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Absolute Mean Graceful Labeling of Jewel Graph and Jelly Fish Graph

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Abstract *- In this paper, we present absolute mean graceful labeling for some graphs. We have proved that jewel graph J_n, jewel graph without prime edge J_n^*, extended jewel graph EJ_n, jelly fish graph $J_{m,n}$, jelly fish graph without prime edge $J_{m,n}^*$, extended jelly fish graph $EJ_{m,n}$ are absolute mean graceful graphs.*

Keywords - *Labeling, Graceful labeling, Absolute mean graceful labeling.*

I. INTRODUCTION

Graph theory is one of the branch of mathematics with many applications in different disciplines. Labeling of graph is the assignment of values to vertices or edges or both subject in certain conditions. A. Rosa[2] initiated the concept of labeling with the name of β-valuation. S. Golomb[3] named such labeling as graceful labeling. Kaneria and Chudasama[6] introduced a new graph labeling namely absolute mean graceful labeling. A function f is said to be *absolute mean graceful labeling* of a graph G, if $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ is injective and edge labeling function $f^*: E(G) \longrightarrow \{1, 2, \dots, q\}$ defined as $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is bijective, for every edge $e = uv \in E(G)$. A graph is called *absolute mean graceful graph*, if it admits an absolute mean graceful labeling. We begin with a simple, connected and undirected graph $G = (V, E)$ with p vertices and q edges. For all terminology and notations, we follow F. Harary[1]. First of all we recall and define some definitions, which are used in this paper.

Definition 1.1 : The *jewel graph*, J_n is obtained from a 4-cycle with vertices $x, y, u, v,$ by joining x and y with a prime edge and also by appending an edge from u and v which meets at common vertices v_i, $1 \leq i \leq n$.
The prime edge in a jewel graph is defined to be the edge joining the vertices x and y. It is shown in the Fig. 1.

Definition 1.2 : The *jewel graph without prime edge*, J_n^* is obtained by removing prime edge in jewel graph J_n.

Definition 1.3 : The *extended jewel graph*, EJ_n is obtained from jewel graph without prime edge J_n^* by appending arbitrary vertices in J_n^* such a way that they all are connected to vertex x and vertex y.

Definition 1.4 : The *jelly fish graph*, $J_{m,n}$ is obtained from a 4-cycle with vertices $x, y, u, v,$ by joining x and y with a prime edge and appending m pendent edges to u and n pendent edges to v.
The prime edge in jelly fish graph is defined to be the edge joining the vertices x and y. It is shown in the Fig. 2.

Definition 1.5 : The *jelly fish graph without prime edge*, $J_{m,n}^*$ is obtained by removing prime edge in jelly fish graph $J_{m,n}$.

Definition 1.6 : The *extended jelly fish graph*, $EJ_{m,n}$ is obtained from jelly fish graph without prime edge $J_{m,n}^*$ by appending arbitrary vertices in J_n^* such a way that they all are connected to vertex x and vertex y.

