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Mathematical Creativity as a Core Tool in Restoring Euclidean Geometry: Perception and Creative/Cognitive Mathematical Skills Of Teachers And Future Teachers

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Abstract - *The present research examine the perceptions of teachers and future teachers about creativity in teaching geometry but also how their cognitive and perceptual skills in geometry can influence the appearance of creativity in their teaching. Data were collected from 116 future teachers and two secondary school teachers with different years of practice. The analysis of the results showed that: a) Teachers' perceptions of creativity in the teaching of Mathematics converge and aim to make themselves and their students creative by choosing the appropriate teaching practices and b) the perceptions and the way that future teachers apprehend the geometrical figure seem to influence the way they will introduce mathematical creativity to the teaching of geometry..*

Keywords - *Euclidean Geometry, mathematical creativity, teachers perception, teacher skills, fluency* **-** *flexibility – originality – elaboration.*

I. INTRODUCTION

Several views report that the teacher has limited creative ideas [1], while also having trouble finding suitable activities that will expose students to creativity. Some research into mathematical creativity has so far been carried out ([2], [3]), but there are no studies linking future teachers perception for creativity with their mathematical creative ability in geometry. Therefore, in order to develop the ability of future teachers to teach mathematics with creativity and to promote mathematical creativity to students, firstly should understand what their perceptions of it are. In a second step, should understand how these perceptions are integrated into their teaching practices which they adopt and how their "cognitive" mathematical abilities can influence the emergence of mathematical creativity in geometry.

Creativity is a complex phenomenon, for which there several definitions are given in the research bibliography ([4], [5]). Mann [6] argues that there are more than 100 contemporary definitions of creativity. Some definitions address the properties of the creative act and product [7] and others concern the stages of the creative processes [8]. Guilford [9] distinguished between convergent and divergent thinking (production). Convergent thinking involves aiming for a single, correct solution to a problem, whereas divergent thinking involves the creative generation of multiple answers to a problem or phenomena and is described more frequently as flexible thinking. Runco [10] described creativity as a multifaceted construct involving “divergent and convergent thinking, problem finding and problem solving, self-expression, intrinsic motivation, a questioning attitude, and self-confidence” (p. ix). Haylock [4] summarized many of the attempts to define creativity. One view “includes the ability to see new relationships between techniques and areas of application and to make associations between possibly unrelated ideas”. Bolden, et al. [1], define creativity primarily as an individual activity that aims to produce something new as "the multidimensional ability or possibility of human to think of something new".

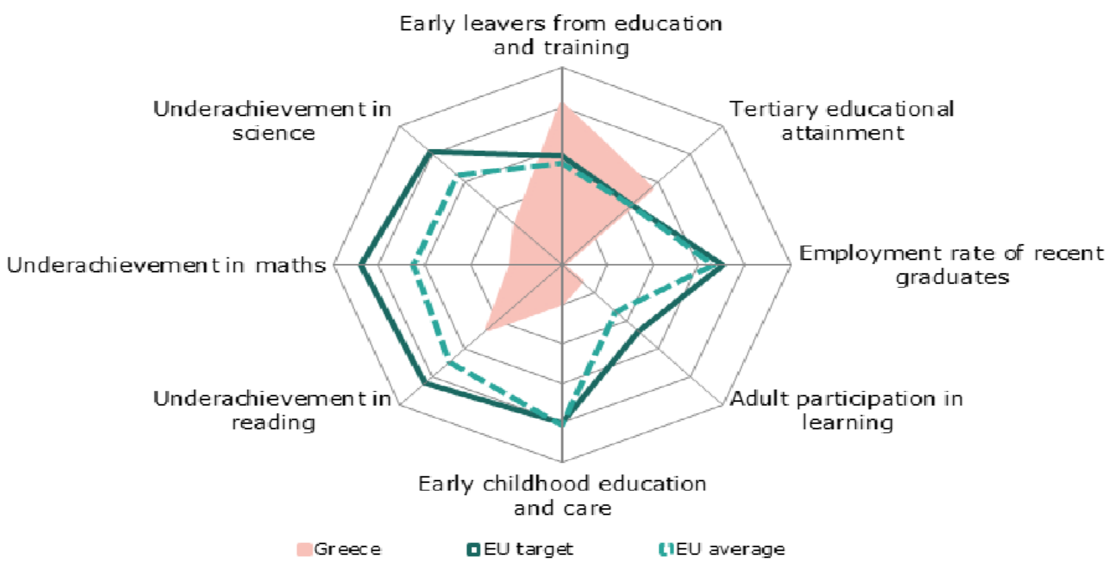
The most useful definition and applicable in real-world classroom conditions proposed by Torrance [11]. Torrance [11], defines creativity as multidimensional: fluency, flexibility, originality, and elaboration are all aspects of creativity. In the field of mathematics education, only the three dimensions are often used: fluency, flexibility and originality. In more detail, fluency is related to the flow of ideas, flexibility has to do with the ability to shift between different ideas and originality is associated with the innovation of the individual’s ideas or products ([5], [7]).

