International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373/doi:10.14445/22315373/IJMTT-V68I1P509
Volume 68 Issue 1, 77-85, January, 2022

Computation of Degree Based Indices of Basava Star Windmill Graph

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***Abstract** - In this paper, we have proposed new windmill graph, that is Basava star windmill graph. The Basava star windmill graph $B_{n+2}^{(m)}$ is the graph obtained by taking $m \geq 2$ copies of the graph $K_1 + K_{1,n}$ for $n \geq 1$ with a vertex K_1 in common. Further-more, we have proposed the general Sombor index of graph G. Inspired by recent work on degree-based topological indices, we have obtained first and second Zagreb index, F-index, first and second hyper-Zagreb index, harmonic index, Randic$'$ index, general Randic$'$ index, sum connectivity index, general sum connectivity index, atom-bond connectivity index, geometric-arithmetic index, arithmetic-geometric index, Symmetric division deg index, SK indices, general $SK_\alpha(G)$ index, general $SK_1^\alpha(G)$ index, Sombor index and general Sombor index of Basava star windmill graph.*

Keywords – *ABC index, hyper-Zagreb index, Randic$'$ connectivity index, SK indices and Sombor indices, sum-connectivity index, Zagreb index, Windmill graph.*

I. INTRODUCTION

Topological indices are designed on the grounds of the transformation of a molecular graph into a number that characterizes the topology of the molecular graph. We study the relationship between the structure, properties, and activity of chemical compounds in molecular modeling. Molecules and molecular compounds are often modeled by molecular graphs. A topological index is known as a connectivity index, is a type of a molecular descriptor that is calculated based on the molecular graph of a chemical compound. A chemical graph is a model used to characterize a chemical compound. A molecular graph is a simple graph whose vertices correspond to the atoms and edges corresponds to the bonds.

In this paper, graph G be a finite undirected graph without loops and multiple edges on n vertices and m edges and is called (n, m) graph. We denote vertex set and edge set of graph G as $V(G)$ and $E(G)$, respectively. For a graph G, the degree of a vertex v is the number of edges incident to v and is denoted by $d_G(v)$. For unexplained graph terminology and notation refer [13, 15]. Now a days, the topological indices are extensively used in mathematical chemistry. In the literature many researchers are defined degree based topological indices are in[10, 11, 12, 14, 23, 24, 25].

First and second Zagreb indices were defined by Gutman and Trinajstic$'$ [8] in 1972 as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2, \qquad M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v).$$

The first Zagreb index [19] can also be defined as

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)].$$

The forgotten topological index or F-index was introduced in [6], which is defined as

$$F(G) = \sum_{v \in V(G)} d_G(u)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2].$$

The first hyper-Zagreb index was introduced in [22] and second hyper-Zagreb index was introduced in [4] which are defined as

$$HM_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^2, \quad HM_2(G) = \sum_{uv \in E(G)} [d_G(u) d_G(v)]^2.$$

The harmonic index of a graph G was introduced by Fajtlowicz in [5] is defined as

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d_G(u) + d_G(v)}.$$

The Randic' index of a graph G was proposed in [20] and defined as

$$\chi(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

The sum connectivity index of a graph G was defined in [28] as

$$X(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}.$$

The general Randic' index of a graph G is defined as

$$\chi^{\alpha}(G) = \sum_{uv\in E(G)} [d_G(u)d_G(v)]^{\alpha}.$$

The general sum connectivity index of a graph G is defined as

$$X^{\alpha}(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v)]^{\alpha}.$$

The above two topological indices were proposed in [2, 8, 16, 29].

The atom-bond connectivity index, which is defined in [3] as

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u)d_G(v)}}.$$

The Geometric-arithmetic index of a graph G is in [26] defined as

$$GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u) + d_G(v)}.$$

The Arithmetic-geometric index of a graph G is in [21] defined as

$$AG_1(G) = \sum_{uv\in E(G)} \frac{d_G(u) + d_G(v)}{2\sqrt{d_G(u)d_G(v)}}.$$

The symmetric division deg index of graph G is in [27], which is defined as

$$SDD(G) = \sum_{uv\in E(G)} \frac{d_G(u)^2 + d_G(v)^2}{d_G(u)d_G(v)}.$$

Recently, Basavanagoud et al. [1], proposed new degree based topological indices of general SK_{α} and general SK_1^{α} indices of a graph G.