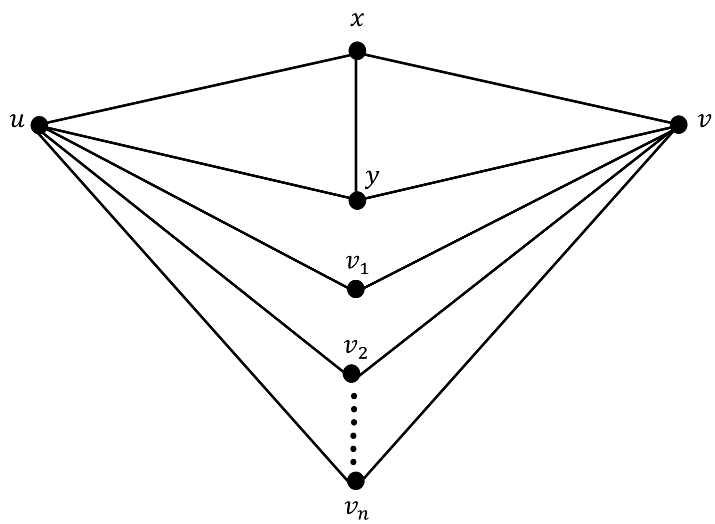

Fig. 1 Jewel graph J_n with prime edge xy

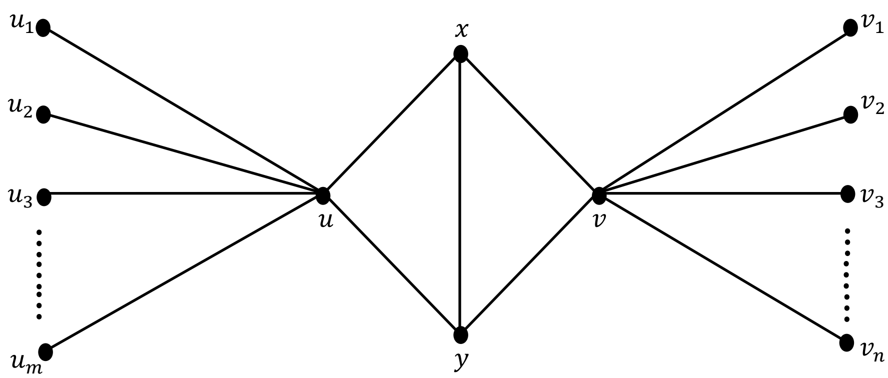

Fig. 2 Jelly fish graph $J_{m,n}$ with prime edge xy

Kaneria and Chudasama[5,6] proved that path graphs P_n, cycles C_n, complete bipertite graphs $K_{m,n}$, grid graphs $P_m \times P_n$, step grid graphs St_n and double step grid graphs DSt_n are absolute mean graceful graphs and they also proved that path union of finite copies in trees T, path graphs P_n, cycles C_n, complete bipertite graphs $K_{m,n}$, grid graphs $P_m \times P_n$, step grid graphs St_n and double step grid graphs DSt_n are absolute mean graceful graphs. In this paper our aim is to study the absolute mean graceful labeling for jewel graph, jelly fish graph and its related graphs. For comprehensive learning of graph labeling, we reffered Gallian[4].

II. MAIN RESULTS

Theorem 2.1 : The jewel graph, J_n is an absolute mean graceful.

Proof : Let $G = J_n$ be any jewel graph. Jewel graph is a $4-$cycle graph with vertices $x, y, u, v,$ including the prime edge connecting to x and y and also by appending an edge from u and v which meets at common vertex v_i, where i is according to the size n of the jewel graph J_n.

i.e. $V(G) = \{x, y, u, v, v_i / 1 \leq i \leq n\}$ and $E(G) = \{xy, ux, vx, uy, vy, uvi, vvi / 1 \leq i \leq n\}$.

Here, $|V(G)| = n + 4 = p$ and $|E(G)| = 2n + 5 = q$.
We defined vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$, as follows.
$f(x) = -q, f(y) = q - 4, f(u) = q - 3, f(v) = q - 1$ and $f(v_i) = 2n - 4i$, if $i = 1, 2, \dots, n$.

By defined pattern of function f, it can be observe that f is one-one, as there is no repeated vertex labels. Now we have to prove that induced edge labeling function f^* is a bijection. First of all we compute range of f^*.
i.e. $f^*(E(G))$
Observe that,
$f^*(vx) = q, f^*(ux) = q-1, f^*(xy) = q-2, f^*(uy) = 1, f^*(vy) = 2$ and
$\{f^*(uv_i)/1 \le i \le n\} \cup \{f^*(vv_i)/1 \le i \le n\} = \{3, 4, \dots, q-3\}$.
i.e. $f^*(E(G)) = \{1, 2, \dots, q = 2n+5\}$.
Hence, f^* is onto map. As domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection.
Thus, f is an absolute mean graceful labeling for $G = J_n$.
Therefore, jewel graph, J_n is absolute mean graceful graph.

Illustration 2.2 : Absolute mean graceful labeling for jewel graph, J_3 with $p = 7$ and $q = 11$ is shown in following Fig. 3.

