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# CASP – CUSUM Schemes Based on Truncated Nadarajah-Haghighi Distribution

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***Abstract**: Acceptance sampling plans are introduced mainly to accept or reject the lots of finished products. There are several techniques available to control the quality. Some of the techniques are popularly used where testing involves destruction, for instance, in the manufacturing of crackers, bullets, batteries, bulbs and so on, it is impossible to go for 100% inspection. In this paper we optimized CASP-CUSUM Schemes based on the assumption that the continuous variable under consideration follows a Truncated Nadarajah-Haghighi Distribution. The Nadarajah-Haghighi Distribution is continuous distribution generally used in Life-time Analysis of products, particularly in estimating reliability by considering its distribution. Optimization of CASP-CUSUM Schemes is suggested based on numerical result obtained by changing the values of the parameters of the Nadarajah-Haghighi Distribution, finally based on the obtained results we determine the maximum values of Average Run Length and Probability of Acceptance*

**Keywords -** *CASP – CUSUM Schemes, Truncated Nadarajah-Haghighi Distribution, Optimal Truncated Point, Upper and lower Truncated Point, Average Run Length.*

## I. INTRODUCTION

In modern days consumers are sensitive with regard to the quality and reliability of the products. In ordered to meet consumer's satisfaction or demand, the manufacturer has to produce the items with high quality and reliability. Further, they have to make sure that the process should be free from assignable causes and eliminate assignable causes from the production process. For this statistical device have been introduced by many researchers. In the beginning, Dodge and Roming [20] introduced acceptance sampling plans for product control. Quality of the products is being maintained in two distinct phases namely 1. Process control 2. Product control. Maintaining the quality of the products by using quality control charts is called process control while maintaining the quality of the products by the sampling inspection method is called product control. As per as product control is concerned much research work has been carried out by Hawkins [3], Vardeman [8] etc.

The acceptance sampling plan was first applied in the US military for testing the bullets during World War II. For instance, if every bullet tested in advance, no bullets are available for shipment, and on the other hand, if no bullets are tested, then disaster may occur in the battlefield at the crucial time. An acceptance sampling plan is a middle path between 100% inspection and no inspection.

An acceptance sampling plan is an essential device in Statistical Quality Control. In most of the statistical quality control experiments 100 % inspection of the items produced is not possible due to various reasons. For instance, 100% inspection involves, more time, more money, manpower, material, machinery etc. even the sample is finite, hundred percent inspection practically not feasible in case of explosive type materials like crackers, bombs, batteries, bulbs etc. The test can be performed without waiting until all the products fail, and then testing time can be reduced significantly. For the purpose of reduces test time and cost, obviously truncated life model is used.

The item which is confirming the quality specifications required by the consumer is referred to as quality items. Quality of an item is subjected to the reliability, one should adopt certain measures such as life testing through various probability models, preventing measures, sampling inspection CUSUM Schemes etc. In the process of improving the quality of products, it should be examined whether the items produced performing their intended duties or not. The items are available up to the warranty time, and how best they satisfy the consumer needs.

Life tests experiments are carried out in order to obtain the lifetime of an item (i.e., time to its failure or the stops working satisfactorily). Sometimes, it may time consuming process as we have to wait until all the products fail in a life test if the lifetimes of products are high. Hawkins, D. M. [3] Proposed a fast accurate approximation for ARL‟s of a CUSUM Control Charts. This approximation can be used to evaluate the ARL‟s for Specific parameter values and the out-of-control ARL‟s of location and scale CUSUM Charts. Kakoty. S., Chakravarthy A.B. [5] Determined CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal Distribution. Generally, truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production engineering items, the sorting procedure eliminates items above or bellows designated tolerance limits. It is worthwhile to note that any continuous variable is first approximated as an exponential variable.