The general SK_{α} index of a graph G [1], is defined as

$$SK_{\alpha}(G) = \sum_{uv\in E(G)} \left(\frac{d_G(u) + d_G(v)}{2}\right)^{\alpha}.$$

Put $\alpha = 1$, we get, the SK index of a graph G [21], is defined as

$$SK(G) = SK_1(G) = \sum_{uv \in E(G)} \frac{d_G(u) + d_G(v)}{2}.$$

Put $\alpha = 2$, we get, the SK_2 index of a graph G [21], is defined as

$$SK_2(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u) + d_G(v)}{2}\right)^2.$$

The general $SK_1^{\alpha}(G)$ index of a graph G [1], is defined as

$$SK_1^{\alpha}(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u) d_G(v)}{2}\right)^{\alpha}.$$

Put $\alpha = 1$, we get, the SK_1 index of a graph G [21], is defined as

$$SK_1(G) = SK_1^1(G) = \sum_{uv \in E(G)} \frac{d_G(u) d_G(v)}{2}.$$

Put $\alpha = 2$, we get, the $SK_1^2(G)$ index of a graph G [1], is defined as

$$SK_1^2(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u) d_G(v)}{2}\right)^2.$$

The concept of Sombor index (SO) was recently introduced by Gutman is in [9], which is defined as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}.$$

Further, we have extend to the new degree based topological indices of general Sombor index $SO_{\alpha}(G)$.

The general $SO_{\alpha}(G)$ index of a graph G, is defined as

$$SO_{\alpha}(G) = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2]^{\alpha}.$$

Put $\alpha = 1$, we get, F-index of a graph G [6].
Put $\alpha = 2$, we get, the second Sombor $SO_2(G)$ index of a graph G, is defined as

$$SO_2(G) = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2]^2.$$

Motivated by recent results on Kulli path and Kulli cycle windmill graphs [17, 18]. We now introduce the Basava star windmill graph which is defined as follow.

Definition 1 The Basava star windmill graph $B_{n+2}^{(m)}$ is the graph obtained by taking $m \geq 2$ copies of the graph $K_1 + K_{1,n}$ for $n \geq 1$ with a vertex K_1 in common. This graph is shown in Figure-1. The Basava star windmill graph $B_{1+2}^{(m)}$ is a friendship graph and is denoted by $F_3^{(m)}$. The Basava star windmill graph $B_{2+2}^{(m)}$ is the Kulli path windmill graph and denoted by $P_4^{(m)}$. For more details on windmill graphs, one can refer [7, 17, 18].

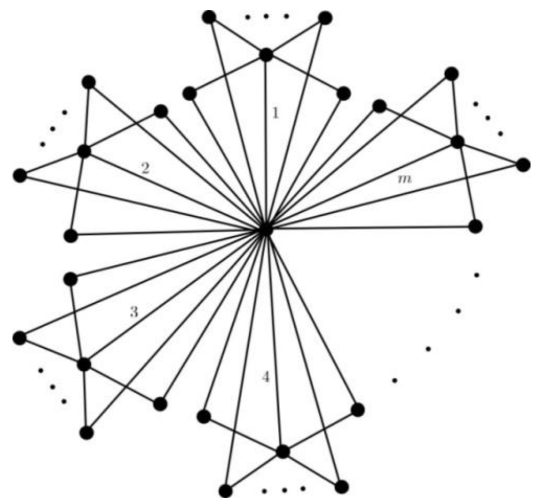

Fig. 1 Basava star windmill graph $B_{n+2}^{(m)}$.

The partitions of the vertices with respect to their degree of vertices of Basava star windmill graph $B_{n+2}^{(m)}$ is given in Table 1.

Table 1. Vertex set partitions of basava star windmill graph.

