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Signed Unidomination Numberof a Rooted Product Graph of a Path with a Cycle

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Abstract *- The field of graph theory has grown exponentially in importance in Mathematics. The theory of graph domination has recently emerged as an exciting new area of graph theory research, attracting the attention of a wide range of scientists. The idea Dunbar et al. [3] introduced the concept of a signed dominating function. Siva Parvathi [9] investigates the signed dominating functions of corona product graphs* $C_n \odot K_m$ *and* $P_n \odot K_{1,m}$*. In 2019, Aruna [1] defined and studied the signed unidominating function, and the results on the signed unidomination number and the upper signed unidomination number of some corona product graphs are discussed here. Godsil and Mckay [4] introduced the rooted product, a new two-graph product denoted by* $G\&H$*, in 1978. Shobha Rani [8] studied by signed edge dominance on a rooted product graph. In this paper, we present the concepts signed unidominating function and upper signed unidomination number of a graph and studied these concepts for a rooted product graph of a path with a cycle.*

Keywords - *Rooted Product graph, Graph theory, unidominating parameters, applications*

I. INTRODUCTION

Due to its applications in various fields of science and technology, graph theory is currently undergoing and being motivated to a greater extent, and it has piqued the interest of many researchers. In their two books [5, 6], Haynes et al. provide an overview and extensive summary of domination in graphs and related topics. The theory of domination in graphs was introduced by Ore [7] and Berge [2], and it has become a popular area of graph theory research in the last three decades. This paper discovers the signed unidomination number and upper signed unidomination number of a rooted product graph of a path with a cycle. This graph also includes the number of minimal signed unidominating functions with maximum weight and the number of signed unidominating functions with minimum weight.

II. ROOTED PRODUCT OF P_n AND C_m

The rooted product of a path $P_n$with a cycle $C_m$is a graph formed by taking one replica of a *n*– vertex graph $P_n$and n replicas of C_m and then joining the *i*^th^ – vertex of $P_n$with any one vertex in *i*^th^ replica of C_m. Every *i*^th^ vertex of P_n is merging with any one vertex in every ith replica of C_m to identify the root. The rooted product of two graphs $P_n$and C_m is represented by P_no C_m, where $P_n$has *n* vertices and $C_m$has $(m-1)$ vertices in each replica of the graph C_m.

The vertices in path P_n are denoted by $v_i$for $i = 1,2, \dots \dots, n$ and the vertices of complete graph C_m are denoted respectively by u_{ij} for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$.

III. SIGNED UNIDOMINATION NUMBER OF P_no C_m

Signed unidomination function and number are defined in this section. One gets the signed unidomination number and the minimally weighted signed unidominating functions of P_no C_m.

Definition 1: Suppose $G(V,E)$ is a connected graph, $g: V \to \{-1,1\}$ is signed unidominating function is a function if

$$\sum_{u \in N[v]} g(u) \geq 1 \quad \forall v \in V \; if \; g(v) = 1$$

$$\text{and} \sum_{u \in N[v]} g(u) = 1 \quad \forall v \in V \; if g(v) = -1.$$

Definition 2: It is known as the signed unidomination number of a connected graph $G(V,E)$ by the formula $\min\{g(V)/g$ is a signed unidominating function$\}$.
It's represented by the identifier $\gamma_{su}(G)$. The weight of the signed unidominating function g is denoted by

$$g(V) = \sum_{u \in V} g(u)$$

.
Theorem 3.1: The following signed unidomination number of rooted product graph P_no C_m is

$$\begin{cases} \dfrac{nm}{3} \; if \;\; m \equiv 0(mod\ 3) \\ \dfrac{n(m+2)}{3} \; if \;\; m \equiv 1(mod\ 3) \\ \dfrac{n(m+4)}{3} \; if \;\; m \equiv 2(mod\ 3) \end{cases}$$

Proof: Consider the rooted product graph P_no C_m.

State a function $g: V \to \{-1,1\}$ through

$$g(v_i) = 1$$

$v_i = u_{ij}$for $i = 1,2, \ldots \ldots, n$ where u_{ij} is merged with the vertex v_i

and $g(u_{ij}) = \begin{cases} -1 \; if \; j \equiv 0(mod\ 3), \\ 1 \qquad otherwise \end{cases}$

for $i = 1,2, \ldots \ldots, n$ and $j = 1,2, \ldots \ldots, m$.

The vertices of P_n should have the functional value 1 assigned to them, and the vertices of C_m should have the functional values assigned in the following order: $1, 1, -1; 1, 1, -1; \ldots \ldots$...upto we get set of triple vertices u_{ij-1}, u_{ij}, u_{ij+1} and the remaining vertices the value 1.

