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Designing of Double Sampling Plan for Truncated Life Tests Based on Percentiles using Kumaraswamy Exponentiated Rayleigh Distribution

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Abstract *- This Paper develops the Double Sampling Plan for life test using percentiles under the Kumaraswamy Exponentiated Rayleigh Distribution. A truncated life test may be conducted to evaluate the smallest sample size to insure certain percentile life time of products. The minimum sample size, specified ratio and operating characteristic values are calculated for various quality levels. The plan parameters and the measures are also studied for the proposed sampling plan which is suitable for the manufacturing industries for the selection of samples. Numerical illustrations and tables are also provided.*

Keywords - *Double Sampling Plan (DSP), Kumaraswamy Exponentiated Rayleigh distribution, Life Tests, Percentiles, Producer's risk.*

I. INTRODUCTION

Quality Control is a system to achieve, assist and enhance the quality of a product or service. A statistical method employed to control, improve and maintain the quality of a product or to determine quality problem is termed as Statistical Quality Control, which negates personal bias and expose poor judgment. A most commonly used technique in statistical quality control is the Acceptance Sampling. Acceptance sampling Plan is the process of randomly inspecting a sample of items and deciding whether to accept or reject the whole lot. The sampling plans based on time-truncated life tests have been used to study the product reliability. In acceptance sampling, the concept of time-truncated life test is gaining more popularity. The life test acceptance sampling plan consists of sampling, inspection and decision making in determining the acceptance or rejection of a lot or batch of products by examining the continuous usage time of the items.

Many authors studied the designing of Single sampling plan and double sampling plan based on the truncated life test. Acceptance Sampling (AS) based on the truncated life test was first introduced by Epstein (1954) assuming the lifetime of the item follows the exponential distribution. Gupta and Groll (1961) promoted the reliability acceptance sampling under the gamma distribution. Kantam and Rosaiah (1998) established half logistic distribution for life test and developed sampling plan. Baklizi and EI Masri (2004) further designed reliability acceptance plan assuming the life time distribtion follows Birnbaum-Saunders distribution. Wu and Tsai (2005) progressed the acceptance sampling truncated life test plan assuring mean lifetime under Birnbaum-Saunders distribution. Balakrishnan et al. (2007) developed the truncated life tests for acceptance sampling under generalized Birnbaum-Saunders distribution.

Percentiles gives more information regarding the life time distribution than mean life does. When the life time distribution is symmetric, the 50th percentile or median is identical to the mean life. Hence, developing acceptance sampling plans based on percentiles for the life time distributions can be used as a generalization of developing acceptance sampling plans based on the mean life of products or items. Lio .et.al (2009) studied acceptance sampling for generalized Birnbaum-Saunders distribution using percentiles. Rosaiah et al. (2012) developed an acceptance sampling procedure for the inverse Rayleigh distribution percentile under a truncated life test. Pradeepa Veera Kumari and Ponneswari (2016) proposed the designing of acceptance sampling plan for life tests based on Percentiles of Exponentiated Rayleigh Distribution. Jayalakshmi and Neena Krishna (2021) studied the designing of Special Type Double Sampling Plan for life tests based on percentiles using Exponentiated Frechet Distribution.

Lord Rayleigh (1880) derived the Rayleigh distribution on the resultant of a large number of vibrations of the same pitch and of arbitrary phase. Kundu et al. (2005) gave different methods and estimations of Generalized Rayleigh distribution. Nasr Ibrahim Rashwan(2016) developed the Kumaraswamy Exponentiated Rayleigh Distribution. This paper describes the designing of double sampling plan for life test based on percentiles using Kumaraswamy Exponentiated Rayleigh Distribution. The purpose of proposing this plan through percentiles may satisfy the customer's expectation by rejecting the lot of low percentile, even when the lot is accepted when mean life of lifetime is considered.

II. KUMARASWAMY EXPONENTIATED RAYLEIGH DISTRIBUTION

Nasr Ibrahim Rashwan(2016) defined the cumulative distribution function and probability density function of Kumaraswamy Exponentiated Rayleigh distribution.

The Cdf of Kumaraswamy Exponentiated Rayleigh Distribution is given by

$$F(t,\lambda,\theta,a,b) = 1-[1-(1-e^{-(\lambda t)^2})^{\theta a}]^{b} \;.........(1)$$

And the corresponding pdf is given by

$$f(t,\lambda,\theta,a,b) = 2ab\theta\lambda^2 te^{-(\lambda t)^2}[1-e^{-(\lambda t)^2}]^{\theta a-1}[1-e^{-(\lambda t)^2}]^{b-1}, t,\lambda,\theta,a,b>0.........(2)$$

Where λ is the scale parameter and θ, a, b are the shape parameters.