Creativity in school mathematics naturally differs from that of professional mathematicians; however, students can offer new insights or solutions to mathematical problems based on the mathematics learned, their previous experience in problem solving and the performance of other students’ contributions. From this point of view, in the mathematics classroom in order to develop these dimensions of creativity many researchers ([8], [9], [12]) recommend: (a) problem posing, and (b) open-ended problems.

There are two definitions of problem posing, at least one of which is used or referred to in the majority of research papers on the topic. The first definition was proposed by Silver [13], who describes problem posing as the activities of generating new problems and reformulating given problems. Both activities can occur before, during, or after a problem-solving process. The second definition comes from Stoyanova and Ellerton [14], who refer to problem posing as the "process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems". Some researchers [15] claim that problem posing may stimulate creativity, possibly even more than problem solving. Singer et al.[16] advocate that problem posing can enhance students' engagement in authentic mathematical activity and open students' thinking towards new ideas and approaches.

One other recognised way to elicit mathematical creativity among mathematics students is by engaging them with open-ended tasks ([7], [12], [17]). When describing the difference between closed and open tasks, Sullivan, Warren, and White [18] stated that "Closed implies there is only one acceptable pathway, response, approach, or line of reasoning. Open refers to the existence of more than one (preferably many more than one) possible pathways, responses, approaches, or lines of reasoning." (p. 3). At times, an open-ended problem may not have a single exact answer but rather a range of solutions [7]. In other words, an open-ended task may have more than one correct final answer in addition to several possible solution pathways. In this way, engaging students with open-ended tasks provides a route for disciplined improvisation in the sense that some parts of the activity are fixed, while others are more or less fluid [19].

Among the various areas of mathematics, geometry can be used as a vehicle to develop different ways of thinking in mathematics [20]. Geometry provides opportunities for investigation and proving activities that resemble the work of mathematicians [21], allowing the smooth integration of multiple approaches to one problem. Almost every geometry problem found in a standard textbook can be turned into a multiple solution problem or open-ended task. However, in recent years the importance of geometry in Greek schools has been downgraded, which is also confirmed in international competitions (e.g Pisa 2015, 2018 – Figure 1). One can clearly see that underachievement in maths is far the worst compared to language and science. Findings from the analyzes of the Pisa results show for our country that mathematics programs are deficient and obsessive with outdated methods, leading to stagnation. There is a huge need to bring it back to modern curricula, with modern approaches and teaching methods, as visual information (figures, charts, etc.) is prevalent nowadays.



Source: DG Education and Culture calculations, based on data from Eurostat (LFS 2017, UOE 2016) and OECD (PISA 2015). Note: all scores are set between a maximum (the strongest performers represented by the outer ring) and a minimum (the weakest performers represented by the centre of the figure).

Fig 1: Position in relation to strongest (outer ring) and weakest performers (centre)

Based on results the evaluation of the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA), it is known that Greek students have low math skills. One of the biggest problems causing low mathematics achievement is that mathematics is presented as a ready-made, abstract, and mechanically taught product. This case can be lead to the creativity of the less developed students because students are not given the opportunity to think and use their ideas in solving mathematical problems. Good creativity will facilitate students in solving any mathematical problems given, therefore creativity is a very important thing that must be owned by students.