The following scenarios arise when trying to figure out the signed unidomination number for P_noC_m

Case 1: Suppose $m \equiv 0(mod\ 3)$.

Let $i \neq 1$ and $i \neq n$.

If $v_i \in P_n$ then

$$\sum_{u\in N[v_i]} g(u) = g(v_{i-1}) + g(v_i) + g(v_{i+1}) + g(u_{i2}) + g(u_{im}) = 1+1+1+1+(-1) = 3.$$

Let $i = 1$ and $i = n$.

If $v_1 \in P_n$ then

$$\sum_{u\in N[v_1]} g(u) = g(v_1) + g(v_2) + g(u_{12}) + g(u_{1m}) = 1+1+1+(-1) = 2.$$

If $v_n \in P_n$ then

$$\sum_{u\in N[v_n]} g(u) = g(v_{n-1}) + g(v_n) + g(u_{n2}) + g(u_{nm}) = 1+1+1+(-1) = 2.$$

If $u_{ij} \in C_m$ then $g(u_{ij}) = 1$ or $g(u_{ij}) = -1$.

Sub Case 1: Let $u_{ij} \in C_m$ where $g(u_{ij}) = 1$, $g(u_{ij+1}) = -1$.

If $g(u_{ij-1}) = 1$ then

$$\sum_{u\in N[u_{ij}]} g(u) = g(u_{ij-1}) + g(u_{ij}) + g(u_{ij+1}) = 1+1+(-1) = 1.$$

Sub Case 2: Let $u_{ij} \in C_m$ where $g(u_{ij}) = -1$, $g(u_{ij+1}) = 1$.

If $g(u_{ij-1}) = 1$ then

$$\sum_{u\in N[u_{ij}]} g(u) = g(u_{ij-1}) + g(u_{ij}) + g(u_{ij+1}) = 1+(-1)+1 = 1.$$

Sub Case 3: Let $u_{ij} \in C_m$ where $g(u_{ij}) = 1$, $g(u_{ij+1}) = 1$.

If $g(u_{ij-1}) = -1$ then

$$\sum_{u\in N[u_{ij}]} g(u) = g(u_{ij-1}) + g(u_{ij}) + g(u_{ij+1}) = (-1)+1+1 = 1.$$

That is g is satisfying the conditions of a signed unidominating function.

Hence g is a signed unidominating function.

$$\text{Now } g(V) = \sum_{u\in P_n} g(u) + \sum_{u\in C_m} g(u)$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-times)} + \underbrace{\underbrace{\{(1+(-1)+1)+(1+(-1)+1)+\cdots+(1+(-1)+1)\}}_{(\frac{m-3}{3}-\text{times})} + (1+(-1))}_{n-times}$$

$$= n + \left(\frac{m-3}{3}\right)n + \big(1 + (-1)\big)n = \frac{nm}{3}.$$

Thus $g(V) = \frac{nm}{3}$.

The resulting functions are not signed unidominating functions for all other assignments of functional values 1 and -1 to P_n and vertices in each copy of C_m.

As a result, there is only one signed unidominating function, which is the one described above.

Therefore $\gamma_{su}(P_n \mathrm{o}\, C_m) = \frac{nm}{3}$ when $m \equiv 0(mod\ 3)$.

Case 2: Assume $m \equiv 1(mod\ 3)$.

Let $i \neq 1$ and $i \neq n$.

If $v_i \in P_n$ then

$$\sum_{u \in N[v_i]} g(u) = g(v_{i-1}) + g(v_i) + g(v_{i+1}) + g(u_{i2}) + g(u_{im}) = 1 + 1 + 1 + 1 + 1 = 5.$$

Let $i = 1$ and $i = n$.

If $v_1 \in P_n$ then

$$\sum_{u \in N[v_1]} g(u) = g(v_1) + g(v_2) + g(u_{12}) + g(u_{1m}) = 1 + 1 + 1 + 1 = 4.$$

If $v_n \in P_n$ then

$$\sum_{u \in N[v_n]} g(u) = g(v_{\mathrm{n}-1}) + g(v_{\mathrm{n}}) + g(u_{n2}) + g(u_{nm}) = 1 + 1 + 1 + 1 = 4.$$

As above case we prove for $u_{ij} \in C_m$.

That is, g is a signed unidominating function that meets all the requirements.

