International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373/doi:10.14445/22315373/IJMTT-V68I1P505

Volume 68 Issue 1, 43-52, January, 2022
© 2022 Seventh Sense Research Group®

K_1 and K_2 Indices

V.R.Kulli

Professor, Department of Mathematics, Gulbarga University, Kalaburgi (Gulbarga), India

Abstract - *In this paper, we introduce the K_1 and K_2 graphical indices of a graph. We compute these two indices for some standard graphs and certain important chemical structures such as nanostar dendrimers. Also we establish some bounds on these two indices.*

Keywords - *molecular structure, K_1 index, K_2 index, nanostar demdrimer.*

Mathematics Subject Classification: *05C05, 05C09, 05C92.*

I. INTRODUCTION

Let G be a simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of edges incident to u. We refer [1], for other undefined notations and terminologies.

A molecular graph is a graph such that its vertices correspond to the atoms and edges to the bonds. Chemical Graph Theory is a branch of mathematical chemistry, which has an important effect on the development of Chemical Sciences. Several graphical indices [2] have been considered in Theoretical Chemistry and have found some applications, see [3, 4, 5].

The Randic index [6] of a graph G was defined as

$$R(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

Details of its Mathematical theory may be found in [7, 8].

This equation consists from 1 as numerator and geometric mean of end vertex degrees of an edge uv, $\sqrt{d_G(u)d_G(v)}$ as denominator.

Motivated by Randic index, we introduce the following graphical indices:

The K_1 index of a graph G is defined as

$$K_1(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\left(d_G(u)^2+d_G(v)^2\right)/2}}=\sum_{uv\in E(G)}\frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2+d_G(v)^2\right)}}.$$

This equation consists from 1 as numerator and quadratic mean of end vertex degrees of an edge uv, $\sqrt{\left(d_G(u)^2+d_G(v)^2\right)/2}$ as denominator.

The K_2 index of a graph is defined as

$$K_2(G)=\sum_{uv\in E(G)}\frac{1}{\left(d_G(u)^2+d_G(v)^2\right)/\left(d_G(u)+d_G(v)\right)}=\sum_{uv\in E(G)}\frac{d_G(u)+d_G(v)}{d_G(u)^2+d_G(v)^2}.$$

This equation consists from 1 as numerator and contraharmonic mean of end vertex degrees of an edge uv, $\left(d_G(u)^2+d_G(v)^2\right)/\left(d_G(u)+d_G(v)\right)$ as denominator.

The harmonic index of a graph G is defined as

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_G(u)+d_G(v)}.$$

This index was studied by Favaron et al. [9] and Zhong [10].

Recently, some new graphical indices were studied, for example, in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

In this paper, we compute the K_1 and K_2 indices for some standard graphs and some nanostar dendrimers. Also we establish some properties of these indices. For dendrimers, see [25].

II. RESULTS FOR SOME STANDARD GRAPHS

A. K_1 index

Proposition 1. Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$, and $s \geq 2$ vertices. Then

$$K_1(K_{r,s})=\frac{rs\sqrt{2}}{\sqrt{r^2+s^2}}.$$

Proof: Let $K_{r,s}$ be a complete bipartite graph with $r + s$ vertices and rs *edges* such that $|V_1| = r$, $|V_2| = s$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq r \leq s$, and $s \geq 2$. Every vertex of V_1 is incident with s edges and every vertex of V_2 is incident with r edges.

$$K_1(K_{r,s})=\frac{rs\sqrt{2}}{\sqrt{r^2+s^2}}.$$

Corollary 1.1. Let $K_{r,r}$ be a complete bipartite graph with $r \geq 2$. Then

$$K_1(K_{r,r})=r.$$

Corollary 1.2. Let $K_{1,r-1}$ be a star with $r \geq 2$. Then

$$K_1(K_{1,r-1})=\frac{(r-1)\sqrt{2}}{\sqrt{(r^2-2r+2)}}$$

Proposition 2. If G is r-regular with n vertices and $r \geq 2$, then $K_1(G)=\frac{n}{2}.$

Proof: Let G is r-regular with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then

$$K_1(G)=\frac{nr}{2}\frac{\sqrt{2}}{\sqrt{(r^2+r^2)}}=\frac{n}{2}.$$

Corollary 1.1. Let C_n be a cycle with $n\geq 3$ vertices. Then $K_1(C_n)=\frac{n}{2}.$