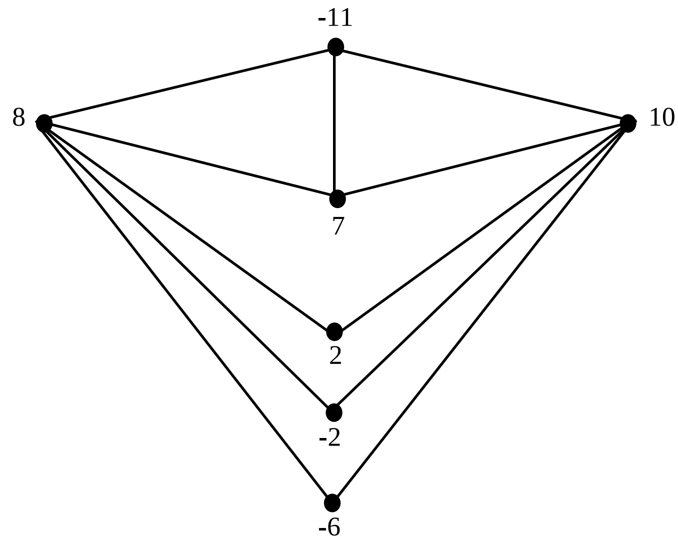

Fig. 3 Absolute mean graceful labeling for jewel graph, J_3

Theorem 2.3 : The jewel graph without prime edge, J_n^* is an absolute mean graceful.

Proof : Let J_n be any jewel graph.
Let $V(J_n) = \{x, y, u, v, v_i/1 \le i \le n\}$ and $E(J_n) = \{xy, ux, vx, uy, vy, uv_i, vv_i/1 \le i \le n\}$.
Let $G = J_n^*$ is obtained by removing prime edge xy from jewel graph J_n.
i.e. $V(G) = \{x, y, u, v, v_i/1 \le i \le n\}$ and $E(G) = \{ux, vx, uy, vy, uv_i, vv_i/1 \le i \le n\}$.
Here, $|V(G)| = p = n+4$ and $|E(G)| = q = 2n+4$.
We defined vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$, as follows.
$f(x) = -q, f(y) = -(q-3), f(u) = q-3, f(v) = q-1$ and $f(v_i) = 4i - 2n - 1$, if $i = 1, 2, \dots, n$.
By defined pattern of function f, it can be observe that f is one-one, as there is no repeated vertex labels. Now we have to prove that induced edge labeling function f^* is a bijection. First of all we compute range of f^*.
i.e. $f^*(E(G))$
observe that,
$f^*(ux) = q-1, f^*(vx) = q, f^*(uy) = q-3, f^*(vy) = q-2$ and $\{f^*(uv_i/1 \le i \le n\}$
$\cup f^*(vv_i)/1 \le i \le n\} = \{1, 2, \dots, q-4\}$
i.e. $f^*(E(G)) = \{1, 2, \dots, q = 2n+4\}$
Hence, f^* is onto map. As domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection.

Thus, f is an absolute mean graceful labling for $G = J_n^*$.
Therefore, jewel graph without prime edge, J_n^* is absolute mean graceful graph.

Illustration 2.4 : Absolute mean graceful labeling for jewel graph without prime edge, J_4^* with $p = 8$ and $q = 12$ is shown in following Fig. 4.

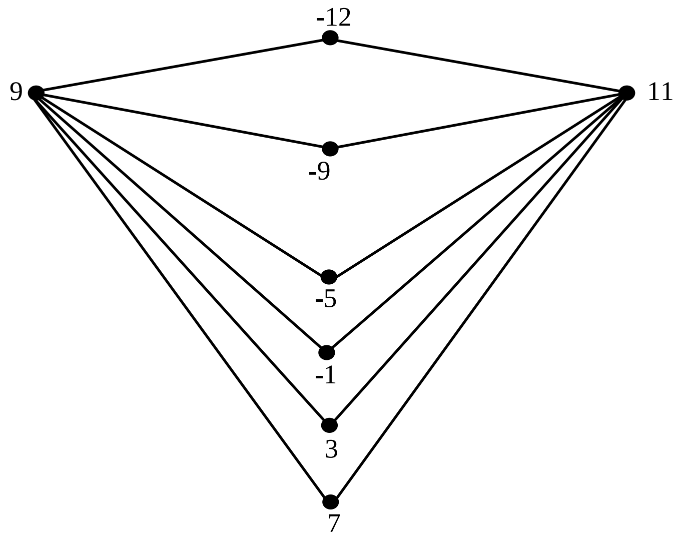

Fig. 4 Absolute mean graceful labeling for jewel graph without prime edge, J_4^*

Theorem 2.5 : The extended jewel graph, EJ_n is an absolute mean graceful.