$d_G(v)$	2	$n+1$	$m(n+1)$
Number of vertices	mn	m	1

The partitions of the edges with respect to their degree of end vertices of Basava star windmill graph $B_{n+2}^{(m)}$ is given in Table 2.

Table 2. Edge set partitions of basava star windmill graph.

$(d_G(u), d_G(v))$	$(2, n+1)$	$(2, m(n+1))$	$(n+1, m(n+1))$
Number of edges	mn	mn	m

II. DEGREE BASED TOPOLOGICAL INDICES OF THE BASAVA STAR WINDMILL GRAPH

In the following theorems, we compute some degree based topological indices of Basava star windmill graph.

Theorem 2.1 The F-index of Basava star windmill graph is

$$F(B_{n+2}^{(m)}) = 8mn + m(n+1)^3(1+m^2).$$

Proof. By using the definition of F-index and Table 1, we derive

$$\begin{aligned} F(B_{n+2}^{(m)}) &= \sum_{v\in V(G)} d_G(v)^3 \\ &= \sum_{v\in V_2} 2^3 + \sum_{v\in V_{n+1}} (n+1)^3 + \sum_{v\in V_{m(n+1)}} m(n+1)^3 \\ &= 8 \times mn + (n+1)^3 \times m + m^3(n+1)^3 \\ &= 8mn + m(n+1)^3(1+m^2). \end{aligned}$$

Theorem 2.2 The harmonic index of Basava star windmill graph is

$$H(B_{n+2}^{(m)}) = 2m[\frac{n}{n+3} + \frac{n}{mn+m+2} + \frac{1}{(n+1)(m+1)}].$$

Proof. By using the definition of harmonic index and Table 2, we derive

$$
\begin{aligned}
H((m)_{n+2}) &= \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)}\\
&= \sum_{uv\in E_{n+3}} \frac{2}{2+n+1} + \sum_{uv\in E_{2+m(n+1)}} \frac{2}{2+m(n+1)} + \sum_{uv\in E_{n+1+m(n+1)}} \frac{2}{n+1+m(n+1)}\\
&= \frac{2}{n+3}\times mn + \frac{2}{2+m(n+1)}\times mn + \frac{2}{n+1+m(n+1)}\times m\\
&= 2m[\frac{n}{n+3} + \frac{n}{mn+m+2} + \frac{1}{(n+1)(m+1)}].
\end{aligned}
$$

Theorem 2.3 The Randic′ index of Basava star windmill graph is

$$\chi(B_{n+2}^{(m)}) = \frac{mn}{\sqrt{2(n+1)}}[1+\frac{1}{\sqrt{m}}] + \frac{m}{(n+1)\sqrt{m}}.$$

Proof. By using the definition of Randic′ index and Table 2, we derive

$$
\begin{aligned}
\chi((m)_{n+2}) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}\\
&= \sum_{uv\in E_{n+3}} \frac{1}{\sqrt{2\times(n+1)}} + \sum_{uv\in E_{2+m(n+1)}} \frac{1}{\sqrt{2\times m(n+1)}} + \sum_{uv\in E_{n+1+m(n+1)}} \frac{1}{\sqrt{(n+1)\times m(n+1)}}\\
&= \frac{1}{\sqrt{2(n+1)}}\times mn + \frac{1}{\sqrt{2m(n+1)}}\times mn + \frac{1}{\sqrt{(n+1)m(m+1)}}\times m\\
&= \frac{mn}{\sqrt{2(n+1)}}[1+\frac{1}{\sqrt{m}}] + \frac{m}{(n+1)\sqrt{m}}.
\end{aligned}
$$

Theorem 2.4 The general Randic′ index of Basava star windmill graph is

$$\chi^{\alpha}(B_{n+2}^{(m)}) = 2^{\alpha}(n+1)^{\alpha}mn[1+m^{\alpha}] + m^{\alpha+1}(n+1)^{2\alpha}.$$

