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Designing of Special Type Double Sampling Plan for Truncate Life Test using Gompertz Frechet Distribution

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Abstract - *In this Paper deals with designing of Special Type Double Sampling plan for truncated life test using Gompertz Frechet Distribution. The lifetime test is truncated at pre-determined time t_0 and Gompertz Frechet Distribution is considered as a life time random variable. The minimum sample size, Operating Characteristic function and values, Producer's risk is also discussed. The results are illustrated by an example.*

Keyword - *Gompertz Frechet distribution, Producer's risk, Special type double sampling plan, Lifetime test.*

I. INTRODUCTION

Acceptance sampling plan can be ensured the decision from accept or reject the lot. The decision of the sample results is subject to the risks to the Producer and Consumer. The producer's risk and consumer's risk lead to minimize to a certain level and it will increase the cost of inspection. Truncation of life test helps to reduce the cost inspection. In various sampling plan use to take the decision, rejecting or accepting the lot. In special type double sampling plan is effective and economic friendly model of producer and consumer is also called zero-one double sampling plan.

The Reliability Acceptance Sampling Plan for percentiles using various distribution has been developed by Epstein (1954), Sobel and Tischendrof (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Kantam and Rosaiah (1998), Kantam et al. (2001), Rosaiah and Kantam (2005), Rosaiah et al. (2006), Rao et al. (2008) and Rao et.al (2009a). Ram Kumar (2011) has developed by the four parameters burr distribution using Reliability Acceptance test. Muthulakshmi(2015) has developed the Special Type Double Sampling plan for truncated lifetests using Marshall-olkin Extended Exponential distribution.

Some other studies related with the extending the various distributions using Gompertz family distribution can be listed as Alizadeh, Cordeiro, Pinho, and Ghosh (2017), Merovci, Khaleel, Ibrahim, and Shitan (2016), Oguntunde, Adejumo, Okagbue, and Rastogi (2016), Oguntunde et al.(2018). Jayalakshmi (2009) has designing a single sampling plan using various procedure of Quick Switching System. Jayalakshmi.S, Neena Krishna P.K (2020) has developed a Special Type Double Sampling Plan using percentiles based on Exponentiated Frechet distribution.

This paper describing the designing of a Special Type Double Sampling Plans truncated life tests based on Percentiles using Gompertz Frechet Distribution. Oguntunde et.al (2019) has developed by Gompertz Frechet distribution Properties and applications. The real life application of Gompertz Frechet distributions are hydrology, finance, reliability Studies and so on.

II. GOMPERTZ FRECHET DISTRIBUTION

The life time distribution of the product follows as Gompertz Frechet distribution with the α be a scale parameter and β, γ, θ be a shape parameters. The Cumulative Distribution Function (cdf) and Probability Density Function (pdf) of the Gompertz Frechet distribution is given by,

$$F(x,\alpha,\beta,\gamma,\theta) = 1 - \exp\left[\frac{\theta}{\gamma}\left(1-\left\{1-exp\left[\frac{\alpha}{x}\right]^{\beta}\right\}^{-\gamma}\right)\right] \quad (1)$$

and

$$f(x,\alpha,\beta,\gamma,\theta) = \theta\beta\alpha^{\beta}X^{-\beta-1}\exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]\left\{\exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]\right\}^{-\gamma-1} * \exp\left[\frac{\theta}{\gamma}\left(1-\left\{1-exp\left[\frac{\alpha}{x}\right]^{\beta}\right\}^{-\gamma}\right)\right] \quad (2)$$

where $\alpha > 0, \beta > 0, \gamma > 0, \theta > 0$.