Vardeman.S, Di-ou Ray [8] was introduced CUSUM control charts under the restriction that the values are regard to quality is exponentially distributed. Further, the phenomena under study are the occurrence of the rate of rare events and the inter-arrival times for a homogenous poison process are identically independently distributed exponential random variables. Lonnie. C. Vance [6], considered Average Run Length of Cumulative Sum Control Charts for controlling for normal means and to determine the parameters of a CUSUM Chart. To determine the parameters of the CUSUM Chart the acceptable and unacceptable quality levels along with the desired respective ARL's are considered. Mohammed Akhtar. P and Sarma K.L.A.P [1] analyzed and Optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distribution and evaluate L (0), L' (0) and the probability of Acceptance and also Optimized CASP-CUSUM Schemes based numerical results. Narayana Murthy, B.R. and Mohammed Akhtar.P [9] proposed an Optimization of CASP CUSUM Schemes based on Truncated Log-logistic distribution and evaluate the probability of acceptance for different parameter values. Sainath. B and Mohammed Akhtar. P [11] studied an Optimization of CASP-CUSUM Schemes based on truncated Burr distribution and the results were analyzed at different values of the parameters. Venkatesulu.G and Mohammed Akhtar.P [12] Determined Truncated Gompertz Distribution and its Optimization of CASP-CUSUM Schemes by changing the values of the parameters and finally, critical comparisons are drawn based on the obtained numerical results. S. Dhanunjaya and P. Mohammed Akhtar [18] also studied "'Continuous Acceptance Sampling plans for Truncated Lindley Distribution Based on CUSUM Schemes

In the present paper, it is determined type-C OC Curves of CASP-CUSUM Schemes when the variable under study follows truncated Nadarajah-Haghighi Distribution. Thus, it is more worthwhile to study some interesting characteristics of Type-C OC Curves based on this distribution.

## II. NADARAJAH-HAGHIGHI DISTRIBUTION

A continuous random variable X assuming non-negative values is said to have Nadarajah-Haghighi distribution with shape and scale parameters α, λ > 0 and its probability density function is given by

$$f(x) = \alpha\lambda(1+\lambda x)^{\alpha-1}.\exp\{1-(1+\lambda x)^{\alpha}\} \ldots\ldots\ldots x>0;\ \alpha>0,\ \lambda>0 \ldots\ldots\ldots (1)$$

In the lifetime theory, we study the life of a system or an item. The word system is defined as an arbitrary device performing its intended task. A system can be an, electronic, mechanical, electrical or chemical device. Apart from manmade systems such as human beings, plants, animals, etc. that are included in the lifetime analysis.

A lifetime distribution represents an attempt to describe, mathematically, the length of the life of a system or device, with the advancement in technology new and complex types of consumable items are being produced every day. To model the failure data of such types of new devices we need more possible statistical distributions to be considered as lifetime models.

Nadarajah-Haghighi distribution is proposed as a model for lifetime. The problem of acceptance sampling plan when the life test is truncated at a predetermined time is studied. The Nadarajah-Haghighi Distribution was first proposed by Saralees Nadarajah and Firoozeh Haghighi [19]. The problem of acceptance sampling plan when the life test is truncated at a predetermined time is studied. In reliability studies, a sample of n items is put on the test and the experiment is terminated when all of them fail. This procedure may take a long time when the lifetime distribution of items has a long tail. Moreover, if the items are expensive such as medical equipment's, it is costly to gather the whole sample information. There are many situations where experimental units are lost or removed from the test, before the complete failure.

In view of the above, truncation is used in life testing to save time and cost testing units. The removal of units in a test may be pre-planned. Data obtained from such experiments are called Truncated sample. Life data are collected for estimating the parameters and reliability functions when complete sample information is not available, we use the information that "the Truncated experimental units did not fail up to a specific time" in the estimation procedure.

## III. TRUNCATED NADARAJAH-HAGHIGHI DISTRIBUTION

It is the ratio of the probability density function of the Nadarajah-Haghighi distribution to their corresponding cumulate distribution function at point T. The random variable x is said to follow a Truncated Nadarajah-Haghighi distribution define as

$$\boldsymbol{F_T(x)} = \frac{\alpha\lambda(1+\lambda x)^{\alpha-1}.\exp\{1-(1+\lambda x)^{\alpha}}{1-\exp\{1-(1+\lambda T)^{\alpha}\}} \ldots\ldots\ldots\ldots \quad (2)$$

Where "T' is the Truncated point of the Nadarajah-Haghighi distribution.