Hence g is a signed unidominating function.

$$\text{Now } g(V) = \sum_{u \in P_n} g(u) + \sum_{u \in C_m} g(u)$$

$$= \underbrace{(1+1+1 \ldots + 1)}_{(n-times)} + \underbrace{\underbrace{\{(1+(-1)+1) + (1+(-1)+1) + \cdots + (1+(-1)+1)\}}_{(\frac{m-1}{3}-\text{times})}}_{n-times}$$

$$= n + \left(\frac{m-1}{3}\right)n = n\left(\frac{m+2}{3}\right).$$

Thus $g(V) = n\left(\frac{m+2}{3}\right)$.

The resulting functions are not signed unidominating functions for all other assignments of functional values 1 and -1 to P_n and vertices in each copy of C_m.

As a result, there is only one signed unidominating function, which is the one described above.

Consequently, $\gamma_{su}(P_n \circ C_m) = n\left(\frac{m+2}{3}\right)$ is in the event that m$\equiv 1(mod\ 3)$.

Case 3: Assume $m \equiv 2(mod\ 3)$.

As above case we prove for $v_i \in P_n$.

If $u_{ij} \in C_m$ then $g(u_{\mathrm{ij}}) = 1$ or $g(u_{\mathrm{ij}}) = -1$.

For $j \neq m$.

Sub Case 1: Let $u_{ij} \in C_m$ where $g(u_{ij}) = 1$, $g(u_{ij+1}) = -1$.

If $g(u_{ij-1}) = 1$ then

$$\sum_{u \in N[u_{ij}]} g(u) = g(u_{ij-1}) + g(u_{ij}) + g(u_{ij+1}) = 1 + 1 + (-1) = 1.$$

Sub Case 2: Let $u_{ij} \in C_m$ where $g(u_{ij}) = -1$, $g(u_{ij+1}) = 1$.

If $g(u_{ij-1}) = 1$ then

$$\sum_{u \in N[u_{ij}]} g(u) = g(u_{ij-1}) + g(u_{ij}) + g(u_{ij+1}) = 1 + (-1) + 1 = 1.$$

Sub Case 3: Let $u_{ij} \in C_m$ where $g(u_{ij}) = 1$, $g(u_{ij+1}) = 1$.

If $g(u_{ij-1}) = -1$ then

$$\sum_{u \in N[u_{ij}]} g(u) = g(u_{ij-1}) + g(u_{ij}) + g(u_{ij+1}) = (-1) + 1 + 1 = 1.$$

For $j = m$.

Sub Case 4: Let $u_{im} \in C_m$ where $g(u_{im}) = 1$, $g(v_1) = 1$.

If $g(u_{im-1}) = 1$ then

$$\sum_{u \in N[u_{ij}]} g(u) = g(u_{im-1}) + g(u_{im}) + g(v_1) = 1 + 1 + 1 = 3.$$

That is, g is a signed unidominating function that meets all the requirements.

So g is a signed unidominating function.

$$\text{Now } g(V) = \sum_{u \in P_n} g(u) + \sum_{u \in C_m} g(u)$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-times)} + \underbrace{\underbrace{\{(1+(-1)+1)+(1+(-1)+1)+\cdots+(1+(-1)+1)\}}_{(\frac{m-2}{3}-\text{times})} \quad +1}_{n-times}$$

$$= n + \left(\frac{m-2}{3}\right)n + n = n\left(\frac{m+4}{3}\right).$$

Thus $g(V) = n\left(\frac{m+4}{3}\right)$.

The resulting functions are not signed unidominating functions for all other assignments of functional values 1 and -1 to P_n and vertices in each copy of C_m.

As a result, there is only one signed unidominating function, which is the one described above.

Consequently, $\gamma_{su}(P_n \text{o } C_m) = n\left(\frac{m+4}{3}\right)$ is the correct answer in the event that m$\equiv 2(mod\ 3)$.

Theorem 3.2: If $m \equiv 0(mod\ 3)$, $m \equiv 1(mod\ 3)$, $m \equiv 2(mod\ 3)$ then the number of signed unidominating functions of $P_n \text{o } C_m$ is 1 with minimum weights $\frac{nm}{3}, \frac{n(m+2)}{3}, \frac{n(m+4)}{3}$ respectively.

Proof: Follows by Theorem 3.1.

IV. UPPER SIGNED UNIDOMINATION NUMBER OF $P_n \text{o } C_m$

A minimal signed unidominating function and an upper signed unidomination number are introduced in this section. The number of minimal signed unidominating functions of maximum weight of $P_n \text{o } C_m$ and the upper signed unidomination number are determined.

Definition 1: Let g and h be functions with values ranging from $V$to $\{-1, 1\}$. We say that $h < g$ if $h(u) \leq g(u) \forall u \in V$, with strict inequality for at least one vertex u.