III. PROCEDURE FOR SPECIAL TYPE SAMPLING PLAN

From an accepted lot samples of n_1 items are selected at random and inspected. If the number of defective x_1 found in the sample is equal or not exceeds to c_1 the lot is accepted without further any inspection. If x_1 is greater than c_1 but does not excess c_2, a second random sample of size n_2 is drawn from the remains the lot and inspected. If the number of defective found in the second random sample, x_2 is such a way (x_1+x_2) does not reaches c_2, the lot is accepted without any inspection. Even so, if x_1 excess c_2 or if (x_1+x_2)$>$ c_2, the lot is rejected and the products remaining in the lot are all inspected. The defectives products are reject able. We also create the regular assumption that the examine process is error free. i.e. that reject products are not accepted and accept products are not rejected.

$$P_a = P\,(x_1 \le c_1) + \sum_{r=c_{1+1}}^{c_2} P(x=r).P(0 \le x \le c_2\text{-}1). \quad (3)$$

In case c_1=0 and c_2=1: we assure that n_1 and n_2 are large enough and p small enough so that Binomial and Poisson approximation respectively and enhances sufficiently exact representation of the behavior in which we are fascinated. Then, Binomial approximation is

$$P_a(P) = (1-p)^{n_1}\left[(1-p)^{n_2} + n_2\,p(1-p)^{(n_2-1)}\right]$$

$$=(1-p)^{n_1-1}.[(1-p)+\emptyset np] \quad (4)$$

And the Poisson approximation is

$$P_a = e^{-n_1\,p} + e^{pn_1}.n_1pe^{-n_2p}$$

$$= e^{-n_1\,p} + n_1pe^{-(n_1+n_2)p} \quad (5)$$

For a predetermined value of P_a, as n_1p decreases n_1+n_2 must increase and since n_1p becomes little increases in n_1+n_2 must be substantial than the decreases $n_{1.}$ for large value of c_1 and c_2 necessary same things occurs.

For c_1=1, we wants to find the minimum value of n_1 for which P($x_1 \le 1$)<Pa but c_1=0, we need to find the lowest value of n_1 for which P($x_1 = 0$) ≤Pa. It is very easy to see that this low value of n_1 is always least for c_1=1.

A. Operating Procedure for Special Type Double Sampling Plan

The operating Procedure for Special Type Double Sampling Plan is as follows:

Step 1:

Select a random sample size from the lot n_1 units and detects the number of defectivesx_1. If $x_1 \geq 1$, then reject the lot. If $x_1 = 0$, then go to a randomly selected a second sample n_2 and find the number of defectivex_2.

Step 2:

If $x_2 \leq 1$, then accept the lot or if $x_2 \geq 2$, reject the lot.

B. Minimum Sample size

To determine the minimum sample size for special purpose plan of special type double sampling plan is characterized by $(n_1, n_2, c_{1,} c_2, t/t_q^0)$. The sizes of lots are abundantly large and also success or failures reach in repeat mode binomial distribution is used. The minimum sample size is determined by

$$L(p) = (1-p)^{n_1-1}.[(1-p) + \emptyset np] \leq 1 - p^* \quad (6)$$

Where p is the number of fail items before time t_0 which is given by

$$p = 1 - \exp\left[\frac{\theta}{\gamma}\left(1 - \left\{1 - exp\left[\frac{-1}{\varphi_q \delta_q}\right]^{\beta}\right\}^{-\gamma}\right)\right] \quad (7)$$

To observe the minimum Average sample Number is include the probability of acceptance not exceeds the 1-P^* and $n_2 \leq n_1$.

Detection of minimum sample sizes of STDS as

$$\left.\begin{array}{ll} \text{Minimum} & ASN = n_1 + n_2(1-p)^{n_1} \\ \text{Subject to} & (1-p)^{n_1-1}.[(1-p) + \emptyset np] \leq 1 - p^* \end{array}\right\} \quad (8)$$

- Increments in the consumer's confidence level increments the first and the second sample sizes expeditiously, when test time is very short
- Increments in shape parameters increments sample sizes for any P*.
- An increment in δ slightly decreases the sample size for any P*.

C. Operating Characteristic Function

According to the ensures quality of the accepted items are describes a execution of the sampling plan. If the actual percentile will be gradually increased also specified lifetime rapidly increases.

$$L(p) = (1-p)^{n_1-1}.[(1-p) + \emptyset np] \quad (9)$$

D. Producer's Risk Ratio

The Producer's risk is the probability of refusing the good quality of the lot when $t_q > t_q^0$ and the ratio of actual lifetime to the specified life time t_q/t_q^0.

$$L(p) = (1-p)^{n_1-1}.[(1-p) + \emptyset np] \leq 1 - \alpha \quad (10)$$

∴ α =0.05 , is the probability of rejecting the lot.