A new distribution with one parameter named a Truncated Nadarajah-Haghighi Distribution is proposed which is more flexible than many well-known heavily tailed distributions. The importance of Truncated Nadarajah-Haghighi distribution is illustrated by means of the two real datasets is evaluated. The results indicate that the new distribution can provide better fits than Exponential, Weibull, Gamma, Log-normal, Log-Logistic, Lindley, Gompertz and generalized extreme value distribution. it may provide an interesting alternative to describe income distributions and can also be applied in actuarial science, finance, bioscience, telecommunications and modelling lifetime data. Nadarajah-Haghighi - Distribution has been used in many reliabilities and survival analysis. Therefore, Truncated Nadarajah-Haghighi distribution can be an alternative for modelling the catastrophe loss in insurance applications.

## IV. DESCRIPTION OF THE PLAN AND TYPE- C OC CURVE

Battie [2] has suggested the method for constructing continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum $S_m = \sum(X_i - k_1)X_i's(i = 1,2,3........)$ are distributed independently and $k_1$ is the reference value. If the sum lies in the area of the normal chart, the product is accepted and if it lies on the return chart, then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., h+h'.
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

   When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

   The procedure, in brief, is given below.

1. Start plotting the CUSUM at 0.
2. The product is accepted $S_m = \sum(X_i - k) < h;$ when $S_m < 0$, return cumulative to 0.
3. When $h < S_m < h + h'$ the product is rejected: when $S_m$ crossed h, i.e., when $S_m > h + h'$ and continue rejecting product until $S_m > h + h'$ return cumulative to h+h'.

The Type-C, OC function, which is defined as the probability of acceptance of an item as a function of incoming quality, when the sampling rate is the same in acceptance and rejection regions. Then the probability of acceptance P (A) is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \quad \ldots\ldots\ldots (3)$$

Where L (0) = Average Run Length in acceptance zone and

L' (0) = Average Run Length in rejection zone.

Page E.S. [8] has introduced the formulae for L (0) and L' (0) as

$$L(0) = \frac{N(0)}{1 - P(0)} \quad \ldots\ldots\ldots (4)$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \quad \ldots\ldots\ldots (5)$$

Where P (0) =Probability for the test starting from zero on the normal chart,

N (0) = ASN for the test starting from zero on the normal chart,

P' (0) = Probability for the test on the return chart and

N' (0) = ASN for the test on the return chart

He further obtained integral equations for the quantities

P (0), N (0), P' (0), N' (0) as follows

$$P(z) = F(k_1 - z) + \int_0^h P(y)\, f(y + k_1 - z)\,dy, \quad \ldots\ldots (6)$$

$$N(z) = 1 + \int_0^h N(y)\, f(y + k_1 - z)\,dy, \quad \ldots\ldots (7)$$

$$P'(z) = \int_{k_1+z}^{B} f(y)\,dy + \int_0^h P'(y)\, f(-y + k_1 + z)\,dy \quad \ldots\ldots (8)$$

$$N'(z) = 1 + \int_0^h N'(y)\, f(-y + k_1 + z)\,dy, \quad \ldots\ldots (9)$$

$$F(x) = 1 + \int_A^h f(x)\,dx$$

$$F(k_1 - z) = 1 + \int_A^{k_1 - z} f(y)\,dy$$

and z is the distance of the starting of the test in the normal chart from zero.

The equations (6), (7), (8) & (9) are evaluated by developing computer programs according the to the iterative procedure explained in Jain M.K and et.al [4] by using Gauss-Chebyshev method of Integration and the Numerical results are evaluated and presented in the next section.