The hazard function of any distribution is given by

$$H(t) = \frac{f(t)}{1-F(t)}..........(3)$$

The percentile or the q^{th} quantile of any distribution is given by,

$$p_r(T \le t_q) = q..........(4)$$

$$t_q = \frac{1}{\lambda}\left[-\ln\left[1-\left(1-(1-q)^{1/b}\right)^{1/\theta a}\right]\right]^{1/2}...........(5)$$

t_q and q are directly proportional. Let

$$\eta_q = \left[-\ln\left[1-\left(1-(1-q)^{1/b}\right)^{1/\theta a}\right]\right]^{1/2}..........(6)$$

By changing the parameter $\lambda = \frac{\eta_q}{t_q}$,then the cumulative distribution function can be written in the form

$$F(t) = 1-[1-(1-e^{-\left(\eta_q t/t_q\right)^2})^{\theta a}]^{b}..........(7)$$

Let $\delta_q = t/t_q$

$$F(t) = 1-[1-(1-e^{-\left(\eta_q \delta_q\right)^2})^{\theta a}]^{b}..........(8)$$

Taking partial derivative with respect to δ , we have

$$\frac{\partial F(t)}{\partial \delta} = b[1-(1-e^{-\left(\eta_q \delta_q\right)^2})^{\theta a}]^{b-1}\theta a(1-e^{-\left(\eta_q \delta_q\right)^2})^{\theta a-1}e^{-\left(\eta_q \delta_q\right)^2}2\eta_q\delta_q........(9)$$

Since $\frac{\partial F}{\partial \delta} > 0$, $F(t,\delta)$ is a non-decreasing function of δ.

III. DESIGNING OF DOUBLE SAMPLING PLAN

A double sampling plan is a sampling plan where two samples to be drawn from the lot and not always since the second sample is drawn only when the first one fails. These types of plan put in advantage by dropping inspection cost for a good or bad product. If the product is consistently good or bad, only one sample to be drawn habitually for acceptance or rejection of the lot. Thus in custom, acceptance double sampling plan can be an option for sentencing lot with less sample supply in average for life tests than using an acceptance single sampling plan. Life testing is a process of testing the life of a product to uncover faults and potential modes of failure in a short period of time.

The double acceptance sampling plan is represented as (n_1, n_2, c_1, c_2, δ_q^0). Here, n_i and c_i are the sample size and acceptance number associated with the i^{th} sample respectively, i=1, 2 with $n_1<n_2$ and $c_1<c_2$. In life testing study, the procedure is to terminate the test at a pre-determined time t . The probability of bad lot to be rejected is P^* and the maximum number of defectives allowed can be fixed as acceptance numbers. Now, the acceptance double sampling plan for percentiles based on the truncated life test is to set up the minimum sample sizes n_1 and n_2 for the specified acceptance number $c_1$and c_2 respectively such that the probability of accepting the bad lot that is the consumer's risk does not exceed *1-P**. A bad lot means that the true $100q^{th}$ percentile t_q is below the specified percentile t_q^0. Hence, the probability P^* is a confidence level in the sense that the chance of rejecting a bad lot with $t_q < t_q^0$ is at least equal to P^*.

IV. OPERATING PROCEDURE

The operating procedure of the proposed plan characterized by the acceptance double sampling plan is (n_1, n_2, c_1, c_2, δ_q^0) as follows:

Step 1: Select the first random sample of size n_1 from the lot and put them on test for time $t_0$and count the number of defectives d_1 and compare it with the acceptance number c_1 .

Step 2: If $d_1 \le c_1$, accept the lot.

Step 3*: If* $d_1 > c_2$ reject the lot.

Step 4*: If* $d_1 > c_2$ is obtained before time t_0, terminate the test and reject the lot.

Step 5*: If* $c_1 < d_1 \le c_2$, take a second sample of size n_2 from the remaining lot and put them on test for time t_0 and count the number of non-conformities (d_2).

Step 6: If $d_1 + d_2 \le c_1$, accept the lot.

Step 7: If $d_1 + d_2 > c_2$, reject the lot.