IV. EXAMPLE FOR LIFE TEST USING GOMPERTZ FRECHET DISTRIBUTION

Assume that an inspector conducted a test on the lifetime of the battery for a mobiles and he was focused on using a proposed sampling plan for lifetime of test of the product. It was based on testing lifetime item which observed a Gompertz Frechet distribution with using shape parameters $= 6, \beta = 3, \gamma = 0.06$. Inspector expect to stopping the runtime of experiment is 4,000 hrs but the experimenter has tests to unknown parameter of true percentile time $t_{0.10}$=2000 hrs, c_1=0, c_2=1, α = 0.05, β=0.05, then, φ=0.7785 is computed from the table 4.4 and then the special type double sampling plan is characterized by (2, 2, 0, 1, 1.2).

The respective illustration of operating characteristic curve for L(p) for special type double sampling plan with P^*=0.95 for 10th percentile of Gompertz Frechet distribution.

Fig. 1 Example for mobile battery

The experiment conducting a lifetime test up to 4000 hrs and the inspector was taken a managerial decision as follows:

- If $x_1 \geq 1$, then reject the lot.
- If $x_1 = 0$, then go to a randomly selected a second sample n_2.
- If $x_2 \leq 1$, then accept the lot.
- If $x_2 \geq 2$, reject the lot

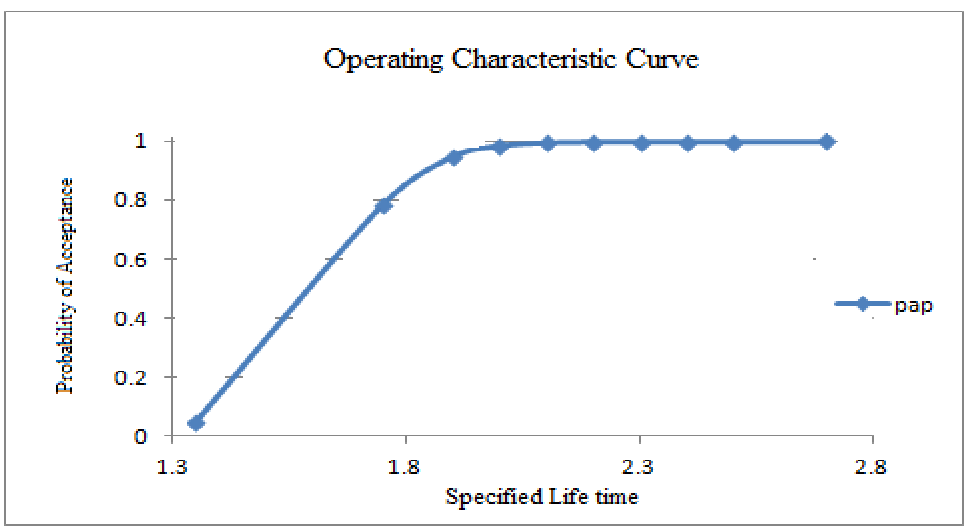

Fig.2 Operating characteristic curve for STDSP using 10th percentile for Gompertz Frechet distribution

Table 1. (n, c_1,c_2 t/$t^0_{0.10}$) = (2, 2, 0, 1.2) with p* = 0.95 under Gompertz Frechet distribution

t_q/t^0_q	1.35	1.75	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6
L(p)	0.0466	0.7859	0.9488	0.9848	0.9962	0.9992	0.9998	0.9999	0.9999	1.000

It indicates that if the true to life 10th percentile is almost equal to the essential 10th percentile ($t_{0.10}/t^0_{0.10}$ = 1.2) the producer's risk nearly 0.9848(1-0.015). The producer's risk is an almost inexact equal to Zero whenever the true to life10th percentile is greater than or equal to 2 times the specified 10th percentile.

V. CONSTRUCTION TABLE FOR MINIMUM SAMPLE SIZE AND OPERATING CHARACTERISTIC FOR SPECIAL TYPE DOUBLE SAMPLING PLAN USING GOMPERTZ FRECHET DISTRIBUTION

The procedure of proposed plan based on life test as given below:

Step 1: To compute the value of φ=0.7785, c_1=0, c_2=1 and t/t^0_q=0.9, 0.95, 1.0, 1.1, 1.25, 1.5, 1.6, 1.65, and 1.7.

