28) into (27), we obtain the following $S_k(Z)$ values successively,

$$\begin{aligned}S_{i1}&=\left(\left(-B+2\right)e^{-2Z}-A\left(B-1\right)e^{-Z}\right)T\\S_{i2}&=\left(\frac{1}{2}(3B^2-15B+18)e^{-3Z}+\frac{1}{2}A(3B^2-10B+8)e^{-2Z}+A(B-1)^2e^{-Z}\right)T^2\\S_{i3}&=(\frac{1}{6}(-16B^3+140B^2-392B+352)e^{-4Z}-\frac{1}{6}A(18B^3-117B^2+243B-162)e^{-3Z}\\&\quad+A(7B^3-32B^2+48B-24)e^{-2z}+A(B-1)^3e^{-Z})T^3\end{aligned}\tag{29}$$

… and so on.

From (29), the approximate solution in a series form is given by

$$
\begin{aligned}
S_i(Z,T) &= e^{-z} + \left(-B+2\right)e^{-2Z} - A\left(B-1\right)e^{-Z}T \\
&+\left(\frac{1}{2}(3B^2-15B+18)e^{-3Z}+\frac{1}{2}A(3B^2-10B+8)e^{-2Z}+A(B-1)^2e^{-Z}\right)T^2 \\
&+\Big(\frac{1}{6}(-16B^3+140B^2-392B+352)e^{-4Z}-\frac{1}{6}A(18B^3-117B^2+243B-162)e^{-3Z} \\
&+A(7B^3-32B^2+48B-24)e^{-2z}+A(B-1)^3e^{-Z}\Big)T^3+\ldots
\end{aligned}
\tag{30}
$$

Equation (30) is the approximate analytic solution of equation (23).

V. RESULTS AND DISCUSSION

In Table II, the different values of saturation produced using RDTM are displayed for different time and distance values. Table II shows that saturation of injected water goes up with respect to T and goes down with respect to Z. Also it determines that, due to time increment, the fingers will arise and oil move to production well through oil formed area during the secondary oil recovery process.

Table II: Numerical values of $S_i(Z,T)$ by RDTM

T $\rightarrow$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Z $\downarrow$	$S_i(Z,T)$									
0.1	0.911712	0.919084	0.927011	0.93555	0.944758	0.954692	0.96541	0.976968	0.989423	1.002832
0.05	0.869184	0.875825	0.882946	0.890595	0.898819	0.907665	0.917183	0.927418	0.93842	0.950235
0.15	0.828657	0.834638	0.841035	0.847887	0.855235	0.863117	0.871573	0.880644	0.890369	0.900788
0.2	0.790034	0.79542	0.801167	0.807306	0.813872	0.820897	0.828415	0.836459	0.845063	0.854258
0.25	0.753224	0.758074	0.763236	0.768737	0.774606	0.78087	0.787557	0.794695	0.802311	0.810433
0.3	0.718141	0.722507	0.727143	0.732073	0.73732	0.742907	0.748857	0.755194	0.76194	0.769119
0.35	0.684702	0.688632	0.692796	0.697214	0.701906	0.70689	0.712187	0.717815	0.723794	0.730144
0.4	0.652831	0.656367	0.660106	0.664065	0.66826	0.672708	0.677424	0.682425	0.687727	0.693347
0.45	0.622452	0.625632	0.628989	0.632537	0.636288	0.640257	0.644458	0.648904	0.653608	0.658584
0.5	0.593495	0.596354	0.599367	0.602545	0.6059	0.609443	0.613185	0.617138	0.621313	0.625722
0.55	0.565892	0.568462	0.571165	0.574012	0.577012	0.580174	0.583508	0.587024	0.590731	0.594638
0.6	0.539579	0.541888	0.544313	0.546863	0.549545	0.552367	0.555338	0.558466	0.561757	0.565222
0.65	0.514496	0.516569	0.518744	0.521027	0.523424	0.525943	0.52859	0.531373	0.534297	0.537369
0.7	0.490584	0.492445	0.494394	0.496437	0.49858	0.500828	0.503187	0.505662	0.508259	0.510985
0.75	0.467789	0.469458	0.471204	0.473032	0.474946	0.476952	0.479053	0.481255	0.483563	0.485981
0.8	0.446057	0.447553	0.449117	0.450751	0.452461	0.45425	0.456122	0.458081	0.460131	0.462276
0.85	0.425339	0.426679	0.428078	0.429539	0.431066	0.432661	0.434328	0.43607	0.437891	0.439795
0.9	0.405587	0.406787	0.408038	0.409343	0.410704	0.412126	0.41361	0.41516	0.416777	0.418466
0.95	0.386755	0.387828	0.388946	0.390111	0.391326	0.392592	0.393913	0.39529	0.396726	0.398224
1.0	0.368801	0.36976	0.370758	0.371797	0.372879	0.374007	0.375182	0.376406	0.377681	0.379009