This degradation is primarily due to the fact that Euclidean geometry is not examined in the final examinations in the last class of High School but also in the "bad teaching practices". There are many bad teaching practices, but in this study, we will address with the following teaching practices:

The purely theoretical character of Euclidean geometry, mainly in the high school but also in lower educational levels, downgrades the role of geometric constructions. Presenting geometry in this way reduces the quality of the lesson, loses much of its application, and traps students' thinking (see figure 2). The substantial marginalization of geometrical problems by current teaching practice has deprived teaching of an essential field of applications that give meaning and interest to geometrical theory. The concept of geometric construction is a dominant epistemological feature of Euclidean geometry and the entire structure of the Elements is saturated with it. In any case, it remains an open teaching problem that Euclidean geometry is developed in high school as a theoretical system of independent mathematical interest, without the motivation of learning that offers a clear scope. If this field cannot be equally theoretical (such as geometrical constructions), then we need to enrich teaching with applications of geometrical propositions to more practical issues. This is an issue of very interesting historical, epistemological and didactical implications.

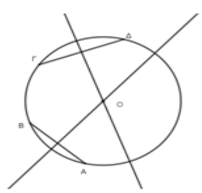
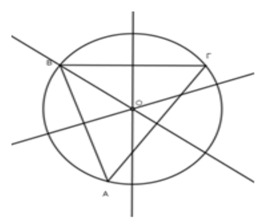
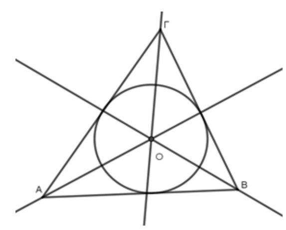
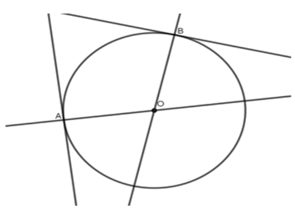
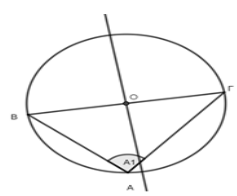
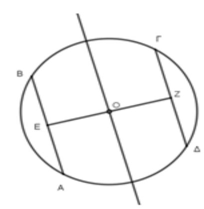
Problem 5: Find the center of a circle in as many ways as you can, if you only know the circumference of the circle**.**		
Proof 1: Construct the perpendicular bisector 	***Proof 2:*** triangle circumscribed circle 	***Proof 3:*** A circle with an inscribed triangle 
Proof 4: Circle tangent 	***Proof 5:*** Angle in a semicircle 	***Proof 6:*** Parallel chords 
		(Gridos et al. [22]

Fig 2: Example of a multiple solution problem with solutions only with geometric constructions

The nature of external representations in geometry, either as an object or as an illustration. According to Mesquita [23], an external representation has the nature of an object when it is possible to infer geometrical relationships from the construction of the figure that may be used in geometrical reasoning and proof and when the visual perception of the figure is consistent with the verbal statements of the problem. On the contrary, when the external representation has the nature of an illustration, it is then impossible to directly extract a geometrical relationship from the construction of the figure, the figure seems to „mislead‟ and the visual perception of the figure is in contradiction within the verbal statements.

Considering that Geometry refers to a model of space [24], spatial visualization and reasoning is prevalent in this mathematical domain and it underlies most geometrical thinking ([25], [26]). Thus, we suggest that two key concepts closely related to creativity in Geometry are visualisation and geometrical figure apprehension. When analyzing a geometrical problem, the utility of the geometrical figure is indisputable as it provides an intuitive / visual presentation of the components and relationships [27]. According to Duval [27], a representation can be recognized in several ways depending on the set of rules applied to visual representations. This shows that in order someone to see the elements geometrically, there is always a set of rules that must be followed to see the given shape as a geometric concept (as a geometric figure). As a result, a significant cognitive leap is required to view the elements geometrically as representations, against their perceptual recognition.

Through cognitive and perceptual approach to geometry, Duval ([26], [27]), distinguishes four apprehensions for a geometrical figure: perceptual, sequential, discursive and operative. In order a drawing to function as a geometrical figure must evoke perceptual apprehension and at least one of the other three. Each has its specific laws of organization and processing of the visual stimulus array. The above four apprehensions for a geometrical figure are tested experimentally in elementary ([28], [29]) and high school students [30]. Below we offer a description of these kinds of apprehension:

1. Perceptual apprehension refers to the recognition of a shape in a plane or in 3Dspace. Perceptual apprehension indicates the ability to identify figures and to recognize several sub-figures in the perceived figure.