Step 2: To compute the minimum sample size value of $n_1 and\ n_2$ satisfying

$$L(p) =(1-p)^{n_1-1}.[(1-p)+\emptyset np] \leq (1-p^*),$$

where p^* is the probability of accepting the good lot.

Step 3: To establish the constant values of t_q/t^0_q= 1.35, 1.75, 1.9,2, 2.1, 2.2, 2.3, 2.4, 2.5 and 2.6. and to extend this $\delta = {t/t^0_q}\Big/{t_q/t^0_q}$.

Step 4: To calculate the values of L(p) using the values of n, c, p in equation.

A. Construction table for Producer's Risk for Special type Double Sampling plan using Gompertz Frechet distribution

Step 1: To compute the value of φ=0.7785, c_1=0, c_2=1 and t/t^0_q=0.9, 0.95, 1.0, 1.1, 1.25, 1.5, 1.6, 1.65, and 1.7.

Step 2: Find the insignificant value of n_1 and $n_2$satisfying

$$(1-p)^{n_1-1}.[(1-p)+\emptyset np] \leq (1-p^*),$$

Where p^* is the probability of accepting the good lot.

Step 3: For n_1 and n_2 value obtained finds the ratio d_q such that

$$(1-p)^{n_1-1}.[(1-p)+\emptyset np] > (1-\alpha)$$

Where $\alpha = 0.05$, p=F$\left(\frac{t_q}{t^0_q}.\frac{1}{d_q}\right)$ and d_q=t_q/t^0_q.

VI. CONCLUSION

In this paper describes Special Type Double Sampling (STDS) Plan using truncated lifetime test for Gompertz Frechet distribution. The STDS plan have a cases of an acceptance number is c_1=0 and c_2=1, and use to construct the table values of minimum sample size, operating characteristic function and ratio between true lifetime and predetermined lifetime.

Table 2. Minimum sample sizes and operating characteristic values for special type double sampling plan (n_1, n_2, c_1=0, c_2=1, $t/t_{0.10}$) for a given p* under Gompertz Freche distribution.