Proof : Let J_n^* be any jewel graph without prime edge. Let $V(J_n^*) = \{x, y, u, v, v_i / 1 \le i \le n\}$ and
$E(J_n^*) = \{ux, vx, uy, vy, uv_i, vv_i / 1 \le i \le n\}$.
Let $G = EJ_n$ be any extended jewel graph obtained from jewel graph without prime edge, J_n^* by appending arbitrary vertices in J_n^* such a way that they all are connected to vertex x and vertex y. Let $w_k (1 \le k \le t)$ be arbitrary t vertices which are connected to x and y.
i.e. $V(G) = \{x, y, u, v, v_i, w_k \ / \ 1 \le i \le n, 1 \le k \le t\}$ and
$E(G) = \{ux, vx, uy, vy, uv_i, vv_i, xw_k, yw_k \ / 1 \le i \le n, 1 \le k \le t\}$
Here, $|V(G)| = p = n + t + 4$ and $|E(G)| = q = 2n + 2t + 4$.
We define vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$, as follows.
$f(x) = q - 1, f(y) = q - 3, f(u) = -(q - 3), f(v) = -q$,
$f(v_i) = 4i - q$, if $i = 1, 2, \dots, n$ and $f(w_k) = 4k - q + 3$, if $k = 1, 2, \dots, t$.
By defined pattern of function f, it can be observe that f is one-one, as there is no repeated vertex labels. Now we have to prove that induced edge labeling function f^* is a bijection. First of all we compute range of f^*.
i.e. $f^*(E(G))$
Observe that,
$f^*(ux) = q - 2, f^*(vx) = q, f^*(uy) = q - 3, f^*(vy) = q - 1$,
$\{f^*(uv_i / 1 \le i \le n\} \cup f^*(vv_i) / 1 \le i \le n\} = \{1, 2, \dots, 2n\}$ and
$\{f^*(xw_k / 1 \le k \le t\} \cup f^*(yw_k) / 1 \le k \le t\} = \{2n + 1, 2n + 2, \dots, q - 4\}$.
i.e. $f^*(E(G)) = \{1, 2, \dots, q = 2n + 2t + 4\}$
Hence, f^* is onto map. As domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection.
Thus, f is an absolute mean graceful labling for $G = EJ_n$.
Therefore, extended jewel graph, EJ_n is absolute mean graceful graph.

Illustration 2.6 : Absolute mean graceful labeling for extended jewel graph, EJ_3 by appending arbitrary vertices $t = 6$ with $p = 13$ and $q = 22$ is shown in following Fig. 5.

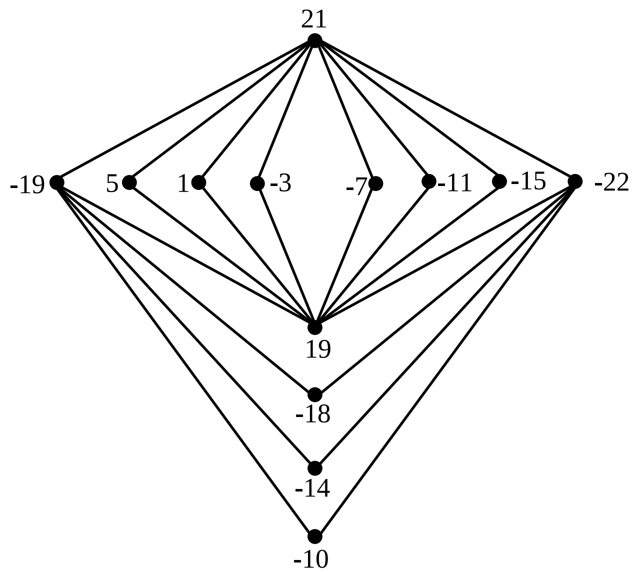

Fig. 5 Absolute mean graceful labeling for jewel graph EJ_3 with arbitrary vertices $t = 6$.

Theorem 2.7: The jelly fish graph, $J_{m,n}$ is an absolute mean graceful.

Proof : Let $G = J_{m,n}$ be any jelly fish graph. Jelly fish graph is a $4-$ cycle graph with vertices x, y, u, v, including the prime edge conncting to x and y and also by appending m pendent edges to u and n pedent edges to v.