2. Sequential apprehension is required whenever one must construct a figure or describe its construction. The organization of the elementary figural units does not depend on perceptual laws and cues, but on technical constraints and mathematical properties.

3. Discursive apprehension is related to the fact that mathematical properties represented in a drawing cannot be determined through perceptual apprehension. In any geometrical figure the perceptual recognition of geometrical properties must be controlled by the statements that define the properties.

4. However, it is through operative apprehension that we can get an insight into a problem solution when looking at a figure. Operative apprehension depends on the various ways of modifying a given figure: the mereologic, the optic and the place way. The mereologic way refers to the mathematical action of dissection, i.e. the division of the whole figure given into parts of various shapes and the combination of them in another figure or sub-figures (reconfiguration), the optic way is when the figure is made larger or narrower, while the place way refers to its position or orientation variation. Each of these different modifications can be performed mentally or physically through various operations. One or more of these operations can highlight a figural modification that gives an insight into the solution of a problem in geometry.

The 4 types of apprehension can be summarized in two approaches [20]. The perceptual approach is the spontaneous recognition of the figure. The mathematical approach is related to operative apprehension of the geometrical figure. That is, it concerns the control of the recognition of the figure through its properties, from which other properties are extracted.

In addition, the teaching of geometry must transform students' apprehension from algorithmic to operative one. This transition is not easy to achieve: the biggest obstacle is the breaking of the habit students have to seek for a formula or a statement to apply in order to solve a geometry problem. Even good students apply an algorithmic formula that results in many operations while the solution in an operative way requires a transformation of the given figure [30]. We believe that research in teaching and learning geometry should explore further didactical approaches that cultivate this transformation.

They serve as a helper and key tool for teaching geometry in educational robotics (STEAM) and consolidate / extend successful processes at all levels of education. However, because creativity is a complex mental process and difficult to analyze, it must be decomposed into distinct parts in order to be studied. In the field of mathematics education, creativity research focuses mainly on: (a) the stages of the creative process, (b) the properties of the creative act and product, (c) the personality of creative individuals, and (d) the cognitive processes involved in creative activities.

According to recent research findings [31], one can examine the mathematical creativity of secondary school students through cognitive and perceptual approach in geometry. To this end, the investigation was carried out in two axes: (a) first was examined the influence of geometrical figure apprehension on the production of multiple solutions, and then (b) how the necessity to construct auxiliary lines in the given shape, influence the production of multiple solutions and the variables of creativity. The analysis of the results primarily shows that the way through which students perceive the geometrical figure and their ability to process it, is an important factor in predicting their mathematical creativity. At a second level, it was found that only perceptual apprehension of geometrical figures is not a reliable predictor of creativity variables, as opposed to operative apprehension of geometrical figures that predict positively the characteristics of creativity: fluency, flexibility, and originality.

II. METHODOLOGY

A. Research Questions

Having examined how mathematical creativity is developed when learning geometry, the question arises as to whether teachers have the right ideas and appropriate cognitive profiles to expose students to situations that enhance mathematical creativity. The purpose of this research is not only to examine the product of creativity or the characteristics of creative individuals, but the cognitive processes required for creative activity. The research will focus on the perceptions and cognitive processes required in creative geometric activities, examining the mathematical creativity that teachers and future teachers can display through a cognitive and perceptual exploration of their profiles. Specifically, the purpose of the research is to show that the creative process in geometry can be related to the perceptions and spatial reasoning of future teachers, and mainly the different way that they apprehend the geometrical figure. For the purposes of research, mathematical creativity in geometry is associated with problem posing and open-ended problem solving.

We address the following research questions:

(a) Which are the perceptions of future educators of creativity, mathematical creativity and creativity in the educational process?

(b) How the mathematical - creative abilities of potential teachers can influence the introduction of mathematical creativity in their teaching in geometry?

(c) What teaching practices do teachers follow in order to introduce their perceptions of creativity in teaching mathematics?