Corollary 1.1. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$K_1(K_n)=\frac{n}{2}.$$

Proposition 3. If G is a path with $n \geq 3$ vertices, then $K_1(P_n)=\frac{n}{2}+\frac{2\sqrt{2}}{\sqrt{5}}-\frac{3}{2}.$

B. K_2 index

Proposition 4. Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$, and $s \geq 2$ vertices. Then

$$K_2(K_{r,s}) = \frac{rs(r+s)}{r^2+s^2}.$$

Proof: Let $K_{r,s}$ be a complete bipartite graph with $r + s$ vertices and *rs edges* such that $|V_1| = r$, $|V_2| = s$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq r \leq s$, and $s \geq 2$. Every vertex of V_1 is incident with s edges and every vertex of V_2 is incident with r edges.

$$K_2(K_{r,s}) = \frac{rs(r+s)}{r^2+s^2}.$$

Corollary 4.1. Let $K_{r,r}$ be a complete bipartite graph with $r \geq 2$. Then

$$K_2(K_{r,r}) = r.$$

Corollary 4.2. Let $K_{1,r-1}$ be a star with $r \geq 2$. Then

$$QGK_2(K_{1,r-1}) = \frac{r(\mathrm{r}-1)}{r^2-2r+2}.$$

Proposition 5. If G is r-regular with n vertices and $r \geq 2$, then $K_2(G) = \frac{n}{2}.$

Proof: Let G is r-regular with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then

$$K_2(G) = \frac{nr(r+r)}{2(r^2+r^2)} = \frac{n}{2}.$$

Corollary 5.1. Let C_n be a cycle with $n \geq 3$ vertices. Then $K_2(C_n) = \frac{n}{2}.$

Corollary 5.1. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$QGK_2(K_n) = \frac{n}{2}.$$

Proposition 6. If G is a path with $n \geq 3$ vertices, then $K_2(P_n) = \frac{n}{2} - \frac{3}{10}.$

III. BOUNDS ON K_1 INDEX OF GRAPHS

Theorem 1. Let $H(G)$ be the harmonic index of a graph G. Then

$$\frac{\sqrt{2}}{2} H(G) \leq K_1(G) \leq H(G).$$

Proof. Let $a \geq b \geq 1$ be real numbers. Then $(a-b)^2 \geq 0.$

$a^2 + b^2 \geq 2ab$, this implies $2a^2 + 2b^2 \geq a^2 + b^2 + 2ab = (a+b)^2$

implying $$\frac{2}{a+b} \geq \frac{\sqrt{2}}{\sqrt{a^2+b^2}}.$$

For $a = d_G(u)$, $b = d_G(v)$, then the above inequality becomes

$$\frac{2}{d_G(u)+d_G(v)} \geq \frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2+d_G(v)^2\right)}}$$

By definitions, we have

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_G(u)+d_G(v)} \geq \sum_{uv\in E(G)}\frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2+d_G(v)^2\right)}}=K_1(G).$$

Equality holds if and only if G is a regular graph.

We have

$$\frac{1}{\sqrt{a^2+b^2}}=\frac{1}{\sqrt{(a+b)^2-2ab}} \geq \frac{1}{\sqrt{(a+b)^2}}=\frac{1}{(a+b)}$$

Implying $$\frac{\sqrt{2}}{\sqrt{a^2+b^2}} \geq \frac{\sqrt{2}}{2}\frac{2}{(a+b)}.$$

For $a = d_G(u)$, $b = d_G(v)$, by using the above inequality and definitions, we have

$$K_1(G)=\sum_{uv\in E(G)}\frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2+d_G(v)^2\right)}} \geq \frac{\sqrt{2}}{2}\sum_{uv\in E(G)}\frac{2}{d_G(u)+d_G(v)}=\frac{\sqrt{2}}{2}H(G).$$

Theorem 2. Let $R(G)$ be the Randic index of a graph G. Then

$$K_1(G) \leq R(G).$$

Proof. Let $a \geq b \geq 1$ be real numbers. Then $(a-b)^2 \geq 0$.