P^*	t/t_q	n_1	n_2	t_q/t_q^0									
				1.35	**1.75**	**1.9**	**2**	**2.1**	**2.2**	**2.3**	**2.4**	**2.5**	**2.6**
0.75	0.9	2	2	0.8597	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.95	2	2	0.7022	0.9993	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	2	2	0.5075	0.9962	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
	1.1	2	2	0.1846	0.9538	0.9944	0.9989	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000
	1.2	2	1	0.0646	0.7141	0.8814	0.9397	0.9713	0.9873	0.9947	0.9979	0.9992	0.9998
	1.25	1	1	0.2472	0.8915	0.9723	0.9908	0.9974	0.9994	0.9998	0.9999	0.9999	0.9999
	1.5	1	1	0.0352	0.4395	0.6739	0.8012	0.8915	0.9469	0.9767	0.9908	0.9967	0.9989
	1.6	1	1	0.0150	0.2747	0.4898	0.6373	0.7652	0.8620	0.9265	0.9644	0.9843	0.9994
	1.65	1	1	0.0098	0.2109	0.4034	0.5491	0.6869	0.8012	0.8848	0.9391	0.9706	0.9869
	1.7	1	1	0.0064	0.1595	0.3257	0.4631	0.6037	0.7305	0.8317	0.9036	0.9493	0.9755
0.90	0.9	3	2	0.6975	0.9961	0.9995	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
	0.95	3	2	0.4944	0.9878	0.9979	0.9995	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000
	1	3	2	0.2966	0.9684	0.9935	0.9979	0.9994	0.9998	0.9999	0.9999	0.9999	1.0000
	1.1	3	2	0.0657	0.8561	0.9604	0.9843	0.9942	0.9979	0.9994	0.9998	0.9999	1.0000
	1.2	2	2	0.0466	0.7859	0.9488	0.9848	0.9962	0.9992	0.9998	0.9999	0.9999	1.0000
	1.25	2	1	0.0327	0.5978	0.8106	0.8959	0.9462	0.9739	0.9881	0.9949	0.9979	0.9993
	1.5	1	1	0.0352	0.4395	0.6739	0.8012	0.8915	0.9469	0.9767	0.9908	0.9967	0.9989
	1.6	1	1	0.0150	0.2747	0.4898	0.6373	0.7652	0.8620	0.9264	0.9644	0.9843	0.9937
	1.65	1	1	0.0098	0.2109	0.4034	0.5491	0.6869	0.8012	0.8848	0.9391	0.9706	0.9869
	1.7	1	1	0.0064	0.1595	0.3257	0.4631	0.6037	0.7305	0.8317	0.9036	0.9493	0.9755
0.95	0.9	4	3	0.5947	0.9959	0.9995	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
	0.95	3	2	0.4944	0.9878	0.9979	0.9995	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000
	1	2	1	0.4836	0.9713	0.9937	0.9979	0.9994	0.9999	0.9999	0.9999	0.9999	1.0000
	1.1	2	1	0.2081	0.8882	0.9647	0.9851	0.9943	0.9979	0.9994	0.9998	0.9999	0.9999
	1.2	2	1	0.0646	0.7141	0.8814	0.9398	0.9713	0.9873	0.9947	0.9979	0.9992	0.9998
	1.25	2	1	0.0327	0.5978	0.8106	0.8959	0.9463	0.9739	0.9881	0.9949	0.9979	0.9993
	1.5	1	1	0.0352	0.4395	0.6739	0.8012	0.8915	0.9469	0.9767	0.9908	0.9967	0.9989
	1.6	1	1	0.0150	0.2765	0.4898	0.6373	0.7652	0.8620	0.9265	0.9644	0.9843	0.9937
	1.65	1	1	0.0098	0.2109	0.4034	0.5491	0.6869	0.8012	0.8848	0.9391	0.9706	0.9869
	1.7	1	1	0.0064	0.1595	0.3257	0.4631	0.6037	0.7305	0.8317	0.9036	0.9493	0.9755
0.99	0.9	8	6	0.2624	0.9916	0.9990	0.9997	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000
	0.95	6	5	0.1660	0.9841	0.9978	0.9995	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000
	1	5	4	0.0908	0.9571	0.9929	0.9979	0.9994	0.9998	0.9999	0.9998	0.9999	1.0000
	1.1	4	3	0.0199	0.8098	0.9533	0.9829	0.9939	0.9979	0.9993	0.9999	0.9999	1.0000
	1.2	2	2	0.0466	0.7859	0.9488	0.9848	0.9962	0.9992	0.9998	0.9999	0.9999	1.0000
	1.25	2	2	0.0215	0.6483	0.8877	0.9592	0.9877	0.9969	0.9993	0.9998	0.9999	0.9999
	1.5	1	1	0.0352	0.4395	0.6739	0.8012	0.8915	0.9469	0.9767	0.9908	0.9967	0.9989
	1.6	1	1	0.0150	0.2747	0.4898	0.6373	0.7652	0.8620	0.9265	0.9644	0.9843	0.9936
	1.65	1	1	0.0098	0.2109	0.4034	0.5491	0.6869	0.8012	0.8848	0.9391	0.9706	0.9869
	1.7	1	1	0.0064	0.1595	0.3257	0.4631	0.6037	0.7305	0.8317	0.9036	0.9493	0.9755

Table 3. Ratio $d_{0.10}$ for accepting the lot with the producer's risk of 0.05 for Special type double sampling plan for Gompertz Frechet distribution:

P^* \ t/t_q^0	**0.9**	**0.95**	**1**	**1.1**	**1.2**	**1.25**	**1.5**	**1.6**	**1.65**	**1.7**
0.75	1.4586	1.5216	1.5250	1.5856	1.7752	1.6305	1.9564	2.0869	2.1521	2.2174
0.90	1.5263	1.5864	1.6358	1.7825	1.7934	1.9936	2.1124	2.2543	2.3237	2.3945
0.95	1.6243	1.6493	1.7076	1.8916	1.8617	1.8626	2.5624	2.3842	2.4570	2.5335
0.99	2.5335	2.1000	2.1000	2.1000	2.2658	2.4894	2.1856	2.8002	2.9153	2.9691

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