Proof. By using the definition of general Randic′ index and Table 2, we derive

$$
\begin{aligned}
\chi^{\alpha}((m)_{n+2}) &= \sum_{uv\in E(G)} [d_G(u)d_G(v)]^{\alpha}\\
&= \sum_{uv\in E_{n+3}} [2\times(n+1)]^{\alpha} + \sum_{uv\in E_{2+m(n+1)}} [2\times m(n+1)]^{\alpha} + \sum_{uv\in E_{n+1+m(n+1)}} [(n+1)\times m(n+1)]^{\alpha}\\
&= (2(n+1))^{\alpha}\times mn + (2m(n+1))^{\alpha}\times mn + ((n+1)m(n+1))^{\alpha}\times m\\
&= 2^{\alpha}(n+1)^{\alpha}mn[1+m^{\alpha}] + m^{\alpha+1}(n+1)^{2\alpha}.
\end{aligned}
$$

By using Theorem 2.4, we establish the following results.

Corollary 2.5 The second Zagreb index of Basava star windmill graph is

$$M_2(B_{n+2}^{(m)}) = 2mn(n+1)(1+m) + m^2(n+1)^2.$$

Corollary 2.6 The second hyper Zagreb index of Basava star windmill graph is

$$HM_2(B_{n+2}^{(m)}) = 4mn(n+1)^2(1+m^2) + m^3(n+1)^4.$$

Theorem 2.7 The sum connectivity index of Basava star windmill graph is

$$X(B_{n+2}^{(m)}) = mn[\frac{1}{\sqrt{n+3}} + \frac{1}{\sqrt{mn+m+2}}] + \frac{m}{\sqrt{(n+1)(m+1)}}.$$

Proof. By using the definition of sum connectivity index and Table 2, we derive

$$
\begin{aligned}
X((m)_{n+2}) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}\\
&= \sum_{uv\in E_{n+3}} \frac{1}{\sqrt{2+n+1}} + \sum_{uv\in E_{2+m(n+1)}} \frac{1}{\sqrt{2+m(n+1)}} + \sum_{uv\in E_{n+1+m(n+1)}} \frac{1}{\sqrt{n+1+m(n+1)}}\\
&= \frac{1}{\sqrt{n+3}}\times mn + \frac{1}{\sqrt{mn+m+2}}\times mn + \frac{1}{\sqrt{mn+m+n+1}}\times m\\
&= mn[\frac{1}{\sqrt{n+3}} + \frac{1}{\sqrt{mn+m+2}}] + \frac{m}{\sqrt{(n+1)(m+1)}}.
\end{aligned}
$$

Theorem 2.8 The general sum connectivity index of Basava star windmill graph is

$$X^{\alpha}(B_{n+2}^{(m)}) = mn[(n+3)^{\alpha} + (mn+m+2)^{\alpha}] + m(n+1)^{\alpha}(m+1)^{\alpha}.$$

Proof. By using the definition of general sum connectivity index and Table 2, we derive

$$\begin{aligned}
X^{\alpha}(B_{n+2}^{(m)}) &= \textstyle\sum_{uv\in E(G)} [d_G(u)+d_G(v)]^{\alpha} \\
&= \textstyle\sum_{uv\in E_{n+3}} [2+n+1]^{\alpha} + \sum_{uv\in E_{2+m(n+1)}} [2+m(n+1)]^{\alpha} + \sum_{uv\in E_{n+1+m(n+1)}} [n+1+m(n+1)]^{\alpha} \\
&= [n+3]^{\alpha} \times mn + [mn+m+2]^{\alpha} \times mn + [(n+1)(m+1)]^{\alpha} \times m \\
&= mn[(n+3)^{\alpha} + (mn+m+2)^{\alpha}] + m(n+1)^{\alpha}(m+1)^{\alpha}.
\end{aligned}$$

By using Theorem 2.8, we establish the following results.