## V. NUMERICAL RESULTS FOR ARL's and P(A)

### TABLE 5.1

Values of ARL's and Type-C OC curve when λ =0.4 α =0.1 K= 1 h=0.04, h'= 0.04

| T | L (0) | L'(0) | P(A) |
|---|---|---|---|
| 5.0 | 2.25778 | 1.2165388 | 0.6498487 |
| 4.9 | 2.25571 | 1.2130654 | 0.6502904 |
| 4.8 | 2.25430 | 1.2096236 | 0.6507937 |
| 4.7 | 2.25358 | 1.2062131 | 0.6513626 |
| 4.6 | 2.25360 | 1.2028340 | 0.6520016 |

### TABLE 5.3

Values of ARL's and Type-C OC curve when λ =0.4 α =0.1 K= 3 h=0.04, h'= 0.04

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.0 | 5.87745 | 1.2165388 | 0.8284406 |
| 4.9 | 6.04150 | 1.2130654 | 0.8327860 |
| 4.8 | 6.23016 | 1.2096236 | 0.8374115 |
| 4.7 | 6.44454 | 1.2062131 | 0.8423406 |
| 4.6 | 6.68974 | 1.2028340 | 0.8475994 |

### TABLE 5.5

Values of ARL's and Type-C OC curve when λ =0.4 α =0.1 K=4 h=0.04, h'= 0.04

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.5 | 19.56835 | 1.0475469 | 0.9491874 |
| 4.4 | 24.41269 | 1.0468199 | 0.958830 |
| 4.3 | 32.99446 | 1.0460963 | 0.9692691 |
| 4.2 | 52.34325 | 1.0453757 | 0.9804195 |
| 4.1 | 136.74786 | 1.0446582 | 0.9924186 |

### TABLE 5.2

Values of ARL's and Type-C OC curve when λ =0.5 α =0.1 K= 1 h=0.04, h'= 0.04

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.0 | 3.43403 | 1.2165388 | 0.7384108 |
| 4.9 | 3.46301 | 1.2130654 | 0.7405806 |
| 4.8 | 3.49527 | 1.2096236 | 0.7429011 |
| 4.7 | 3.53116 | 1.2062131 | 0.7453836 |
| 4.6 | 3.57109 | 1.2028340 | 0.7480406 |

**TABLE 5.4**
Values of ARL's and Type-C OC curve when λ =0.4 α =0.1 K=2 h=0.04, h'= 0.04

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.0 | 14.35426 | 1.2165388 | 0.9218705 |
| 4.9 | 15.85386 | 1.2130654 | 0.9289230 |
| 4.8 | 17.81213 | 1.2096236 | 0.9364390 |
| 4.7 | 20.51195 | 1.2062131 | 0.9444606 |
| 4.6 | 24.40891 | 1.2028340 | 0.9530358 |

**TABLE 5.6**
Values of ARL's and Type-C OC curve when λ =0.4 α =0.1 K=4 h=0.02, h'= 0.02

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.5 | 19.36761 | 1.0871676 | 0.9468502 |
| 4.4 | 24.50413 | 1.0858995 | 0.9575655 |
| 4.3 | 33.96795 | 1.0846379 | 0.9690568 |
| 4.2 | 57.19342 | 1.0833833 | 0.9814097 |
| 4.1 | 203.98227 | 1.0821356 | 0.9947230 |

**TABLE 5.7**
Values of ARL's and Type-C OC curve when λ =0.4 α =0.1 K=5 h=0.02, h'= 0.02

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.5 | 27.39655 | 1.1002004 | 0.9613921 |
| 5.4 | 35.39157 | 1.0988693 | 0.9698861 |
| 5.3 | 50.85103 | 1.0975443 | 0.9788725 |
| 5.2 | 93.35766 | 1.0962254 | 0.9883941 |
| 5.1 | 728.31421 | 1.0949126 | 0.9984989 |

**TABLE 5.9**
Values of ARL's and Type-C OC curve when λ =0.5 α =0.1 K= 4 h=0.02, h'= 0.02

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.5 | 28.17401 | 1.1045029 | 0.9622760 |
| 4.4 | 36.46195 | 1.1027527 | 0.9706439 |
| 4.3 | 52.98985 | 1.1010147 | 0.9796451 |
| 4.2 | 102.07392 | 1.0992892 | 0.9893452 |
| 4.1 | 6079.56445 | 1.0975760 | 0.9998195 |