V. MINIMUM SAMPLE SIZE

The Sampling Plan is exemplified by (n_1, n_2, c_1, c_2, δ_q^0). Since the lots considered are sufficiently large also success and failure item attains in a frequent mode binomial distribution is applied. Here the problem is to find the sample size n_1 which is the least for the known P^*, t_q^0 and c. The smallest positive integer n_1 should satisfy

$$\sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} \le 1 - p^* \text{........(10)}$$

Here is the failure probability at time t given a specified 100qth percentile life time t_q^0 and p depends only on $p = F(t, \delta_q^0)$ using equation (8) . Thus the second sample size $n_2$can be computed using the following procedure for various *P**, acceptance numbers and $\frac{t}{t_q^0}$ using the inequality to determine the probability of acceptance for the proposed sampling plan as follows:

$$L(P)=\sum_{d_1=0}^{c1}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1}+[\sum_{d_1=c_1+1}^{c2}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1}]*[\sum_{d_1=0}^{c2-d_1}\binom{n2}{d2}p^{d_2}(1-p)^{n_2-d_2}]\le 1-p^* ....(11)$$

VI. OPERATING CHARACTERISTIC FUNCTION AND PRODUCER'S RISK

The operating characteristic function of the double sampling plan follows *B (n, c, p)* gives the probability of accepting the lot *L(p)* as

$$L(P)=\sum_{d_1=0}^{c1}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1}+[\sum_{d_1=c_1+1}^{c2}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1}]*[\sum_{d_1=0}^{c2-d_1}\binom{n2}{d2}p^{d_2}(1-p)^{n_2-d_2}] ....(12)$$

The producer's risk '*α'* is the probability of rejecting a lot when $t_q > t_q^0$. And for the given producer's risk, *p* as a function of d_q should be evaluated from the condition given by Cameron (1952) as

$$L(P)=\sum_{d_1=0}^{c1}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1}+[\sum_{d_1=c_1+1}^{c2}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1}]*[\sum_{d_1=0}^{c2-d_1}\binom{n2}{d2}p^{d_2}(1-p)^{n_2-d_2}]\ge 1-\alpha ....(13)$$

Where $p = F(t, \delta_q^0)$ and *F(.)* can be determined as a function of d_q. For the proposed sampling plan developed, the $d_{0.25}$ values are obtained at the producer's risk *α=0.05*.

VII. NUMERICAL EXAMPLE

Assume that the experimenter is interested in testing the life time of an electric lamp follows Kumaraswamy Exponentiated Rayleigh Distribution and setting up the true unknown 10^{th} percentile life $t_{0.25}$ is at least 1000hrs. Let $\theta = 2, a = 1, b = 0.5$, $\alpha = 0.05, \beta = 0.10$. It is desire to stop the experiment at time *t*=1500hrs. Then for the acceptance numbers c_1=0 and c_2=2 then from table 1, one can obtain the double sampling plan (n_1 n_2 c_1, c_2, t/t_q^0) = (4,7,0,2,1.50).

This explains, the experiment is done up to 1500hrs and the following decision is made

1) *d=0*, the lot is accepted.
2) $d \ge 2$, the lot is rejected and the inspector should advice the management to concentrate on the production process for better quality products.
3) d=1, the inspector is suggested to go for second sample.

The respective operating characteristic values *L(p)* for double sampling plan with P*=0.95 for 25^{th} percentile of Kumaraswamy Exponentiated Rayleigh Distribution obtained from table 1 is tabulated below.

t_q/t_q^0	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
L(P)	0.01202	0.02439	0.04496	0.30195	0.63552	0.83644	0.92715	0.96596