We get $a^2+b^2 \geq 2ab$. This implies $$\frac{\sqrt{2}}{\sqrt{a^2+b^2}} \leq \frac{1}{\sqrt{ab}}.$$

For $a = d_G(u)$, $b = d_G(v)$, by using the above inequality and definitions, we have

$$K_1(G)=\sum_{uv\in E(G)}\frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2+d_G(v)^2\right)}} \leq \sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}=R(G).$$

Equality holds if and only if G is a regular graph

In literature, there exist many upper bounds on the Randic index. Thus one can establish many upper bounds on the K_1 index by using Theorem 2. For example, It is a well-known fact that if G is a graph without isolated vertices then

$$R(G) \leq \frac{n}{2}.$$

Corollary 2.1. Let G be a graph with n vertices and minimum degree at least 1. Then

$$K_1(G) \leq \frac{n}{2}.$$

with equality if and only if G is a regular graph

IV. BOUNDS ON K_2 INDEX OF GRAPHS

Theorem 3. Let G be a connected graph G with m edges *and* minimum degree δ . Then

$$K_2(G) \le \frac{m}{\delta}.$$

Proof. For any edge *uv in E* (G), we can easily see that

$$\frac{d_G(u)+d_G(v)}{d_G(u)^2+d_G(v)^2} \le \frac{1}{\delta}.$$

with equality if and only if $d_G(u) = d_G(v) = \delta$. That is, equality holds if and only if G is regular.

By using the above inequality and definitions, we have

$$K_2(G) = \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{d_G(u)^2+d_G(v)^2} \le \sum_{uv\in E(G)} \frac{1}{\delta} = \frac{m}{\delta}$$

Theorem 4. Let G be a connected graph. Then

$$K_2(G) > \frac{1}{2}H(G).$$

Proof. Let $a \ge b \ge 1$ be real numbers. Then $(a+b)^2 = a^2+b^2+2ab > a^2+b^2.$

implying $\dfrac{a+b}{a^2+b^2} > \dfrac{2}{2(a+b)}.$

For $a = d_G(u)$, $b = d_G(v)$, by using the above inequality and definitions, we have

$$K_2(G) = \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{d_G(u)^2+d_G(v)^2} > \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)} = \frac{1}{2}H(G).$$

Theorem 5. Let G be a connected graph with m edges. Then

$$K_2(G) \le \frac{m}{2}.$$

Proof. Let $a \ge b \ge 2$ be real numbers. Then $(a+b)^2 = a^2+b^2+2ab > a^2+b^2.$

Implying $\dfrac{a+b}{a^2+b^2} \le \dfrac{1}{2}.$

For $a = d_G(u)$, $b = d_G(v)$, by using the above inequality and definitions, we have

$$K_2(G) = \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{d_G(u)^2+d_G(v)^2} \le \sum_{uv\in E(G)} \frac{1}{2} = \frac{m}{2}.$$

V. RESULTS FOR POLY ETHYLENE AMIDE AMINE DENDRIMER *PETAA*

We consider the family of poly ethylene amide amine dendrimers. This family of dendrimers is denoted by *PETAA*. The molecular graph of *PETAA* is presented in Figure 1.

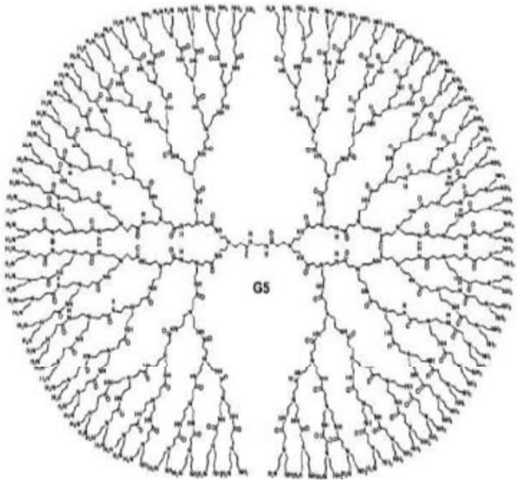

Fig 1. The molecular graph of *PETAA*

Let G be the molecular graph of *PETAA*. By calculation, we find that G has $44\times 2^n - 18$ vertices and $44\times 2^n - 19$ edges. In *PETAA*, there are three types of edges based on degrees of end vertices of each edge as given in Table 1.