Let $u_i (1 \leq i \leq m)$ be m vertices which are connected to u and $v_j (1 \leq j \leq n)$ be n vertices which are connectes to v.

i.e. $V(G) = \{x, y, u, v, u_i, v_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{xy, ux, vx, uy, vy, uu_i, vv_j / 1 \leq i \leq m, 1 \leq j \leq n\}$

Here, $|V(G)| = p = m + n + 4$ and $|E(G)| = q = m + n + 5$.

We define vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$, as follows.

$f(x) = -(q-2), f(y) = -(q-1), f(u) = q-4, f(v) = q,$

$f(u_i) = -(q - 2i - 2)$, if $i = 1, 2, \dots, m$ and $f(v_j) = q - 2j - 1$, if $j = 1, 2, \dots, n$.

By defined pattern of function f, it can be observe that f is one-one, as there is no repeated vertex labels. Now we have to prove that induced edge labeling function f^* is a bijection. First of all we compute range of f^*.

i.e. $f^*(E(G))$

Observe that,

$f^*(xy) = 1, f^*(vy) = q, f^*(vx) = q-1, f^*(uy) = q-2, f^*(ux) = q-3,$

$\{f^*(uu_i)/1 \leq i \leq m\} = \{n+2, n+3, \dots, q-4\}$ and $\{f^*(vv_j)/1 \leq j \leq n\} = \{2, 3, \dots, n+1\}$.

i.e. $f^*(E(G)) = \{1, 2, \dots, q = m + n + 5\}$.

Hence, f^* is onto map. As domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection. Thus, f is an absolute mean graceful labling for $G = J_{m,n}$.

Therefore, jelly fish graph, $J_{m,n}$ is absolute mean graceful graph.

Illustration 2.8 : Absolute mean graceful labeling for jelly fish graph, $J_{4,6}$ with $p = 14$ and $q = 15$ is shown in following Fig. 6.

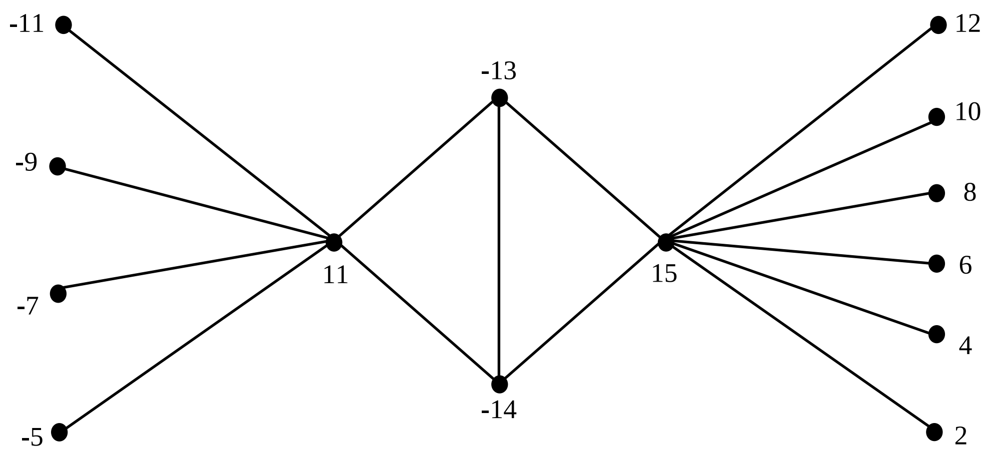

Fig. 6 Absolute mean graceful labeling for jelly fish graph, $J_{4,6}$

Theorem 2.9 : The jelly fish graph without prime edge, $J^*_{m,n}$ is an absolute mean graceful graph.