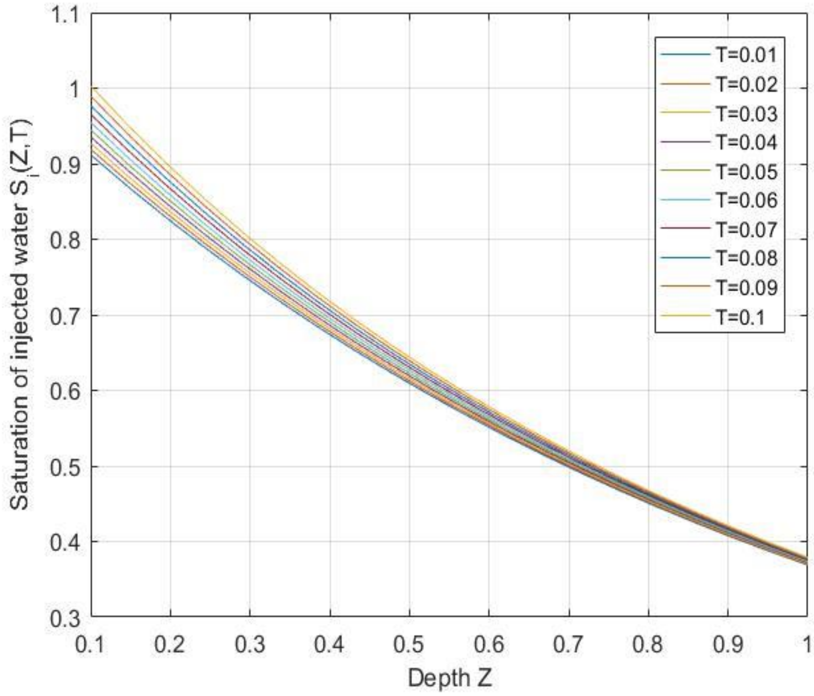

Figure-3: Saturation $S_i(Z,T)$ verses Z

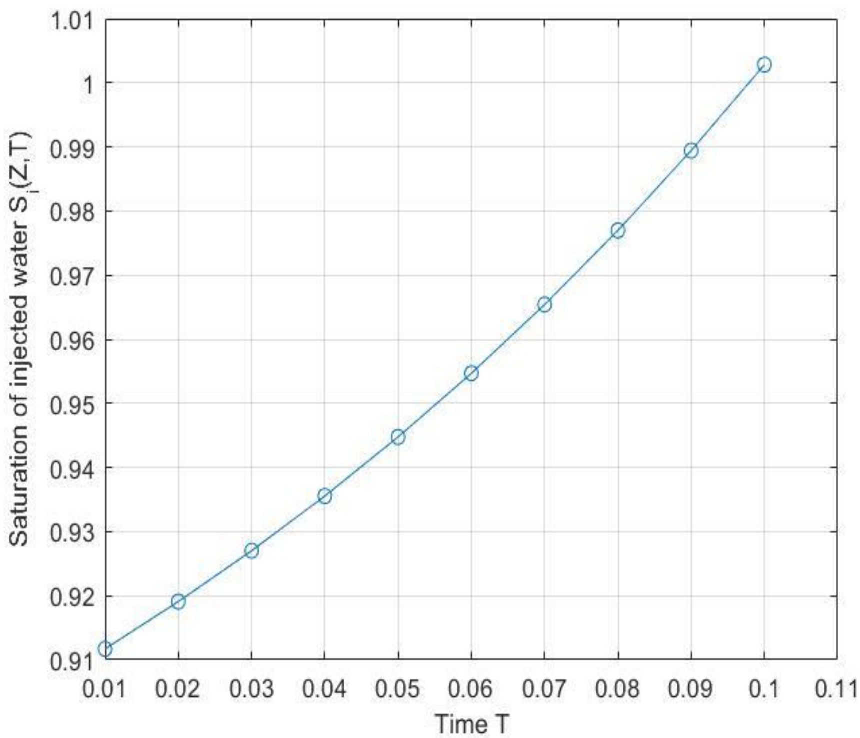

Figure- 4: Saturation $S_i(Z,T)$ verses T for fixed Z = 0.1

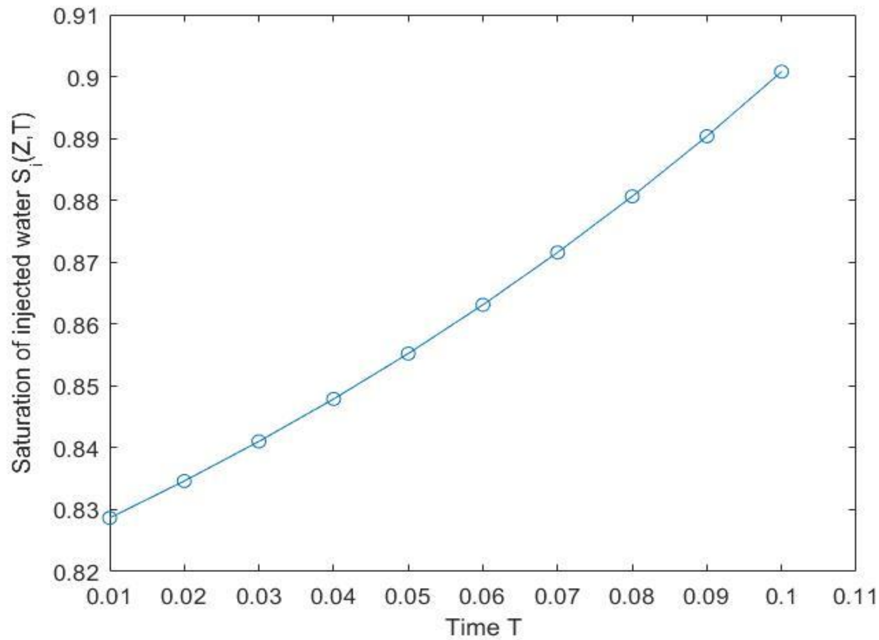

Figure- 5: Saturation $S_i(Z,T)$ verses T for fixed Z = 0.3

Table II: Comparison of $S_i(Z,T)$ by RDTM and VIM [7]

	T →	0.02	0.04	0.06	0.08	0.1
	Z ↓	$S_i(Z,T)$				
RDTM	0.1	0.9191	0.9357	0.9558	0.9810	1.0142
VIM		0.9210	0.9369	0.9525	0.9678	0.9827
RDTM	0.2	0.8302	0.8434	0.8590	0.8779	0.9021
VIM		0.8320	0.8450	0.8578	0.8703	0.8825
RDTM	0.3	0.7500	0.7605	0.7726	0.7871	0.8049
VIM		0.7517	0.7623	0.7728	0.7830	0.7930
RDTM	0.4	0.6777	0.6860	0.6955	0.7066	0.7199
VIM		0.6792	0.6879	0.6965	0.7049	0.7131
RDTM	0.5	0.6124	0.6190	0.6265	0.6351	0.6451
VIM		0.6138	0.6210	0.6280	0.6348	0.6415
RDTM	0.6	0.5535	0.5588	0.5646	0.5713	0.5789
VIM		0.5548	0.5606	0.5664	0.5720	0.5775
RDTM	0.7	0.5004	0.5045	0.5091	0.5143	0.5201
VIM		0.5015	0.5063	0.5109	0.5155	0.5201
RDTM	0.8	0.4523	0.4556	0.4592	0.4632	0.4677
VIM		0.4533	0.4572	0.4611	0.4649	0.4685
RDTM	0.9	0.4090	0.4115	0.4144	0.4175	0.4210
VIM		0.4098	0.4131	0.4162	0.4193	0.4223
RDTM	1.0	0.3698	0.3718	0.3740	0.3764	0.3791
VIM		0.3706	0.3732	0.3758	0.3783	0.3808