B. Population, Data Collection & Data Analysis

Data were collected from 116 students of thε department of primary level education and two mathematical teacher. All participants were asked to complete a two-part questionnaire and the two mathematical teacher called for a semi-structured interview and monitoring of their teaching. The first teacher has been involved in education for about 28 years and has been working in public secondary education for the last 10 years. The second teacher has been in the field of education for about 10 years and in the last year he has been working in private secondary education. The first part of questionnaire designed based on the model of Lev - Zamir and Leikin [32] (see figure 3) regarding the perceptions of future teachers and contained open-ended questions. The design of the questionnaire was based on two axes. The first axis contained questions concerning the personal data of each examinee, while the second axis was divided into two directions. In the first direction the future teachers were asked to mention an exercise from the textbook or their own which they consider creative, to justify why they consider it creative in terms of mathematical content and to identify what they generally consider creativity in Mathematics. This direction was designed with the aim of in-depth detection of teachers' perceptions of creativity in Mathematics. The second direction concerned teachers' perceptions of creativity in the teaching of Mathematics and teachers were asked to identify the characteristics of a creative teaching and to justify them with an example of a creative lesson, to state what is different from formal and creative teaching and to comment on whether creativity is the goal of their teaching.

	Teachers' conceptions of creativity in teaching mathematics	
	Teacher-directed conceptions	**Student-directed conceptions**
Flexibility	Mathematical flexibility ● Teacher transforms mathematical tasks: changing operations, changing the numbers ● Doing mathematics in different ways ● Using mathematical models Pedagogical flexibility ● Transforming instructional settings ● Adjusting the planned learning trajectory to students' needs and responses ● Suiting the content to students at different stages of learning	● Students generate various solutions to a particular problem, different from those presented/generated previously
Elaboration		● Students generalize mathematical ideas, and raise the level of mathematical discussion

Originality	Mathematical originality ● Generating mathematical tasks beyond the textbook Pedagogical originality ● Generating new* ideas suited to the teachers' own preferences ● Generating new* ideas in order to make the lesson interesting and enjoyable for students to stimulate students' mathematical reasoning	● Students generate new** ideas, new exercises, and discover new** facts ● Students suggest rare/insightful solutions to a problem
* new for teachers ** new for students	relatively new	

Fig 3: The model of teachers' conception of creativity in teaching mathematics (Lev - Zamir & Leikin, 2011)

The second part that concerns geometrical figure apprehension and geometrical creative abilities of future teachers includes five tasks. The first 3 questions of the questionnaire was structured according to the Duval theory and is characterized by three categories of tasks. The first category includes a tasks concerning students' geometrical figure perceptual ability and their recognition ability, the second category includes a task and examine the sequential apprehension of a geometrical figure the third category includes a task and examine students' operative apprehension to reconfiguration a geometrical figure. Regarding the future teachers creativity in the last two tasks of the test, we examine it through on a open – ended task and one problem to posed in geometry focusing on three components: fluency, flexibility and originality. Then, the aspects of creativity (fluency – flexibility - originality) were evaluated based on: (a) number of correct solutions to each task (for fluency), (b) the ability to shift between different ideas, i.e. the number of the ideas (for flexibility) and (c) the number of conventional solutions (for originality) and on the basis of the frequency of the solution across all students [5].

III. RESULTS

A. Results from the future teachers

Which are the perceptions of future educators of creativity, mathematical creativity and creativity in the educational process?

To the question «*how in your opinion would the concept of creativity be adapted to Mathematics?*», the 32% of teachers said through diagrams and drawings, 29% through problem solving in general, 6% through problem which solved in many ways, 14% through problem posing, 11% through examples from everyday life, while 8% either did not answer or their answer does not fall into any category.

When asked by future teachers to mention a math exercise / problem / task from their experience so far that they consider creative, the majority of students (73%) chose problems from the textbooks they remembered being the same students. The 15% of students suggested problems arising from geometry and using geometric instruments, 3% problems arising from the history of mathematics, and 9% of students suggest as creative open – ended problems, multiple solutions problem and problems in which the student is asked to pose a problem. In the reasons why they chose this problem, 57% of the participants said that it stimulates the students' interest, 21% that it makes the student to think, while 9% said that it makes the lesson more fun.

The data mentioned by the participants that they promote creativity in mathematics mainly concern the creation and solution of a problem by the students themselves (43%) and the connection of mathematics with examples from the daily life of students (19%). A large percentage of participants (29%), however, presented as elements that promote creativity the application of algorithmic techniques or the application of specific steps to solve a problem.

To the question *"Do you think that creativity should play a central role in the teaching practice or should it just be inherent?"*, the 45% of the participants answered that it should play a central role as it offers new experiences and helps in the personal development of each student, 22% answered that it should play a central role because it stimulates the student's interest, 18% answered that it should not holds a central role but is important, while 15% answered that they should not play a central role but take part sometimes during the school year.