Table 1. Edge partition of *PETAA*

$d_G(u), d_G(v) \setminus uv \in E(G)$	(1, 2)	(1, 3)	(2, 2)	(2, 3)
Number of edges	4×2^n	$4 \times 2^n - 2$	$16 \times 2^n - 8$	$20 \times 2^n - 9$

In the following theorem, we determine the K_1 and K_2 indices of *PETAA*.

Theorem 6. Let *PETAA* be the family of poly ethylene amide amine dendrimers. Then

(i) $$K_1(PETAA) = \left(\frac{4\sqrt{2}}{\sqrt{5}} + \frac{4}{\sqrt{5}} + 8 + \frac{20\sqrt{2}}{\sqrt{13}}\right)2^n - \frac{2}{\sqrt{5}} - 4 - \frac{9\sqrt{2}}{\sqrt{13}}.$$

(ii) $$K_2(PETAA) = \frac{256\times 2^n}{13} - \frac{537}{65}.$$

Proof: By using definitions and Table 1, we obtain

(i) $$K_1(PETAA) = \sum_{uv\in E(G)} \frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2 + d_G(v)^2\right)}}$$

$$= \frac{4\times 2^n\sqrt{2}}{\sqrt{(1^2+2^2)}} + \frac{(4\times 2^n - 2)\sqrt{2}}{\sqrt{(1^2+3^2)}} + \frac{(16\times 2^n - 8)\sqrt{2}}{\sqrt{(2^2+2^2)}} + \frac{(20\times 2^n - 9)\sqrt{2}}{\sqrt{(2^2+3^2)}}$$

After simplification, we obtain the desired result.

(ii) $$K_2(PETAA) = \sum_{uv\in E(G)} \frac{d_G(u) + d_G(v)}{d_G(u)^2 + d_G(v)^2}$$

$$= \frac{4\times 2^n(1+2)}{1^2+2^2} + \frac{(4\times 2^n - 2)(1+3)}{1^2+3^2} + \frac{(16\times 2^n - 8)(2+2)}{2^2+2^2} + \frac{(20\times 2^n - 9)(2+3)}{2^2+3^2}$$

giving the desired result

VI. RESULTS FOR PROPYL ETHER IMINE DENDRIMER *PETIM*

We consider the family of propyl ether imine dendrimers. This family of dendrimers is denoted by *PETIM*. The molecular graph of *PETIM* is depicted in Figure 2.

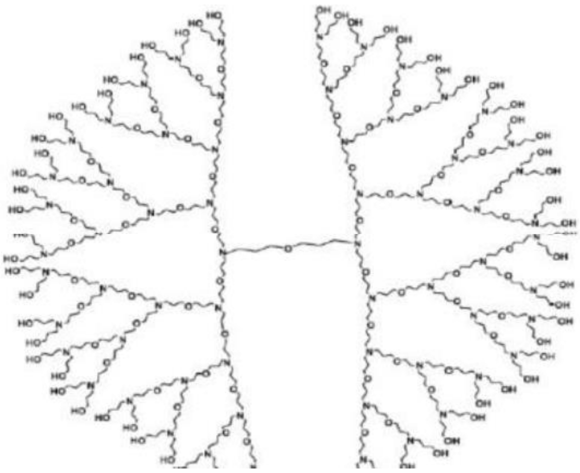

Fig 2. The molecular graph of *PETIM*

Let G be the molecular graph of *PETIM*. By calculation, we find that G has $24\times2^n - 23$ vertices and $24\times2^n - 24$ edges. In *PETIM*, there are three types of edges based on degrees of end vertices of each edge as given in Table 2.

Table 2. Edge partition of *PETIM*

$d_G(u), d_G(v) \setminus uv \in E(G)$	(1, 2)	(2, 2)	(2, 3)
Number of edges	2×2^n	$16 \times 2^n - 18$	$6 \times 2^n - 6$

In the following theorem, we compute the K_1 and K_2 indices of *PETIM*.