**TABLE 5.11**
Values of ARL's and Type-C OC curve when λ =0.5 α =0.1 K=4 h=0.03, h'= 0.03

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.6 | 37.81148 | 1.1767136 | 0.9698187 |
| 4.5 | 48.22946 | 1.1734812 | 0.9762467 |
| 4.4 | 68.52177 | 1.1702819 | 0.9832078 |
| 4.3 | 125.05375 | 1.1671156 | 0.9907534 |
| 4.2 | 1098.86047 | 1.1639818 | 0.9989418 |

**TABLE 5.13**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.1 K= 5 h=0.01, h'= 0.01

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.7 | 19.59743 | 1.0564915 | 0.9488479 |
| 5.6 | 22.75591 | 1.0557309 | 0.9556633 |
| 5.5 | 27.34324 | 1.0549731 | 0.9628507 |
| 5.4 | 34.60933 | 1.0542181 | 0.9704399 |
| 5.3 | 47.86192 | 1.0534656 | 0.9784635 |

**TABLE 5.8**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.1 K=5 h=0.03, h'= 0.03

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.7 | 61.70486 | 1.2145211 | 0.9806972 |
| 5.6 | 79.53914 | 1.2109079 | 0.9850042 |
| 5.5 | 115.17626 | 1.2073311 | 0.9896263 |
| 5.4 | 221.15071 | 1.2037901 | 0.9945862 |
| 5.3 | 13206.60840 | 1.2002845 | 0.9999091 |

**TABLE 5.10**

Values of ARL's and Type-C OC curve when λ =0.4 α =0.15 K= 4 h=0.02, h'= 0.02

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.5 | 19.50175 | 1.0878097 | 0.9471669 |
| 4.4 | 24.68477 | 1.0865113 | 0.957840 |
| 4.3 | 34.24879 | 1.0852203 | 0.9692868 |
| 4.2 | 57.79967 | 1.0839369 | 0.9815919 |
| 4.1 | 209.29443 | 1.0826614 | 0.9948537 |

**TABLE 5.12**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.1 K= 4 h=0.04, h'= 0.04

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.8 | 57.02152 | 1.2744859 | 0.9781377 |
| 4.7 | 69.89861 | 1.2689784 | 0.9821692 |
| 4.6 | 93.22216 | 1.2635484 | 0.9866271 |
| 4.5 | 147.58810 | 1.2581949 | 0.9915470 |
| 4.4 | 412.11450 | 1.2529167 | 0.9969690 |

**TABLE 5.14**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.1 K= 5 h=0.02, h'= 0.02

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.7 | 30.27943 | 1.1264583 | 0.9641322 |
| 5.6 | 36.04489 | 1.1245614 | 0.9697450 |
| 5.5 | 45.07286 | 1.1226768 | 0.9756973 |
| 5.4 | 61.19890 | 1.1208045 | 0.9820153 |
| 5.3 | 98.14566 | 1.1189445 | 0.9887276 |

**TABLE 5.15**

Values of ARL's and Type-C OC curve when λ =0.4 α =0.15 K=1 h=0.04, h'= 0.04

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 1.5 | 4.74934 | 1.1160530 | 0.8097224 |
| 1.4 | 5.62326 | 1.1141999 | 0.8346262 |
| 1.3 | 7.16622 | 1.1125067 | 0.856186 |
| 1.2 | 10.60449 | 1.1110085 | 0.9051676 |
| 1.1 | 24.87087 | 1.1097534 | 0.9572853 |

**TABLE 5.17**

Values of ARL's and Type-C OC curve when λ =0.4 α =0.15 K=4 h=0.03, h'= 0.03

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.6 | 19.6635 | 1.1428870 | 0.9450700 |
| 4.5 | 24.00372 | 1.1406502 | 0.9546360 |
| 4.4 | 31.28109 | 1.1384304 | 0.9648844 |
| 4.3 | 45.98552 | 1.362276 | 0.9578874 |
| 4.2 | 91.27106 | 1.134023 | 0.9877275 |