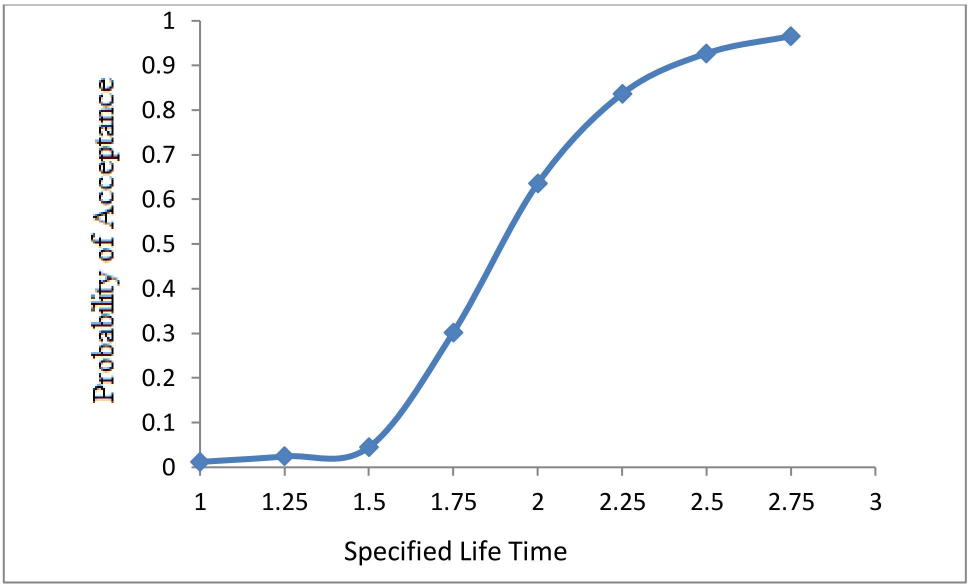

Figure 1: Operating Characteristic curve of DSP for 25^{th} percentile

This shows that if the actual 25^{th} percentile is equal to the required 25^{th} percentile the producer's risk is approximately 0.98789 (1- 0.01202). The producer's risk is almost equal to 0.05 or less when the actual 25^{th} percentile is greater than or approximately equal to 2.75 times the specified 25^{th} percentile.

Table 2 gives the values of $d_{0.25}$ for c_1=0 and c_2=2 and $t/t_{0.25}=1.50$ in order to assert that the producer's risk is less than or equals 0.05. In this example, the value of $d_{0.25}$ is 2.18861 for $c_1=0$, c_2=2, $t/t_{0.25}$ =1.5 and α =0.05 . This means the product can have a 25th percentile life of 2.18861 times the required 25^{th} percentile lifetime in order that under the above Double Sampling Plan the product is accepted with probability of at least 0.95.

VIII. CONSTRUCTION OF TABLE

Step 1 :- Find the value of η_q for an fixed acceptance number c_1=0, c_2=2.

Step 2: Set the value of $\frac{t}{t_q^0}$ =0.80, 0.85, 0.90, 1.00, 1.50, 2.00, 2.50, 3.00, 3.50 and 4.00.

Step 3: Find n_1 and n_2 by satisfying $L(P) \le 1-P^*$ when P^*=0.99, 0.95, 0.90 and 0.75 is the probability of rejecting the bad lot.

Step 4: Insert the constant t_q/t_q^0 =1.00, 1.25, 1.50, 1.75, 2.00, 2.25 , 2.50, 2.75 such that $\delta_q = (t/t_q^0)/(t_q/t_q^0)$ .Step 5: For the values of n_1, n_2, c_1, c_2 and p , L(P) is calculated using equation (12).

Step 6: For the $n_1 and\, n_2$ value obtained find the ratio $d_{0.25}$ such that $L(P) \ge 1-\alpha$, where $\alpha = 0.05$, p= F $(\frac{t}{t_q^0}, \frac{1}{d_q})$ and $d_q = t_q/t_q^0$.

Table 1:-The minimum sample sizes and OC values for double acceptance sampling plan (n_1 n_2 c_1, c_2, t/t_q^0)when c_1=0, and c_2=2 for 25th percentile of Kumaraswamy Exponentiated Rayleigh Distribution