Proof : Let $J_{m,n}$ be any jelly fish graph.
Let $V(J_{m,n}) = \{x, y, u, v, u_i, v_j \,/ 1 \le i \le m, 1 \le j \le n\}$ and
$E(J_{m,n}) = \{xy, ux, vx, uy, vy, uu_i, vv_j / 1 \le i \le m, 1 \le j \le n\}$.
Let $G = J^*_{m,n}$ is obtained by removing prime edge xy from jelly fish graph $J_{m,n}$.
i.e. $V(G) = \{x, y, u, v, u_i, v_j / 1 \le i \le m, 1 \le j \le n\}$ and
$E(G) = \{ ux, vx, uy, vy, uu_i, vvj / 1 \le i \le m, 1 \le j \le n\}$.
Here, $|V(G)| = m + n + 4$ and $|E(G)| = q = m + n + 4$
We define vertex labeling function $f: V(G) \longrightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$, as follows.
$f(x) = -(q - 2), f(y) = -(q - 1), f(u) = q - 4, f(v) = q,$
$f(u_i) = -(q - 2i - 2)$, if $i = 1, 2, \dots, m$ and
$f(v_i) = q - 2j + 1$, if $j = 1, 2, \dots, n$.
By defined pattern of function f, it can be observe that f is one-one, as there is no repeated vertex labels. Now we have to prove that induced edge labeling function f^* is a bijection. First of all we compute range of f^*.
i.e. $f^*(E(G))$
Observe that,
$f^*(vy) = q, f^*(vx) = q - 1, f^*(uy) = q - 2, f^*(ux) = q - 3$
$\{f^*(uu_i / 1 \le i \le m\} = \{n + 1, n + 2, \dots, q - 4\}$ and $f^*(vv_j / 1 \le j \le n\} = \{1, 2, \dots, n\}$
i.e. $f^*(E(G)) = \{1, 2, \dots, q = m + n + 4\}$
Hence, f^* is onto map. As domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection.
Thus, f is an absolute mean graceful labling for $G = J^*_{m,n}$.
Therefore, jelly fish graph without prime edge, $J^*_{m,n}$ is absolute mean graceful graph.

Illustration 2.10 : : Absolute mean graceful labeling for jelly fish graph without prime edge, $J^*_{7,3}$ with $p = 14$ and $q = 14$ is shown in following Fig. 7.

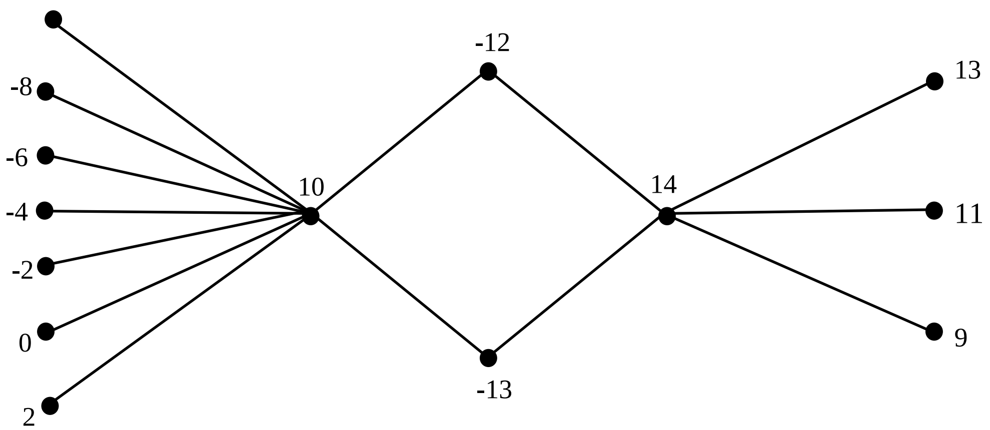

Fig. 7 Absolute mean graceful labeling for jelly fish graph without prime edge, $J^*_{7,3}$

Theorem 2.11 : The extended jelly fish graph, $EJ_{m,n}$ is an absolute mean graceful.