**TABLE 5.19**

Values of ARL's and Type-C OC curve when λ =0.4 α =0.15 K=5 h=0.02, h'= 0.02

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.5 | 27.69443 | 1.1011873 | 0.9617585 |
| 5.4 | 35.81343 | 1.0998181 | 0.9702053 |
| 5.3 | 51.56527 | 1.0984559 | 0.9791421 |
| 5.2 | 95.23612 | 1.0971005 | 0.9886114 |
| 5.1 | 817.23767 | 1.0957520 | 0.9986610 |

**TABLE 5.21**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.15 K=4 h=0.02, h'= 0.02

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.5 | 28.34934 | 28.34934 | 0.9624698 |
| 4.4 | 36.72370 | 1.1036458 | 0.9708241 |
| 4.3 | 53.47683 | 1.1018611 | 0.9798115 |
| 4.2 | 103.64549 | 1.1000903 | 0.9894975 |
| 4.1 | 25731.98047 | 1.0983329 | 0.9999573 |

**TABLE 5.16**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.05 K=4 h=0.01, h'= 0.01

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.6 | 16.40638 | 1.0480212 | 0.9399566 |
| 4.5 | 19.50578 | 1.0473028 | 0.9490440 |
| 4.4 | 24.33060 | 1.0465873 | 0.9587587 |
| 4.3 | 32.87287 | 1.0458746 | 0.9691653 |
| 4.2 | 52.11005 | 1.0451647 | 0.9803375 |

**TABLE 5.18**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.05 K=4 h=0.03, h'= 0.03

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.6 | 37.73491 | 1.1757687 | 0.9697829 |
| 4.5 | 48.09731 | 1.1725880 | 0.9762007 |
| 4.4 | 68.23575 | 1.1694387 | 0.9831505 |
| 4.3 | 124.02540 | 1.1663207 | 0.9906837 |
| 4.2 | 1017.99994 | 1.1632336 | 0.9988586 |

**TABLE 5.20**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.15 K= 3 h=0.04, h'= 0.04

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 3.6 | 31.38621 | 1.2145771 | 0.9627439 |
| 3.5 | 39.61244 | 1.2098382 | 0.9703633 |
| 3.4 | 55.79157 | 1.2051722 | 0.9788554 |
| 3.3 | 101.67709 | 1.2005789 | 0.9883300 |
| 3.2 | 1103.21399 | 1.1960579 | 0.9989170 |

**TABLE 5.22**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.15 K=5 h=0.01, h'= 0.01

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 5.5 | 27.60112 | 1.0556234 | 0.9631632 |
| 5.4 | 34.95918 | 1.0548437 | 0.9707102 |
| 5.3 | 48.40855 | 1.0540671 | 0.9786896 |
| 5.2 | 80.83660 | 1.0532938 | 0.9871377 |
| 5.1 | 268.46677 | 1.0525237 | 0.9960948 |

**TABLE 5.23**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.05 K=4 h=0.02, h'= 0.02

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.5 | 28.08806 | 1.1039684 | 0.9621825 |
| 4.4 | 36.33166 | 1.1022452 | 0.9705549 |
| 4.3 | 52.74288 | 1.1005337 | 0.9795605 |
| 4.2 | 101.26151 | 1.0988337 | 0.9892650 |
| 4.1 | 4282.07031 | 1.0971456 | 0.9997438 |

**TABLE 5.24**

Values of ARL's and Type-C OC curve when λ =0.5 α =0.05 K=4 h=0.04, h'= 0.04

| T | L (0) | L' (0) | P(A) |
|---|---|---|---|
| 4.8 | 57.13726 | 1.2728604 | 0.9782082 |
| 4.7 | 70.00655 | 1.2674471 | 0.9822173 |
| 4.6 | 93.28511 | 1.2621080 | 0.9866510 |
| 4.5 | 147.39223 | 1.2568418 | 0.9915449 |
| 4.4 | 407.62677 | 1.2516474 | 0.9969388 |

## VI. CONCLUSIONS

At the hypothetical values of the parameters, λ, α, K, h and h' are given at the top of each table, we determine optimum truncated point T at which P (A) the probability of accepting an item is maximum and also obtained ARL's values which represent the acceptance zone L (0) and rejection zone L'(0) values. The values of truncated point T of random variable X, L (0), L'(0) and the values for Type-C Curve, i.e., P (A) are given in columns I, II, III, and IV respectively. From the above tables 5.1 to 5.24 we made the following conclusions.