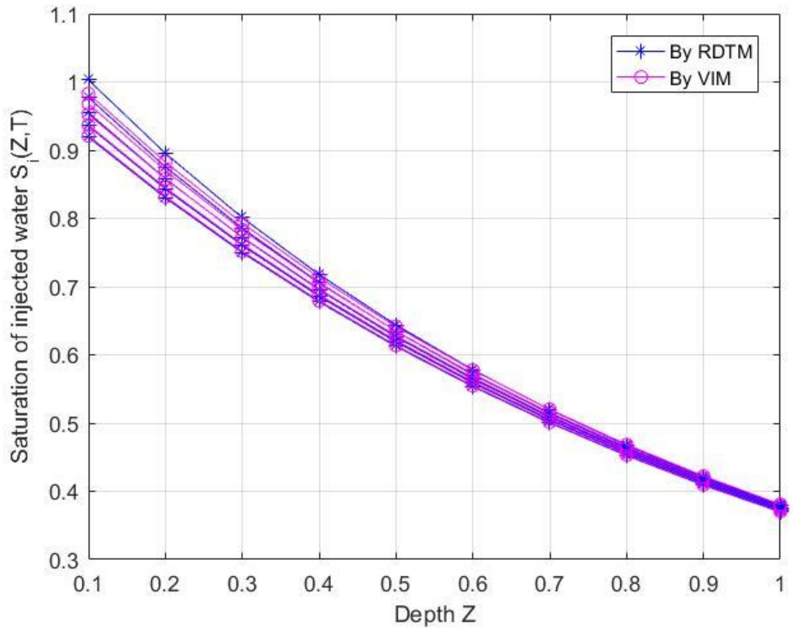

Figure-6: Saturation $S_i(Z,T)$ by RDTM and VIM

Table IV: Error between RDTM and VIM

ERROR Z/T	0.02	0.04	0.06	0.08	0.1
0.1	0.001907	0.001169	0.003267	0.013153	0.031509
0.2	0.001809	0.001615	0.001151	0.007642	0.019551
0.3	0.001677	0.001805	0.000159	0.004102	0.01187
0.4	0.001496	0.001881	0.000973	0.001721	0.006793
0.5	0.001355	0.001953	0.001496	0.000264	0.003583
0.6	0.001259	0.001822	0.001765	0.000733	0.001391
0.7	0.001142	0.001791	0.00179	0.001238	2.91E-07
0.8	0.000969	0.001586	0.001872	0.001663	0.000775
0.9	0.00085	0.001558	0.001823	0.001802	0.001342
1.0	0.00084	0.001401	0.001781	0.001853	0.001682

Figure-3 shows graphical representation of the saturation of the injected fluid decreases with respect to Z. The obtained results are physically consistent. In Table III, the saturation level of water is compared to the values obtained using Variational iteration method (VIM) [5] in the literature. Figures 4 and 5 illustrate that for fixed depth, saturation of injected water increases with respect to time T. This is consistent with the underlying physical phenomenon.

VI. CONCLUSION

In this paper, proposed model is studied with the appropriate initial condition. For the validation of the method, the obtained results are compared with VIM. From table IV, we can observe that the errors between RDTM with VIM are negligible. Also figure- 6 displays applicability of the present method. Hence we conclude that the assumptions which are considered for the study as well as the proposed method is very effective.

NOMENCLATURE

V_i Velocity of water

V_n Velocity of oil

P Porosity

P_c Capillary pressure

K Variable permeability

K_i Relative permeability of water

K_n Relative permeability of oil

T Time

Z Depth

β, K_c Proportionality Constant

δ_i Kinematic viscosity of water

δ_n Kinematic viscosity of oil

a, b, a_1 Positive constants

C Arbitrary constant

ρ_i Density of water

ρ_n Density of oil

P_o Pressure of oil

$\overline{P}$ Constant mean pressure

g Acceleration due to gravity

L Length of pipe shaped porous matrix

S_i Saturation of injected water

S_n Saturation of native oil

REFERENCES

[1] A. E. Scheidegger and Johnson, The statistical behaviour of instabilities in displacement process in porous media, Canadian Journal of Physics. 39(2) (1961) 326–334.

[2] A. E. Scheidegger, The physics of flow through Porous Media. University of Toronto Press, (1960).

[3] A.P. Verma, Statistical behaviour of fingering in a displacement in heterogeneous porous medium with capillary pressure, Canadian Journal of Physics. 47(3) (1969) 319–324.

[4] A. P. Verma and S. K. Mishra, Similarity solution for instabilities in double-phase flow through porous media, Journal of Applied Physics. 44(4) (1973) 1622–1624.