Proof : Let $J^*_{m,n}$ be any jelly fish graph. Let $V\left(J^*_{m,n}\right) = \{x, y, u, v, u_i, v_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and
$E\left(J^*_{m,n}\right) = \{ ux, vx, uy, vy, uu_i, vvj / 1 \leq i \leq m, 1 \leq j \leq n\}$.
Let $G = EJ_{m,n}$ be any extended jelly fish graph obtained from jelly fish graph without prime edge, $J^*_{m,n}$ by appending arbitrary t vertices in $J^*_{m,n}$ such a way that they all are connected to vertex x and y. Let $w_k (1 \leq k \leq t)$ be arbitrary t vertices which are connected to x and y.
i.e. $V(G) = \{x, y, u, v, u_i, v_j, w_k \ / \ 1 \leq i \leq m, \ 1 \leq j \leq n, 1 \leq k \leq t \}$ and
$E(G) = \{ux, vx, uy, vy, uu_i, vv_j, xw_k, yw_k \ / \ 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq t\}$
Here, $|V(G)| = p = m + n + t + 4$ and $|E(G)| = q = m + n + 2t + 4$.
We define vertex labeling function $f : V(G) \longrightarrow \{0, \pm 1, \pm 2, \ldots, \pm q\}$, as follows.
$f(x) = q - 1, f(y) = q - 3, f(u) = 4t - q + 3, f(v) = -q$,
$f(u_i) = q - 2i - 4$, if $= 1, 2, \ldots, m$, $f\left(v_j\right) = 2j - q$, if $j = 1, 2, \ldots, n$ and
$f(w_k) = 4k - q - 1$, if $k = 1, 2, \ldots, t$.
By defined pattern of function f, it can be observe that f is one-one, as there is no repeated vertex labels. Now we have to prove that induced edge labeling function f^* is a bijection. First of all we compute range of f^*.
i.e. $f^*(E(G))$
Observe that,
$f^*(vy) = q - 1, f^*(vx) = q, \ f^*(uy) = q - 2t - 3, f^*(ux) = q - 2t - 2$,
$\{f^*(uu_i / \ 1 \leq i \leq m\} = \{n + 1, n + 2, \ldots, q - 2t - 4\}$, $f^*\left(vv_j / \ 1 \leq j \leq n\right\} = \{1, 2, \ldots, n\}$ and
$\{f^*(xw_k / 1 \leq k \leq t\} \cup f^*(yw_k) / 1 \leq k \leq t\} = \{ q - 2t - 1, q - 2t, \ldots, q - 2\}$.
i.e. $f^*\left(E(G)\right) = \{1, 2, \ldots, q = m + n + 2t + 4\}$.
Hence, f^* is onto map. As domain of f^* and range of f^* have same cardinality, gives f^* is one-one. Therefore, f^* is bijection. Thus, f is an absolute mean graceful labling for $G = EJ_{m,n}$.
Therefore, extended jelly fish graph, $EJ_{m,n}$ is absolute mean graceful graph.

Illustration 2.12 : : Absolute mean graceful labeling for extended jelly fish graph, $EJ_{7,4}$ by appending arbitrary vertices $t = 3$ with $p = 16$ and $q = 21$ is shown in following Fig. 8.

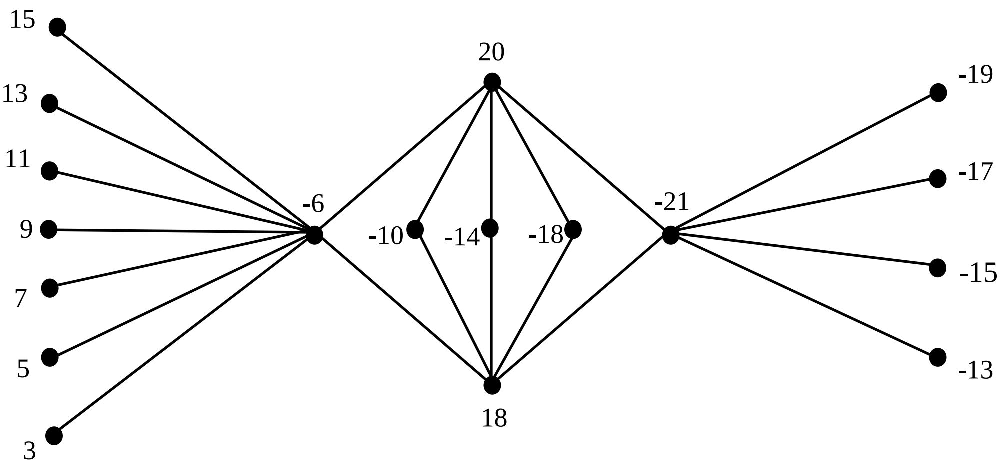

Fig. 8 Absolute mean graceful labeling for extended jelly fish graph, $EJ_{7,4}$ with arbitrary vertices $t = 3$

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