Corollary 2.9 The first Zagreb index of Basava star windmill graph is

$$M_1(B_{n+2}^{(m)}) = mn[mn+m+n+5] + m(m+1)(n+1).$$

Corollary 2.10 The first hyper Zagreb index of Basava star windmill graph is

$$HM_1(B_{n+2}^{(m)}) = mn[(n+3)^2 + (mn+m+2)^2] + m(n+1)^2(m+1)^2.$$

Theorem 2.11 The atom-bond connectivity index of Basava star windmill graph is

$$ABC(B_{n+2}^{(m)}) = \sqrt{2}mn + \frac{m}{n+1}\sqrt{\frac{(n+1)(m+1)-2}{m}}.$$

Proof. By using the definition of atom-bond connectivity index and Table 2, we derive

$$\begin{aligned}
ABC(B_{n+2}^{(m)}) &= \textstyle\sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\
&= \textstyle\sum_{uv\in E_{n+3}} \sqrt{\frac{2+n+1-2}{2\times(n+1)}} + \sum_{uv\in E_{2+m(n+1)}} \sqrt{\frac{2+m(n+1)-2}{2\times m(n+1)}} + \sum_{uv\in E_{n+1+m(n+1)}} \sqrt{\frac{n+1+m(n+1)-2}{(n+1)\times m(n+1)}} \\
&= \sqrt{\frac{n+1}{2(n+1)}} \times mn + \sqrt{\frac{m(n+1)}{2m(n+1)}} \times mn + \sqrt{\frac{(n+1)(m+1)-2}{m(n+1)^2}} \times m \\
&= \sqrt{2}mn + \frac{m}{n+1}\sqrt{\frac{(n+1)(m+1)-2}{m}}.
\end{aligned}$$

Theorem 2.12 The Geometric-arithmetic index of Basava star windmill graph is

$$GA(B_{n+2}^{(m)}) = 2mn\sqrt{2(n+1)}[\frac{1}{n+3} + \frac{\sqrt{m}}{mn+m+2}] + \frac{2m\sqrt{m}}{1+m}.$$

Proof. By using the definition of Geometric-arithmetic index and Table 2, we derive

$$\begin{aligned}
GA(B_{n+2}^{(m)}) &= \textstyle\sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)} \\
&= \textstyle\sum_{uv\in E_{n+3}} \frac{2\sqrt{2\times(n+1)}}{2+n+1} + \sum_{uv\in E_{2+m(n+1)}} \frac{2\sqrt{2\times m(n+1)}}{2+m(n+1)} + \sum_{uv\in E_{n+1+m(n+1)}} \frac{2\sqrt{(n+1)\times m(n+1)}}{n+1+m(n+1)} \\
&= \frac{2\sqrt{2(n+1)}}{n+3} \times mn + \frac{2\sqrt{2m(n+1)}}{2+m(n+1)} \times mn + \frac{2\sqrt{(n+1)m(n+1)}}{n+1+m(n+1)} \times m \\
&= 2mn\sqrt{2(n+1)}[\frac{1}{n+3} + \frac{\sqrt{m}}{mn+m+2}] + \frac{2m\sqrt{m}}{1+m}.
\end{aligned}$$

Theorem 2.13 The Arithmetic-geometric index of Basava star windmill graph is

$$AG_1(B_{n+2}^{(m)}) = \frac{mn}{2\sqrt{2(n+1)}}[n+3+\frac{mn+m+2}{\sqrt{m}}] + \frac{m(m+1)}{2\sqrt{m}}.$$

Proof. By using the definition of Arithmetic-geometric index and Table 2, we derive

$$AG_1(B_{n+2}^{(m)}) = \textstyle\sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{2\sqrt{d_G(u)d_G(v)}}$$

$$= \sum_{uv\in E_{n+3}} \frac{2+n+1}{2\sqrt{2\times(n+1)}} + \sum_{uv\in E_{2+m(n+1)}} \frac{2+m(n+1)}{2\sqrt{2\times m(n+1)}} + \sum_{uv\in E_{n+1+m(n+1)}} \frac{n+1+m(n+1)}{2\sqrt{(n+1)\times m(n+1)}}$$
$$= \frac{n+3}{2\sqrt{2(n+1)}} \times mn + \frac{2+m(n+1)}{2\sqrt{2m(n+1)}} \times mn + \frac{(n+1)(m+1)}{2\sqrt{(n+1)m(n+1)}} \times m$$
$$= \frac{mn}{2\sqrt{2(n+1)}}[n+3+\frac{mn+m+2}{\sqrt{m}}] + \frac{m(m+1)}{2\sqrt{m}}.$$