How the mathematical - creative abilities of potential teachers can influence the introduction of mathematical creativity in their teaching in geometry?

In order to investigate the relationship between geometrical figure apprehension and mathematical creativity in problem posing of future teachers, multiple regression analysis was performed with independent variables geometrical figure apprehension abilities (perceptual apprehension; sequential apprehension; operative apprehension) and with dependent variables the components of mathematical creativity: fluency, flexibility and originality. Using the Enter method, statistically significant prediction models were obtained for each dependent variable (Feature: $R^2 = .14$, $F = 9.781$, $p < .05$; Flexibility: $R^2 = .12$, $F = 17.248$, $p < .05$; Originality: $R^2 = .11$, $F = 14.315$, $p < .05$). Table 1 presents the results of the multiple regression analysis.

Table 1. Multiple regression analysis which explore the relationship between geometrical figure apprehension and mathematical creativity components in problem posing.

		Perceptual Apprehension		Sequential Apprehension		Operative Apprehension	
		b*	p*	b	p	b	p
Mathematical Creativity	**Fluency**	.112	.041	.196	.001	.223	.001
	Flexibility	.094	.024	.168	.001	.117	.037
	Originality	.067	.001	.144	.112	.126	.087

* b = regression coefficient and p = p-value

Regression analysis gives interesting results for the components of the subject. The first result we deduce is that perceptual apprehension positively influences the creativity variables fluency, flexibility and originality ($p < .05$). The second result is that sequential apprehension related positively to the components of mathematical creativity, however, this influence is not statistically significant for the originality ($p > .05$). Specifically, data analysis showed that sequential apprehension is a statistically significant predictor of fluency (b = 196, p = .001) and flexibility (b = .168, p = .001).

Another result of the multiple regression analysis relates to the relationship between operative apprehension and the components of mathematical creativity. In particular, it was found that operative apprehension of a geometric figure is a statistically significant predictor of mathematical creativity components: fluency (b = .223, p = .001), flexibility (b = .117, p = .037), but not for originality ($p > .05$). This means that students who apprehend the geometrical figure operatively through reconfiguration of the figure achieve higher levels of fluency and flexibility in problem posing.

In order to investigate the relationship between geometrical figure apprehension and mathematical creativity in open – ended problem solving of future teachers, multiple regression analysis was performed with independent variables geometrical figure apprehension abilities (perceptual apprehension; sequential apprehension; operative apprehension) and with dependent variables the components of mathematical creativity: fluency, flexibility and originality. Using the Enter method, statistically significant prediction models were obtained for each dependent variable (Feature: $R^2 = .27$, $F = 16.541$, $p < .05$; Flexibility: $R^2 = .19$, $F = 19.862$, $p < .05$; Originality: $R^2 = .09$, $F = 14.431$, $p < .05$). Table 2 presents the results of the multiple regression analysis.

Table 2. Multiple regression analysis which explore the relationship between geometrical figure apprehension and mathematical creativity components in open-ended problem solving.

		Perceptual Apprehension		Sequential Apprehension		Operative Apprehension	
		b*	p*	b	p	b	p
Mathematical Creativity	**Fluency**	.131	.001	.067	.072	.234	.011
	Flexibility	.178	.014	.129	.123	.173	.001
	Originality	.116	.023	.088	.234	.215	.001

* b = regression coefficient and p = p-value

The first result we deduce is that perceptual apprehension positively influences the creativity variables fluency ($b = .131$, $p < .05$), flexibility ($b = .178$, $p < .05$) and originality ($b = .116$, $p < .05$) in solving open – ended tasks. The second result is that sequential apprehenion related positively to the components of mathematical creativity, however, this influence is not statistically significant for all components ($p > .05$). Another result of the multiple regression analysis relates to the relationship between operative apprehension and the components of mathematical creativity. In particular, it was found that operative apprehension of a geometric figure is a statistically significant predictor of mathematical creativity components: fluency ($b = .234$, $p = .011$), flexibility ($b = .173$, $p = .001$) and originality ($b = .215$, $p = .001$) when university students solved open – ended tasks. This means that future teachers who apprehend the geometrical figure operatively through reconfiguration of the figure achieve higher levels of fluency, flexibility and originality.