Theorem 7. Let *PETIM* be the family of porpyl ether imine dendrimers. Then

(i) $$K_1(PETIM) = \left(\frac{2\sqrt{2}}{\sqrt{5}} + 8 + \frac{6\sqrt{2}}{\sqrt{13}}\right)2^n - 9 - \frac{6\sqrt{2}}{\sqrt{13}}.$$

(ii) $$K_2(PETIM) = \frac{748\times2^n}{65} - \frac{147}{13}.$$

Proof: By using definitions and Table 2, we obtain

(i) $$K_1(PETIM) = \sum_{uv\in E(G)} \frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2 + d_G(v)^2\right)}}$$

$$= \frac{2\times2^n\sqrt{2}}{\sqrt{\left(1^2+2^2\right)}} + \frac{\left(16\times2^n-18\right)\sqrt{2}}{\sqrt{\left(2^2+2^2\right)}} + \frac{\left(6\times2^n-6\right)\sqrt{2}}{\sqrt{\left(2^2+3^2\right)}}$$

giving the desired result

(ii) $$K_2(PETIM) = \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{d_G(u)^2 + d_G(v)^2}$$

$$= \frac{2\times2^n(1+2)}{1^2+2^2} + \frac{\left(16\times2^n-18\right)(2+2)}{2^2+2^2} + \frac{\left(6\times2^n-6\right)(2+3)}{2^2+3^2}.$$

After simplification, we obtain the desired result.

VII. RESULTS FOR ZINC PROPHYRIN DENDRIMER DPZ_N

We consider the family of zinc prophyrin dendrimers. This family of dendrimers is denoted by DPZ_n, where n is the steps of growth in this type of dendrimers. The molecular graph of DPZ_n is shown in Figure 3.

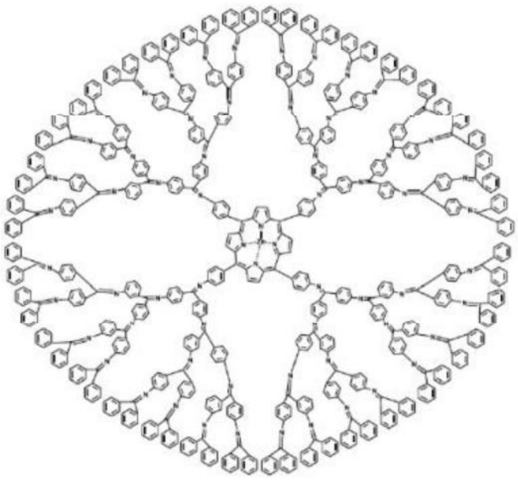

Fig 3. The molecular graph of DPZ_n

Let G be the molecular graph of DPZ_n. By calculation, we obtain that G has $56\times2^n - 7$ vertices $64\times2^n - 4$ edges. In DPZ_n, there are four types of edges based on degrees of end vertices of each edge as given in Table 3.

Table 3. Edge partition of DPZ_n

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)	(3, 4)
Number of edges	$16\times2^n - 4$	$40\times2^n - 16$	$8\times2^n + 12$	4

In the following theorem, we determine the K_1 and K_2 indices of DPZ_n.

Theorem 8. Let DPZ_n be the family of zinc prophyrin dendrimers. Then

(i) $$K_1(DPZ_n) = \left(\frac{32}{3} + \frac{10\sqrt{2}}{\sqrt{13}}\right)2^n + 2 - \frac{16\sqrt{2}}{\sqrt{13}} + \frac{4\sqrt{2}}{5}.$$

(ii) $$K_2(DPZ_n) = \frac{566\times2^n}{39} - \frac{986}{325}.$$

Proof: From definitions and by using Table 3, we deduce

(i) $$K_1(DPZ_n) = \sum_{uv\in E(G)} \frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2 + d_G(v)^2\right)}}$$

$$= \frac{(16\times2^n - 4)\sqrt{2}}{\sqrt{(2^2+2^2)}} + \frac{(40\times2^n - 16)\sqrt{2}}{\sqrt{(2^2+3^2)}} + \frac{(8\times2^n + 12)\sqrt{2}}{\sqrt{(3^2+3^2)}} + \frac{4\sqrt{2}}{\sqrt{(3^2+4^2)}}$$

This gives the desired result after simplification.