[5] A. Kumar, R. Arora, Solutions of the coupled system of Burgers' equations and coupled Klein-Gordon equation by RDT Method, International Journal of Advances in Applied Mathematics and Mechanics. 1(2) (2013) 133-145.

[6] A.K. Parikh, Mathematical model of instability phenomenon in homogeneous porous medium in vertical downward direction, International Journal for Innovative Research in Multidisciplinary Field. 3(1) (2017) 211–218.

[7] D.A. Shah, A.K. Parikh, Mathematical solution of fingering phenomenon in vertical downward direction through heterogeneous porous medium, Advances in Mathematics: Scientific Journal. 10(1) (2021) 483–496.

[8] D. J. Prajapati, N. B. Desai, Application of the Basic Optimal Homotopy Analysis Method to Fingering Phenomenon, Global Journal of Pure and Applied Mathematics. 12(3) (2016) 2011–2022.

[9] D. J. Prajapati, N. B. Desai, Analytic Analysis for Oil Recovery During Cocurrent Imbibition in Inclined Homogeneous Porous Medium, International Journal on Recent and Innovation Trends in Computing and Communication. 5(7) (2017) 189 – 194.

[10] H. Patel, R. Maher, Simulation of Fingering Phenomena in Fluid Flow through Fracture Porous Media with Inclination and Gravitational Effect, Journal of Applied Fluid Mechanics. 9(6) (2016) 3135-3145.

[11] J. Bear, A.H. Chang, Modelling groundwater flow and contaminant transport, Dynamics of fluids in porous media, Springer Science Business, Media B. V., (2010).

[12] M. Rawashdeh, Using the Reduced Differential Transform Method to Solve Nonlinear PDEs Arises in Biology and Physics, World Applied Sciences Journal. 23 (8) (2013) 1037-1043.

[13] M. Rawashdeh, N. A. Obeidat, On Finding Exact and Approximate Solutions to Some PDEs Using the Reduced Differential Transform Method, Applied Mathematics & Information Sciences. 8(5) (2014) 2171-2176.

[14] M. S. Mohamed, K. A. Gepreel, Reduced differential transform method for nonlinear integral member of Kadomtsev–Petviashvili hierarchy differential equations, Journal of the Egyptian Mathematical Society, Journal of the Egyptian Mathematical Society. 25 (2017) 1–7.

[15] P. G. Saffman, G. I. Taylor, The penetration of a fluid into a porous medium or Hele-Show cell containing a more viscous fluid, Proc. R. Soc. London Series A.,245 (1985) 312–329.

[16] P. R. Mistry, V. H. Pradhan, K. R. Desai, Mathematical Model and Solution for Fingering Phenomenon in Double Phase Flow through Homogeneous Porous Media, The Scientific World Journal. 2013(2013).

[17] R. Borana, V. Pradhan, M. Mehta, Numerical solution of instability phenomenon arising in double phase flow through inclined homogeneous porous media, Perspectives in Science. 8 (2016) 225–227.

[18] R. Meher, M. N. Mehta, S. K. Meher, Instability phenomenon arising in double phase flow through porous medium with capillary pressure, International Journal of Applied Mathematics and Mechanics. 7 (15) (2011) 97-112.

[19] S. Mohmoud, M. Gubara, Reduced differential transform method for solving linear and nonlinear Goursat problem, Applied Mathematics. 7(2016) 1049-1056.

[20] S.S. Chen, C.K. Chen, Application to differential transformation method for solving systems of differential equations, Nonlinear analysis: Real world applications. 10 (2) (2009) 881-888.

[21] Y. Keskin, G. Oturanc, Reduced differential transform method for partial differential equations, International Journal of Nonlinear Sciences and Numerical Simulation.10 (6) (2009) 741-749.

[22] Y. Keskin, G. Oturanc, Application of Reduced differential transformation method for solving gas dynamics equation, International Journal of Contemporary Mathematical Sciences. 5(22) (2010) 1091-1096.

[23] Y. Keskin, G. Oturanc, Reduced differential transform method for generalized kdv equations, Mathematical and Computational Applications. 15 (3) (2010) 382-393.

[24] Z. Cheng, Reservoir Simulation: Mathematical techniques in oil recovery, Society for Industrial and Applied Mathematics, Philadelphia, (2007) 1-25.