[1] From the Table 5.1 to 5.24, it is observed that the values of P (A) are increased as the value of truncated point decreases thus the Truncated point of the random variable and the various parameters for CASP-CUSUM are related.

[2] It is observed that increasing the value of K from Tables 5.1 to 5.24 that the value of the truncated point is maximized.

[3] From Table 5.1 to 5.24, it is observed that at the maximum level of probability of acceptance P (A) the Truncated point T from 10.0 to 1.1 as the value of h changes from 0.01 to 0.04.

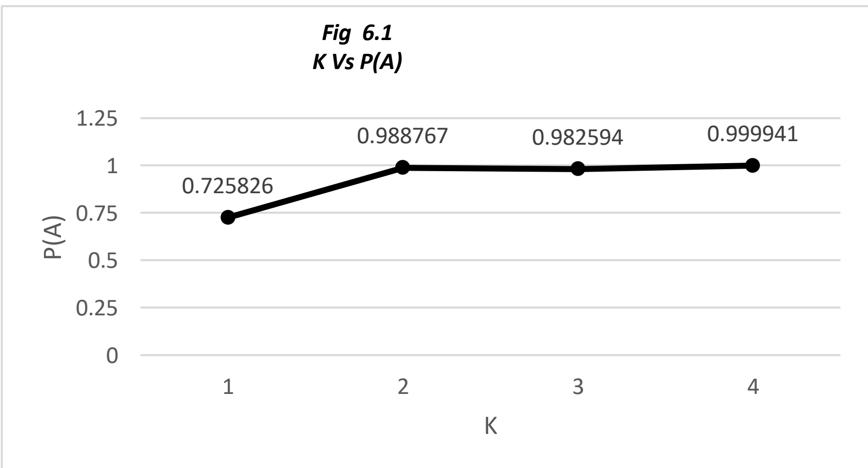
[4] From Table 5.1 to 5.24 it was observed that the Truncated point T changes from 4.5 to 4.1 and P (A) are as h $\to$ 0.02 maximum i.e., 0.9999573. Thus, Truncated point T and h are inversely related and hand P (A) are positively related.

[5] It is observed that the values of probability of acceptance increased as increase values of K as shown in table 6.1 and in figure 6.1.

**TABLE 6.1**

Values of ARL's and Type-C OC curve when λ =0.4 α =0.1 K=4 h=0.04, h'= 0.04

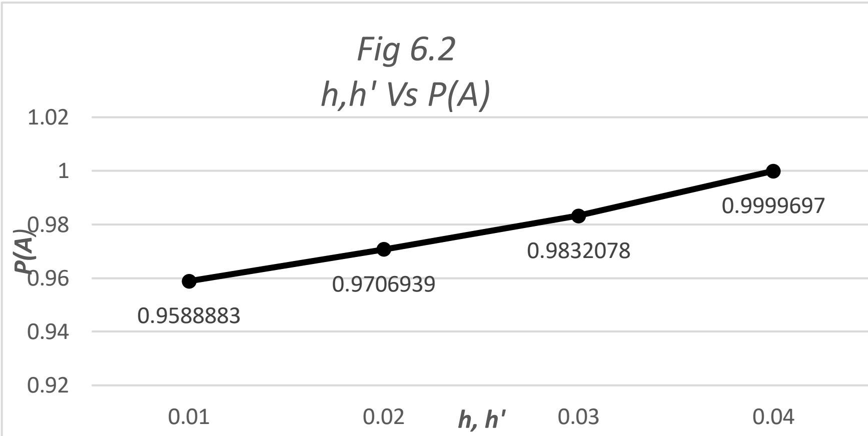
| K | *P(A)* |
|---|---|
| 1 | 0.725826 |
| 2 | 0.988767 |
| 3 | 0.982594 |
| 4 | 0.999941 |



[6] It is observed that the values of Maximum Probabilities increased as the values of h and h' as shown in table 6.2 and is shown as below the Figure 6.2