(ii) $$K_2(DPZ_n) = \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{d_G(u)^2 + d_G(v)^2}$$

$$= \frac{(16\times2^n - 4)(2+2)}{2^2+2^2} + \frac{(40\times2^n - 16)(2+3)}{2^2+3^2} + \frac{(8\times2^n + 12)(3+3)}{3^2+3^2} + \frac{4(3+4)}{3^2+4^2}$$

After simplification, we obtain the desired result.

VIII. RESULTS FOR PORPHYRIN DENDRIMER D_NP_N

We consider the family of porphyrin dendrimers. This family of dendrimers is denoted by D_nP_n. The molecular graph of D_nP_n is shown in Figure 4.

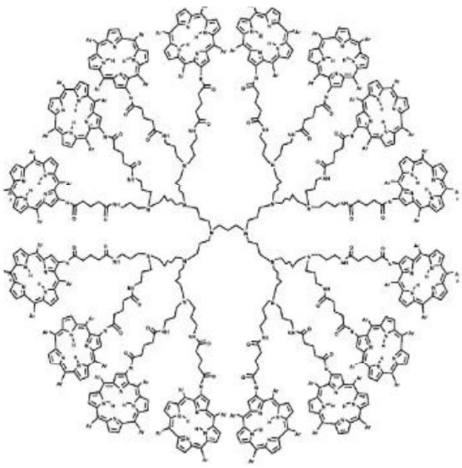

Fig 4. The molecular graph of D_nP_n

Let G be the molecular graph of D_nP_n. By calculation, we find that G has $96n - 10$ vertices and $105n - 11$ edges. In D_nP_n, there are six types of edges based on degrees of end vertices of each edge as given in Table 4.

Table 4. Edge partition of D_nP_n

$d_G(u), d_G(v) \setminus uv \in E(G)$	(1, 3)	(1, 4)	(2, 2)	(2, 3)	(3, 3)	(3, 4)
Number of edges	$2n$	$24n$	$10n - 5$	$48n - 6$	$13n$	$8n$

In the following theorem, we compute the K_1 and K_2 indices of D_nP_n.

Theorem 9. Let D_nP_n be the family of porphyrin dendrimers. Then

(i) $$K_1(D_nP_n) = \left(\frac{2}{\sqrt{5}} + \frac{24\sqrt{2}}{\sqrt{17}} + 5 + \frac{48\sqrt{2}}{\sqrt{13}} + \frac{13}{3} + \frac{8\sqrt{2}}{5}\right)n - \frac{5}{2} - \frac{6\sqrt{2}}{\sqrt{13}}.$$

(ii) $$K_2(D_nP_n) = \left(\frac{4}{5} + \frac{120}{17} + 5 + \frac{240}{13} + \frac{13}{3} + \frac{56}{25}\right)n - \frac{125}{26}.$$

Proof: From definitions and by using Table 4, we deduce

(i) $$K_1(D_nP_n) = \sum_{uv \in E(G)} \frac{\sqrt{2}}{\sqrt{\left(d_G(u)^2 + d_G(v)^2\right)}}$$

$$= \frac{2n\sqrt{2}}{\sqrt{(1^2+3^2)}} + \frac{24n\sqrt{2}}{\sqrt{(1^2+4^2)}} + \frac{(10n-5)\sqrt{2}}{\sqrt{(2^2+2^2)}} + \frac{(48n-6)\sqrt{2}}{\sqrt{(2^2+3^2)}} + \frac{13n\sqrt{2}}{\sqrt{(3^2+3^2)}} + \frac{8n\sqrt{2}}{\sqrt{(3^2+4^2)}}.$$

After simplification, we get the desired result.

(ii) $$K_2(D_nP_n) = \sum_{uv \in E(G)} \frac{d_G(u) + d_G(v)}{d_G(u)^2 + d_G(v)^2}$$

$$= \frac{2n(1+3)}{1^2+3^2} + \frac{24n(1+4)}{1^2+4^2} + \frac{(10n-5)(2+2)}{2^2+2^2} + \frac{(48n-6)(2+3)}{2^2+3^2} + \frac{13n(3+3)}{3^2+3^2} + \frac{8n(3+4)}{3^2+4^2}$$

giving the desired result

IX. CONCLUSION

In this paper, we have introduced two novel graphical indices which are K_1 and K_2 indices and computed exact values of some standard graphs. Furthermore we have determined these two indices for certain nanostar dendrimers. Also we have obtained some properties of these indices.