Theorem 2.14 The Symmetric division deg index of Basava star windmill graph is
$$SDD(B_{n+2}^{(m)}) = 1 + m^2 + \frac{n}{2(n+1)}[m(n+1)^2(1+m) + 4m + 4].$$

Proof. By using the definition of Symmetric division deg index and Table 2, we derive
$$SDD(B_{n+2}^{(m)}) = \sum_{uv\in E(G)} \frac{d_G^2(u)+d_G^2(v)}{d_G(u)d_G(v)}$$
$$= \sum_{uv\in E_{n+3}} \frac{2^2+(n+1)^2}{2\times(n+1)} + \sum_{uv\in E_{2+m(n+1)}} \frac{2^2+(m(n+1))^2}{2\times m(n+1)} + \sum_{uv\in E_{n+1+m(n+1)}} \frac{(n+1)^2+(m(n+1))^2}{(n+1)\times m(n+1)}$$
$$= \frac{4+(n+1)^2}{2(n+1)} \times mn + \frac{4+m^2(n+1)^2}{2m(n+1)} \times mn + \frac{(n+1)^2(1+m^2)}{m(n+1)^2} \times m$$
$$= 1 + m^2 + \frac{n}{2(n+1)}[m(n+1)^2(1+m) + 4m + 4].$$

Theorem 2.15 The general SK_α index of Basava star windmill graph is
$$SK_\alpha(B_{n+2}^{(m)}) = \frac{1}{2^\alpha}[mn((n+3)^\alpha + (mn+m+2)^\alpha) + m(m+1)^\alpha(n+1)^\alpha].$$

Proof. By using the definition of general SK_α index and Table 2, we derive

$$SK_\alpha(B_{n+2}^{(m)}) = \sum_{uv\in E(G)} (\frac{d_G(u)+d_G(v)}{2})^\alpha$$
$$= \sum_{uv\in E_{n+3}} (\frac{2+n+1}{2})^\alpha + \sum_{uv\in E_{2+m(n+1)}} (\frac{2+m(n+1)}{2})^\alpha + \sum_{uv\in E_{n+1+m(n+1)}} (\frac{n+1+m(n+1)}{2})^\alpha$$
$$= (\frac{n+3}{2})^\alpha \times mn + (\frac{2+m(n+1)}{2})^\alpha \times mn + (\frac{n+1+m(n+1)}{2})^\alpha \times m$$
$$= \frac{1}{2^\alpha}[mn((n+3)^\alpha + (mn+m+2)^\alpha) + m(m+1)^\alpha(n+1)^\alpha].$$

By using Theorem 2.15, we establish the following results.

Corollary 2.16 The SK_1 index of Basava star windmill graph is
$$SK_1(B_{n+2}^{(m)}) = \frac{mn}{2}[mn+m+n+5] + \frac{m(m+1)(n+1)}{2}.$$

Corollary 2.17 The SK_2 index of Basava star windmill graph is
$$SK_2(B_{n+2}^{(m)}) = \frac{mn}{4}[(n+3)^2 + (mn+m+2)^2] + \frac{m(m+1)^2(n+1)^2}{4}.$$

Theorem 2.18 The general SK_1^α index of Basava star windmill graph is
$$SK_1^\alpha(B_{n+2}^{(m)}) = mn(n+1)^\alpha[1+m^\alpha] + \frac{m^{\alpha+1}(n+1)^{2\alpha}}{2^\alpha}.$$

Proof. By using the definition of general SK_1^α index and Table 2, we derive

$$SK_1^\alpha(B_{n+2}^{(m)}) = \sum_{uv\in E(G)} (\frac{d_G(u)d_G(v)}{2})^\alpha$$
$$= \sum_{uv\in E_{n+3}} (\frac{2\times(n+1)}{2})^\alpha + \sum_{uv\in E_{2+m(n+1)}} (\frac{2\times m(n+1)}{2})^\alpha + \sum_{uv\in E_{n+1+m(n+1)}} (\frac{(n+1)\times m(n+1)}{2})^\alpha$$
$$= (\frac{2(n+1)}{2})^\alpha \times mn + (\frac{2m(n+1)}{2})^\alpha \times mn + (\frac{m(n+1)^2}{2})^\alpha \times m$$
$$= mn(n+1)^\alpha[1+m^\alpha] + \frac{m^{\alpha+1}(n+1)^{2\alpha}}{2^\alpha}.$$

By using Theorem 2.18, we establish the following results.