Definition 2: If for all $h < g,\ h$ is not a signed unidominating function, the signed unidominating function $g: V \rightarrow \{-1,1\}$ is called a minimal signed unidominating function.

Definition 3: The upper signed unidomination number of a graph $G(V, E)$ is defined as $\max\{g(V)/g\ \ is\ a\ minimal\ signed\ unidominating\ function\}$.

$\Gamma_{su}(G)$ is the symbol for it.

Theorem 4.1: For a rooted product graph $P_n o C_m$, the upper signed unidomination number is

$$\begin{cases} \frac{nm}{3} \ if \ \ m \equiv 0(mod\ 3) \\ \frac{n(m+2)}{3} \ if \ \ m \equiv 1(mod\ 3) \\ \frac{n(m+4)}{3} \ if \ \ m \equiv 2(mod\ 3) \end{cases}$$

Proof: Consider the rooted product graph $P_n o\, C_m$.

Describe a function $g: V \rightarrow \{-1,1\}$ through

$$g(v_i) = 1$$

$v_i = u_{ij}$ for $i = 1,2, \dots \dots, n$ where u_{ij} is merged with the vertex v_i

and $g(u_{ij}) = \begin{cases} -1 \ if \ j \equiv 0(mod\ 3), \\ 1 \qquad otherwise \end{cases}$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$.

By Theorem 3.1 shows that g is a signed unidominating function.

Case 1: Assume $m \equiv 0(mod\ 3)$.

Now we show for the minimality of g.

Describe a function $h: V \rightarrow \{-1,1\}$ through

$$h(v_i) = \begin{cases} -1 \ \ for\ v_1 = \ v_k \in P_n\ for\ some\ k, where\ k = 1 \\ 1 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

$v_i = u_{ij}$ for $i = 1,2, \dots \dots, n$ where u_{ij} is merged with the vertex v_i

and $h(u_{ij}) = \begin{cases} -1 \ if \ j \equiv 0(mod\ 3), \\ 1 \qquad otherwise \end{cases}$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$.

Suppose $1 = k$. Then $h(v_k) = -1$.

For $v_k \in P_n$ we have

$$\sum_{u \in N[v_k]} h(u) = h(v_\mathrm{k}) + h(v_2) + h(u_{k2}) + h(u_{km}) = (-1) + 1 + 1 + (-1) = 0 \neq 1.$$

This is the case when a signed unidominating function fails a vertex $v_k \in P_n$ where $h(v_k) = -1$ because it is in the vicinity of the vertex. Therefore h is not a signed unidominating function.

Since h is defined arbitrarily, there is no $h < g$ such that h is a signed unidominating function.

As a result, g is a minimal signed unidominating function.

g is the only minimal signed unidominating function because assigning the functional values -1,1 to the vertices of P_n and C_m in any other way does not make g any longer a signed unidominating function.

$$\text{Now } g(V) = \sum_{u \in P_n} g(u) + \sum_{u \in C_m} g(u)$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-times)} + \underbrace{\underbrace{\{(1+(-1)+1)+(1+(-1)+1)+\cdots+(1+(-1)+1)\}}_{(\frac{m-3}{3}-\text{times})} + (1+(-1))}_{n-times}$$

$$= n + \left(\frac{m-3}{3}\right)n + \big(1+(-1)\big)n = \frac{nm}{3}.$$

Thus $g(V) = \frac{nm}{3}$.

Because g is the only minimally signed unidominating function, so $\max\{g(V)\} = \frac{nm}{3}$.

Consequently $\Gamma_{su}(P_n \text{o} C_m) = \frac{nm}{3}$ when $m \equiv 0(mod\ 3)$.

Case 2: Assume $m \equiv 1(mod\ 3)$.

Now we demonstrate for the minimality of g.

Define a function $h: V \to \{-1,1\}$ by

$$h(v_i) = \begin{cases} -1 & for\ v_i = v_k \in P_n\ for\ some\ k, where\ k \neq 1, n \\ 1 & otherwise \end{cases}$$

$v_i = u_{ij}$ for $i = 1,2, \ldots \ldots, n$ where u_{ij} is merged with the vertex v_i

and $h(u_{ij}) = \begin{cases} -1 & if\ j \equiv 0(mod\ 3), \\ 1 & otherwise \end{cases}$

for $i = 1,2, \ldots \ldots, n$ and $j = 1,2, \ldots \ldots, m$.

Suppose $i = k$. Then $h(v_k) = -1$.