Many questions are suggested by this research, among them are the following:

1. Characterize the K_1 and K_2 indices in terms of other degree based topological indices.
2. Obtain the extremal values and extremal graphs of K_1 and K_2 indices.
3. Compute these two indices for other chemical nanostructures.

REFERENCES

[1] V.R. Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
[2] V.R.Kulli, Graph indices, in Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2020) 66-91.
[3] I. Gutman and O.E. Polansky, Mathematical Concepts in Organic Chemistry, Springer, Berlin (1986).
[4] V.R.Kulli, Multiplicative Connectivity Indices of Nanostructures, LAP LEMBERT Academic Publishing, (2018).
[5] R.Todeschini and V. Consonni, Handbook of Molecular Descriptors for Chemoinformatics, Weinheim, Wiley-VCH, (2009).
[6] M.Randic, On characterization of molecular branching, Journal of American Chemical Society, 97 (1975) 6609-6615.
[7] X.Li and I.Gutman, Mathematical Aspects of Randic Type Molecular Structure Descriptors, Univ. Kragujevac, Kragujevac, (2006).
[8] I.Gutman and B.Furtula, Recent Results in the Theory of Randic Index, Univ. Kragujevac, Kragujevac, (2008).
[9] O.Favaron, M.Maho and F.J.Sacle, Some eigenvalue properties in graphs (conjectures of Graffiti II), discrete Math. 111(1-3) (1993) 197-220.
[10] L.Zhong, The harmonic index for graphs, Appl. Math Lett 25(3) (2012) 561-566.
[11] I.Gutman, V.R.Kulli and I.Redzepocic, Nirmala index of Kragujevac trees, International Journal of Mathematics Trends and Technology, 67(6) (2021) 44-49.
[12] V.R.Kulli, Computation of distance based connectivity status neighborhood Dakshayani indices, International Journal of Mathematics Trends and Technology, 66(6) (2020) 118-128.
[13] V.R.Kulli, Computation of status neighborhood indices of graphs, International Journal of Recent Scientific Research, 11(4) (2020) 38079-38085.
[14] V.R.Kulli, Hyper Zagreb-K-Banhatti indices of graphs, International Journal of Mathematics Trends and Technology, 66(8) (2020) 123-130.
[15] V.R.Kulli, Some new status neighborhood indices of graphs, International Journal of Mathematics Trends and Technology, 66(9) (2020) 139-153.
[16] V.R.Kulli, Harmonic Zagreb-K-Banhatti index of a graph, International Journal of Mathematics Trends and Technology, 66(10) (2020) 123-132.
[17] V.R.Kulli, Neighborhood Sombor index of some nanostructures, International Journal of Mathematics Trends and Technology, 67(5) (2021) 101-108.
[18] V.R.Kulli, Different versions of Nirmala index of certain chemical structures, International Journal of Mathematics Trends and Technology, 67(7) (2021) 56-63.
[19] V.R.Kulli, B.Chaluvaraju and T.Vidya, Computation ofAdriatic (a, b)-KA index of some nanostructures, International Journal of Mathematics Trends and Technology, 67(4) (2021) 79-87.
[20] V.R.Kulli and I.Gutman, (a, b)-KA indices of benzenoid systems, International Journal of Mathematics Trends and Technology, 67(1) (2021) 17-20.
[21] V.R.Kulli, Some new Kulli-Basava topological indices, Earthline Journal of Mathematical Sciences, 2(2) (2019) 343-354.
[22] V.R.Kulli, Contraharmonic-quadratic index of certain nanostar dendrimers, International Journal of Mathematical Archive, 13(1) (2022).
[23] V.R.Kulli, Geometric-quadratic and quadratic-geometric indices, submitted.
[24] V.R.Kulli, ABC Banhatti and augmented Banhatti indices of chemical networks, Journal of Chemistry and Chemical Sciences, 8(8) (2018) 1018-1025.
[25] V.R.Kulli, B.Chaluvaraju, V.Lokesha and S.A.Basha, Gourava indices of some dendrimers, Research Review International Journal of Multidisciplinary, 4(6) (2019) 212-215.