Corollary 2.19 The SK_1^1 index of Basava star windmill graph is

$$SK_1^1(B_{n+2}^{(m)}) = mn(n+1)(1+m) + \frac{m^2(n+1)^2}{2}.$$

Corollary 2.20 The SK_1^2 index of Basava star windmill graph is

$$SK_1^2(B_{n+1}^{(m)}) = mn(n+1)^2[1+m^2] + \frac{m^3(n+1)^4}{4}.$$

Theorem 2.21 The general Sombor index of Basava star windmill graph is

$$SO_\alpha(B_{n+2}^{(m)}) = mn([4+(n+1)^2]^\alpha + [4+m^2(n+1)^2]^\alpha) + m(n+1)^{2\alpha}(1+m^2)^\alpha.$$

Proof. By using the definition of general Sombor index and Table 2, we derive

$$\begin{aligned} SO_\alpha(B_{n+2}^{(m)}) &= \sum_{uv\in E(G)} [d_G(u)^2 + d_G(v)^2]^\alpha \\ &= \sum_{uv\in E_{n+3}} [2^2 + (n+1)^2]^\alpha + \sum_{uv\in E_{2+m(n+1)}} [2^2 + (m(n+1))^2]^\alpha \\ &\quad + \sum_{uv\in E_{n+1+m(n+1)}} [(n+1)^2 + (m(n+1))^2]^\alpha \\ &= [4+(n+1)^2]^\alpha \times mn + [4+m^2(n+1)^2]^\alpha \times mn + [(n+1)^2 + m^2(n+1)^2]^\alpha \times m \\ &= mn([4+(n+1)^2]^\alpha + [4+m^2(n+1)^2]^\alpha) + m(n+1)^{2\alpha}(1+m^2)^\alpha. \end{aligned}$$

By using Theorem 2.21, we establish the following results.

Corollary 2.22 The Sombor index of Basava star windmill graph is

$$SO(B_{n+2}^{(m)}) = mn(\sqrt{4+(n+1)^2} + \sqrt{4+m^2(n+1)^2}) + m(n+1)\sqrt{(1+m^2)}.$$

Corollary 2.23 The second Sombor index of Basava star windmill graph is

$$SO(B_{n+2}^{(m)}) = mn(n+1)^4(1+m^4) + 8mn(n+1)^2(1+m^2) + m(n+1)^4(1+m^2)^2 + 32mn.$$

III. CONCLUSION

We have proposed new graph transformation, is Basava star windmill graph. And also we have proposed new topological index, i.e., general Sombor index $SO_\alpha(G)$, using this index one can obtain physico-chemical properties of molecular graphs.

In this paper, we have obtained certain degree-based topological indices of Basava star windmill graph, namely, first and second Zagreb, F-index, first and second hyper-Zagreb, harmonic, Randic$'$, general Randic$'$, sum connectivity, general sum connectivity, atom-bond connectivity, geometric-arithmetic, arithmetic-geometric, Symmetric division deg index, SK indices, general $SK_\alpha(G)$, general $SK_1^\alpha(G)$ and Sombor indices, are studied. Further one can investigate different topological indices of some chemical structures by using Sombor index.

ACKNOWLEDGMENT

The second author is supported by Directorate of Minorities, Government of Karnataka, Bangalore, through M.Phil/Ph.D fellowship-2019-20:No.DOM/Ph.D/M.Phil/FELLOWSHIP/CR-01/2019-20 dated 15^{th} October 2019.

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