B. Results from the teachers

What teaching practices do teachers follow in order to introduce their perceptions of creativity in teaching mathematics?

Mathematical flexibility lies for teachers in trying to help students solve a problem in many ways and in a variety of ways. For example, the teacher mentions:

"If I ask why the sum of the angles of a triangle is 180o, it is creative to say that I measured the angles with a protractor, added them and found 180o, it is creative for them to think of the proof with the alternating angles."

During the lesson, they report that an attempt is made to present the problems in various ways, such as by presenting them on the board, in a worksheet formally and verbally in various ways, aiming at adapting the content to the level of the students.

Pedagogical flexibility is seen when students 'needs come first in teachers' priorities. No matter how much time they have devoted to the design of a lesson, they adjust its teaching according to the issues raised by the students.

"As much as I have prepared the lesson or the worksheet or I have in mind some things I want to say,..., I adapt according to the needs that arise in the classroom."

Then they report that the lesson differs from class to class and from student to student and so different teaching approaches appear each time.

Teachers associate creativity and in particular mathematical originality in the teaching of mathematics with the discovery of knowledge and this is done by adapting the content of the material to specially designed worksheets. The flow of matter changes, bringing earlier topics that are later, thus provoking interesting mathematical discussions. Giving an example from the textbook, the teacher states:

"The students have not been taught the secondary elements of a triangle, I left them and went straight here for an interesting discussion."

At the internship level, during the lesson, they choose to invite students to the board and let them make assumptions and come up with ideas, aiming to discover mathematical knowledge on their own.

The pedagogical originality seems to appear as the inspiration that is caused and guided in the lessons. In order to create conditions that favor its development, they try to understand the needs of the students, to ask them appropriate questions and to bring new ideas to the lesson that arise from their continuous training. They mention:

"Everything I consider creative has passed through the students' filter. "You try to lead things so that you fall on the difficulty that the students will face; we are two different entities that learn from each other."

Finally, the elaboration appears as an idea that students should generalize mathematical ideas. From the discussion in the classroom, the teacher considers that important results emerge that lead the students to draw general conclusions, something that would not be possible if he did the teaching without the students participating in it. The teacher states:

"Creativity is the student's ability to remove and generalize; if you want to get the material out then you do not have in mind to do a creative lesson"

IV. CONCLUSIONS

The aim of this study is to explore the perceptions and cognitive processes required in creative geometric activities, examining the mathematical creativity that teachers and future teachers can display through a cognitive and perceptual exploration of their profiles. It seems that their perception of creativity in geometry through the problems they choose is limited. The analysis of the teachers' answers showed that there is a difficulty in inventing problems that enhance creativity in the teaching of geometry, as a large percentage of the problems they suggested are from school textbooks. Elements of creativity are absent from participants' responses, such as ease and originality, findings that are consistent with findings and other research ([1], [33]). In addition, the way future teachers apprehend the geometrical figures seems to influence the way they approach creativity in learning geometry. More specifically, future educators with a developed sequential and operative apprehension of the geometrical figure seem to develop better ideas regarding problem posing, thus enhancing the introduction of fluency and flexibility in their lesson, in contrast to the open-ended problems in which the sequential apprehension of the geometrical figure does not affect the appearance of mathematical creativity.

In addition, two secondary school teachers with different years of teaching each were studied. Teachers adapt their teaching practices according to pre-existing experiences and new ideas they gain from their continuous training. Teachers' perceptions of creativity in teaching Mathematics converge, despite the difference in years of experience. Their main goal is to become creative themselves by bringing appropriate teaching practices in the classroom but also to make students creative by developing new, for them, mathematical skills. To achieve these goals, they design problems that are solved in many ways and different means, adapt the material to provoke interesting discussions, form worksheets and give students space to develop their thoughts and generalize their knowledge.

Its difference with corresponding researches in international literature is, that it is based both on the determination of the gap between students' knowledge (according to the results of the international research program PISA) and the teachings proposed in the Curriculum or the Interdisciplinary Single Framework of Mathematics Curriculum, as well as analysing the perceptions and competencies of teachers and potential teachers for creative teaching in Geometry. Therefore, this study can serve as a bridge for the revival of Geometry and the creative abilities of students. In addition, it can improve the overall development of mathematics teaching, young people's creative perception and release teachers from educational stereotypes.

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