P^*	n_1	n_2	t/t_q^0	t_q/t_q^0							
				1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
0.99	33	50	0.8	0.00135	0.00398	0.00994	0.18789	0.57066	0.82109	0.92670	0.96815
	28	36	0.85	0.00122	0.00376	0.00988	0.19526	0.57093	0.81499	0.92150	0.96503
	24	28	0.9	0.00113	0.00360	0.00970	0.19355	0.56132	0.80513	0.91523	0.96158
	17	23	1	0.00105	0.00387	0.00961	0.17521	0.53464	0.79000	0.90834	0.95836
	6	7	1.5	0.00109	0.00365	0.00894	0.14378	0.44723	0.70619	0.85300	0.92599
	3	5	2	0.00112	0.00283	0.00636	0.09710	0.34983	0.62091	0.80008	0.89639
	2	4	2.5	0.00058	0.00151	0.00347	0.05995	0.25320	0.51368	0.72198	0.84932
	2	3	3	0.00065	0.00128	0.00080	0.02451	0.13735	0.33978	0.55082	0.71435
	1	3	3.5	0.00156	0.00362	0.00742	0.07862	0.26504	0.50584	0.71112	0.84687
	1	3	4	0.00012	0.00178	0.00097	0.02191	0.11495	0.29430	0.50584	0.68915
0.95	24	34	0.8	0.00778	0.02347	0.04908	0.39090	0.74353	0.90203	0.96070	0.98301
	20	28	0.85	0.00573	0.02317	0.04817	0.38151	0.73345	0.89633	0.95791	0.98165
	16	26	0.9	0.01140	0.02419	0.04975	0.37401	0.72952	0.89619	0.95839	0.98198
	11	23	1	0.01395	0.02756	0.04985	0.34127	0.70355	0.88723	0.95593	0.98127
	4	7	1.5	0.01202	0.02439	0.04496	0.30195	0.63552	0.83644	0.92715	0.96596
	2	5	2	0.01088	0.02022	0.03494	0.22639	0.53865	0.77580	0.89868	0.95355
	2	3	2.5	0.00316	0.00730	0.01479	0.13735	0.38392	0.62316	0.78548	0.97969
	2	2	3	0.00688	0.01273	0.02150	0.12103	0.29704	0.48876	0.65064	0.96957
	2	2	3.5	0.00078	0.00181	0.00371	0.04011	0.14270	0.29704	0.56252	0.90868
	2	2	4	0.00087	0.00019	0.00048	0.01101	0.15923	0.35994	0.69704	0.84246
0.90	20	31	0.8	0.02296	0.04852	0.09044	0.49090	0.80543	0.92898	0.97210	0.98806
	20	28	0.85	0.00973	0.02317	0.04817	0.38151	0.73345	0.89633	0.95791	0.98165
	17	23	0.9	0.00961	0.02289	0.04751	0.37359	0.72337	0.89004	0.95467	0.98003
	9	20	1	0.03196	0.05745	0.09564	0.45967	0.78569	0.92314	0.97058	0.98758
	3	7	1.5	0.03652	0.06246	0.09957	0.42680	0.74099	0.89647	0.95759	0.98125
	2	4	2	0.01824	0.03477	0.05995	0.31778	0.62822	0.82409	0.91846	0.96074
	2	2	2.5	0.04286	0.06539	0.09354	0.29704	0.52434	0.70332	0.82208	0.89467
	1	3	3	0.01371	0.02530	0.04255	0.22742	0.50584	0.73863	0.87795	0.94690
	1	3	3.5	0.00156	0.00362	0.00742	0.07862	0.26504	0.50584	0.71112	0.84687
	1	3	4	0.00012	0.00038	0.00097	0.02191	0.11495	0.29430	0.50584	0.68915
0.75	14	23	0.8	0.09466	0.16129	0.24571	0.68907	0.89749	0.91430	0.95620	0.98415
	12	18	0.85	0.09500	0.16156	0.24507	0.68050	0.89045	0.91070	0.94453	0.98336
	11	13	0.9	0.09374	0.15954	0.24100	0.66211	0.77559	0.92300	0.95091	0.98166
	6	16	1	0.11368	0.17357	0.24815	0.67070	0.79399	0.92538	0.95722	0.97468
	2	7	1.5	0.11057	0.15885	0.21806	0.58939	0.74615	0.94785	0.98135	0.97254
	1	5	2	0.10458	0.14298	0.18892	0.49929	0.77794	0.92034	0.97412	0.98182
	1	3	2.5	0.08390	0.12651	0.17834	0.50584	0.77375	0.91198	0.96834	0.98890
	1	3	3	0.01371	0.02530	0.04255	0.22742	0.50584	0.73863	0.87795	0.94690
	1	3	3.5	0.00156	0.00362	0.00742	0.17862	0.26504	0.50584	0.71112	0.84687
	1	3	4	0.00012	0.00038	0.00097	0.12191	0.11495	0.29430	0.50584	0.68915

Table 2: Gives the ratio d0.25for accepting the lot with the producer's risk of 0.05

P^* \ t/t_q^0	0.80	0.85	0.90	1.00	1.50	2.00	2.50	3.00	3.50	4.00
0.99	2.17831	2.20575	2.23534	2.26161	2.48141	2.62883	2.80245	3.29544	2.72055	3.10420
0.95	1.99349	2.01294	2.00945	2.02464	2.18861	2.29896	2.74402	3.25753	3.80338	4.34462
0.90	1.89933	2.01269	2.03472	2.05874	2.09875	2.23970	2.71686	2.32834	2.71338	3.10852
0.75	1.72096	1.74441	1.76940	1.79538	1.81012	1.90175	1.94017	2.32679	2.71934	3.10121

IX. CONCLUSION

This paper indicates the designing of Double Sampling plan based on the percentiles of Kumaraswamy Exponentiated Rayleigh Distribution, when the life test is truncated for a pre-specified time. This plan is very helpful to the engineers and experimenters for saving the cost and time of experiment. The proposed plan is illustrated with certain numerical examples and infers with minimum sample size and experiment time. Useful tables are also given for the required sampling plan.

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