**TABLE 6.2**

| h, h' | P(A) |
|---|---|
| 0.01 | 0.9588883 |
| 0.02 | 0.9706939 |
| 0.03 | 0.9832078 |
| 0.04 | 0.9999697 |




**7**. The various relations exhibited among the ARL's and Type-C OC Curves with the parameters of the CASP -CUSUM based on the above table 5.1 to 5.24 are observed from the following. Table 7.1

**TABLE - 7.1**
**CONSOLIDATED TABLE**

| T | λ | α | h | h' | K | L(O) | P(A) |
|---|---|---|---|---|---|---|---|
| 4.6 | 0.4 | 0.1 | 0.04 | 0.04 | 1 | 2.2536 | 0.6520016 |
| 4.6 | 0.4 | 0.1 | 0.04 | 0.04 | 2 | 24.40891 | 0.9530358 |
| 4.6 | 0.01 | 0.1 | 0.04 | 0.04 | 3 | 6.68974 | 0.8475994 |
| 4.1 | 0.4 | 0.1 | 0.02 | 0.02 | 4 | 203.98227 | 0.994723 |
| 4.1 | 0.4 | 0.1 | 0.04 | 0.04 | 4 | 136.74786 | 0.9924186 |
| 5.1 | 0.4 | 0.1 | 0.02 | 0.02 | 5 | 728.31421 | 0.9984989 |
| 4.6 | 0.5 | 0.1 | 0.04 | 0.04 | 1 | 3.57109 | 0.7480406 |
| 4.1 | 0.5 | 0.1 | 0.02 | 0.02 | 4 | 6079.56445 | 0.9998195 |
| 4.2 | 0.5 | 0.1 | 0.03 | 0.03 | 4 | 1098.86047 | 0.9989418 |
| 4.4 | 0.5 | 0.1 | 0.04 | 0.04 | 4 | 412.1145 | 0.996969 |
| 5.3 | 0.5 | 0.1 | 0.01 | 0.01 | 5 | 47.86192 | 0.9784635 |
| 5.3 | 0.5 | 0.1 | 0.02 | 0.02 | 5 | 98.14566 | 0.987276 |
| **5.3** | 0.5 | 0.1 | 0.03 | 0.03 | 5 | **13206.6084** | **0.9999091** |
| 1.1 | 0.4 | 0.15 | 0.04 | 0.04 | 1 | 24.87087 | 0.9572853 |
| 4.1 | 0.4 | 0.15 | 0.02 | 0.02 | 4 | 209.29443 | 0.9948537 |
| 4.2 | 0.4 | 0.15 | 0.03 | 0.03 | 4 | 91.27106 | 0.9877275 |
| 5.1 | 0.4 | 0.15 | 0.02 | 0.02 | 5 | 817.23767 | 0.986661 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3.2 | 0.5 | 0.15 | 0.04 | 0.04 | 3 | 1103.21399 | 0.998917 |
| 4.1 | 0.5 | 0.15 | 0.02 | 0.02 | 4 | 25731.9805 | 0.999573 |
| 5.1 | 0.5 | 0.15 | 0.01 | 0.01 | 5 | 268.46677 | 0.9960948 |
| 4.2 | 0.5 | 0.05 | 0.01 | 0.01 | 4 | 52.11005 | 0.903375 |
| 4.1 | 0.5 | 0.05 | 0.02 | 0.02 | 4 | 4282.07031 | 0.9997438 |
| 4.2 | 0.5 | 0.05 | 0.03 | 0.03 | 4 | 1017.9994 | 0.9988586 |
| 4.4 | 0.5 | 0.05 | 0.04 | 0.04 | 4 | 407.62677 | 0.9969388 |

By observing the Table 7.1, we can conclude that the optimum CASP-CUSUM Schemes which have the values of ARL's and P(A) reach their maximum i**.e., 13206.6084** and **0.9999091** respectively, is

$$\begin{bmatrix} T = 5.3 \\ \lambda = 0.5 \\ \alpha = 0.1 \\ K = 5 \\ h = 0.03 \\ h' = 0.03 \end{bmatrix}$$

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