For $v_k \in P_n$ we have

$$\sum_{u \in N[v_k]} h(u) = h(v_{\text{k}-1}) + h(v_\text{k}) + h(v_{\text{k}+1}) + h(u_{k2}) + h(u_{km}) = 1 + (-1) + 1 + 1 + 1 = 3 \neq 1.$$

This is the case when a signed unidominating function fails near a vertex $v_k \in P_n$where $h(v_k) = -1$ because it is a local failure.

Thus h is not a signed unidominating function.

Since h is defined arbitrarily, there is no $h < g$ such that h is a signed unidominating function.

The function g is therefore a minimal signed unidominating function.

Other than that, there's no other way to assign the functional values -1,1 to P_n and C_m's vertices without also creating another signed unidominating function, so g is the only minimal one.

$$\text{Now } g(V) = \sum_{u \in P_n} g(u) + \sum_{u \in C_m} g(u)$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-times)} + \underbrace{\underbrace{\{(1+(-1)+1)+(1+(-1)+1)+\cdots+(1+(-1)+1)\}}_{(\frac{m-1}{3}-\text{times})}}_{n-times}$$

$$= n + \left(\frac{m-1}{3}\right)n = n\left(\frac{m+2}{3}\right).$$

Thus $g(V) = n\left(\frac{m+2}{3}\right)$.

Due to the fact that there is only one minimally signed unidominating function g, so max $\{g(V)\} = n\left(\frac{m+2}{3}\right)$.

Consequently, $\Gamma_{su}(P_n \mathrm{o} C_m) = n\left(\frac{m+2}{3}\right)$ when $m \equiv 1(mod\ 3)$.

Case 3: Suppose $m \equiv 2(mod\ 3)$.

Now we prove for the minimality of g.

Define a function $h: V \to \{-1,1\}$ by

$$h(v_i) = \begin{cases} -1 \ for\ v_i = v_k \in P_n\ for\ some\ k, where\ k \neq 1, n \\ 1 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

$v_i = u_{ij}$ for $i = 1,2, \ldots \ldots, n$ where u_{ij} is merged with the vertex v_i

and $h(u_{ij}) = \begin{cases} -1\ if\ j \equiv 0(mod\ 3), \\ 1 \quad otherwise \end{cases}$

for $i = 1,2, \ldots \ldots, n$ and $j = 1,2, \ldots \ldots, m$.

Suppose $i = k$. Then $h(v_k) = -1$.

For $v_k \in P_n$ we have

$$\sum_{u \in N[v_k]} h(u) = h(v_{\mathrm{k}-1}) + h(v_{\mathrm{k}}) + h(v_{\mathrm{k}+1}) + h(u_{k2}) + h(u_{km}) = 1 + (-1) + 1 + 1 + 1 = 3 \neq 1.$$

This is the case when a signed unidominating function fails near a vertex $v_k \in P_n$ where $h(v_k) = -1$ because it is a local failure.

So h is not a signed unidominating function.

Since h is defined arbitrarily, there is no $h < g$ such that h is a signed unidominating function.

The function g is therefore a minimal signed unidominating function.

Other than that, there's no other way to assign the functional values -1, 1 to P_n and C_m's vertices without also creating another signed unidominating function, so g is the only minimal one.

$$\text{Now } g(V) = \sum_{u \in P_n} g(u) + \sum_{u \in C_m} g(u)$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-times)} + \underbrace{\underbrace{\{(1+(-1)+1)+(1+(-1)+1)+\cdots+(1+(-1)+1)\}}_{(\frac{m-2}{3}-\text{times})} \;+1}_{n-times}$$

$$= n + \left(\frac{m-2}{3}\right)n + n = n\left(\frac{m+4}{3}\right).$$

Thus $g(V) = n\left(\frac{m+4}{3}\right)$.

Therefore $\gamma_{su}(P_n o C_m) = n\left(\frac{m+4}{3}\right)$when $m \equiv 2(mod\ 3)$.

Theorem 4.2: If $m \equiv 0(mod\ 3)$, $m \equiv 1(mod\ 3)$, $m \equiv 2(mod\ 3)$then the number of minimal signed unidominating functions of $P_n o C_m$ is 1 with maximum weights $\frac{nm}{3}, \frac{n(m+2)}{3}, \frac{n(m+4)}{3}$ respectively.

Proof: Follows by Theorem 4.1.

V. CONCLUSION

Graph theoretic properties and dominance parameters of a rooted product graph of a path with a cycle are interesting to investigate. There are two functions of this graph that are signed unidominating and minimal signed unidominating function studied by authors. Further research into the total signed unidominating function and the upper total signed unidominating function of the above rooted graph is aided by an understanding of this graph.

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